THE OVERLAPPED TESSELLATION A SUPERVISED NEURAL RULE

Ch. Molina, P. Baylou, M. Najim

Equipe Signal et Image and GDR 143 - CNRS
ENSERB. Université de Bordeaux I
351, Cours de la libération. 33405 Talence cedex. FRANCE
Tel. 56 84 61 40 Fax. 56 84 84 06

EXTENDED ABSTRACT

A three-layer Perceptron employing the *nearest neighbour* pattern classification rule was presented independently by O. Murphy [1] and P. Martin [2]. The main advantages claimed by the authors are a complete design and training process carried out in polynomial time in terms of n , the number of training patterns.

Unfortunately, as set out by P. Martin in (Martin, 92), this rule proves unworkable because of time computation $O(n^{((d+1)/2)})$ and storage $O(n^{[d/2]})$, which depend on the dimension d of the input vectors. Most of this time is employed to calculate the facets of the Voronoï polytopes, which make up the *Voronoï Tessellation*. Some facets discriminate patterns from opposite classes and, in terms of classification, these are essential; the rest separate patterns from the same class, which is unnecessary.

In order to reduce time computation and storage, we propose a new classification rule derived from the *Voronoï Tessellation*. This time, the new rule computes only those facets discriminating patterns from opposite classes. Voronoï polytopes are constructed in this way and optimized at a second stage, in order to obtain polytopes with the minimum possible number of facets. For a given pattern X , considered as a point in d -dimensional space, its polytope should be computed as follows. Construct a line segment from X to every other training pattern belonging to the opposite classes, and compute the hyperplane that bisects each line segment. This operation is computed in linear time. The intersection domain of all these half-planes is the Voronoï polytope of X . In terms of classification, we do not need all these facets to discriminate the pattern. We suggest a method which eliminates the useless facets of the polytope. A polytope facet is useless if, after its elimination, patterns from opposite classes are not included [3][4]. This operation is also computed in linear time. Computed polytopes usually overlap and they may contain several patterns, but only patterns from the same class, which improves the generalization capacity of the Perceptron. The process stops, with no classification error, when all patterns are included in the constructed polytopes.

This new classification rule, called Overlapped Tessellation (OT), designs and trains the three-layer Perceptron in linear time, and discriminates classes with far fewer neurons than the Voronoï Tessellation (VT).

REFERENCES

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