Networks for Learning and Differentiating an Input-Output Mapping

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A scheme is described which allows learning and differentiating an input-output mapping $y=f(x)$ from a set of sparse data points $\{x_i, y_i; i=0,1,2,\dots,n\}$. The key of the method consists in solving the inverted problem of approximation in two steps, as follows: use of parabolic splines $P_i(x)$ to interpolate the unknown nodal gradients $\phi^*(x_i)$ of continuous function $y=f(x)$ being considered as a smooth approximation of input-output relation $y=f(x)$ in the bounded region $x \in [x_0, x_n]$; design of the symmetrical operator for integrating the combined local basis gradients $\phi^*(x) = \sum_{j=0}^n \phi_j^* P_j(x)$ and thus reconstructing the real-valued function of examined mapping. For this purpose the approximating formula (i.e. similar to this developed in [1]) was used in following form:

$$|y_1^*| = \frac{1}{2} \left| (y_0 + y_n) + \int_{x_0}^{x_1} \phi^*(x) dx - \int_{x_1}^{x_n} \phi^*(x) dx \right| \equiv \sqrt{\left[y_0 + \int_{x_0}^{x_1} \phi^*(x) dx \right] \cdot \left[y_n - \int_{x_1}^{x_n} \phi^*(x) dx \right]}$$

This equation was transformed to the linear and vectorizable procedure for differentiating the unknown real-valued function, taking the final form:

$$\begin{cases} \phi_0^* = \frac{y_{-1} - y_0}{h} & \text{or} & \phi_0^* = \frac{y_{-1} - y_1^*}{2h} \\ [\underline{\phi}^*]_{(n+1)} = [D]_{(n+1)} \cdot [2\underline{y} - \underline{y}_0 - \underline{y}_n - \phi_0^* \underline{a}_0]_{(n+1)} \end{cases}$$

where: $h=(x_n-x_0)/n$; n -is an even natural number; y_{-1} - additional "bias" value of output variable measured beyond the range of practical interest. This equation suggests a novel procedure for fitting the function in the sense of first-order proximity and it may be easily outlined as the best approximation feed-forward network with one hidden layer. Due to uniqueness of the best approximation, proposed configuration has constant structure and weights of connections. This novel architecture although being discrete in its own nature, preserves the "internal compactness" of shared information. Such kind of property was admitted as yet for physical continuum (e.g. analog signal) only and it was not considered for discrete information processing. This procedure was used in modelling and simulation of target tracking process [2].

1. Korgul A.: The new computing method for countercurrent exchange phenomena simulation. Proceed. Int. AMSE Conf. "Modelling & Simulation", Sorrento (Italy) Sept.29 - Oct.1; 1986, Vol.3.4, 41-54.
2. Korgul A.: Developing of the detailed algorithms for fighter homing at the air targets. Military Academy of Technology, Warsaw 1992; in polish.