Computing Complexity of Symmetric Quadratic Neural Networks ¹

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Extended Abstract

A neural network of order $p \in \mathbb{N}$ (p-PNN in the sequel) [1, 4] is defined as a set of n neurons taking values in $\{0, 1\}$ and interconnected by a tuple $(A^{(p)}, A^{(p-1)}, \dots, A^{(0)})$ where $A^{(p)} = (a_{i_0, i_1, \dots, i_p}^{(p)}) \in \mathbb{R}^{(p+1)n}, \dots, A^{(0)} = (a_{i_0}^{(0)}) \in \mathbb{R}^n$. The update function is as follows: For $1 \leq i_0 \leq n$

$$y_{i_0} = \mathbb{I}\left(\frac{1}{p!} \sum_{i_1, \dots, i_p=1}^n a_{i_0 i_1 \dots i_p}^{(p)} x_{i_1} \cdots x_{i_p} + \dots + \frac{1}{2} \sum_{i_1, i_2=1}^n a_{i_0 i_1 i_2}^{(2)} x_{i_1} x_{i_2} + \sum_{i_1=1}^n a_{i_0 i_1}^{(1)} x_{i_1} + a_{i_0}^{(0)}\right)$$

where $\mathbb{I}(u) = 1$ iff $u \geq 0$ (0 otherwise).

The most usual dynamics for the previous network are the sequential and the parallel iteration. In the former case all cells are updated one by one in a prescribed order. The parallel iteration consists to update all the cells synchronously. We introduce also some hypothesis on the arrays $(A^{(p)}, \dots, A^{(0)})$; Symmetry: $\forall 1 \leq s \leq p$, for any permutation, σ , of the indices: $i_0, \dots, i_s \in \{1, \dots, n\}$ $a_{i_0 i_1 \dots i_s}^{(s)} = a_{\sigma(i_0) \sigma(i_1) \dots \sigma(i_s)}^{(s)}$ and Non-negativity: $\forall 1 \leq s \leq p$, if at least two indices have the same value; i.e. when $|\{i_0, i_1, \dots, i_s\}| \leq s$, then $a_{i_0, \dots, i_s}^{(s)} \geq 0$. Under these hypothesis one gets, for $p = 1$, the usual Hopfield model [3] of a linear symmetric neural networks: $y_i = \mathbb{I}(\sum_{j=1}^n a_{ij} x_j - \theta_i)$, where $A = (a_{ij})$ is symmetric and $\text{diag}(A) \geq 0$.

Here, we present a simulation of arbitrary (non necessarily symmetric) linear neural networks by symmetric neural networks with quadratic ($q = 2$) arguments (2-PSNN). In order to make the simulation, we introduce a special unit: the Δ -unit which consists of two neurons connected by a quadratic weight. This unit can orientate the information and then it can simulate the interaction a_{ij} between two neurons i and j in the linear network.

Our general result is given by the following theorem whose proof can be seen in [2].

Theorem . Let (A, θ) be a linear neural network of size n , then there exists a 2-PSNN with $2(n+1)$ Δ -units and a morphism $\varphi : \{0, 1\}^n \rightarrow \{0, 1\}^{2(n+1)}$ such that: $\forall x \in \{0, 1\}^n \quad \varphi(F(x)) = G^2(\varphi(x))$, where F and G are the global update functions of the linear and quadratic network respectively.

References

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¹Partially supported by DTI, U. Chile, FONDECYT 91-1211 (EG) and FONDECYT 0047/90 (MM)

** Dept. Ingeniería Matemática, U. de Chile, from December 1993