Author's Pick

C.C. Heyde¹

Paper [M1] shows by example that the lognormal distribution is not determined by its moments. It is referenced in Feller's second volume (Feller 1971) and has become a widely cited result.

Paper [M43] with Ted Hannan has the key time series result that the best linear predictor is the best predictor if the innovations are martingale differences. This has major implications for the linear/nonlinear debate, and in this respect has received much attention, especially from econometricians. This was one of the very early papers which made it clear that martingales would play an important role in statistics.

Paper [M44] illustrates a realization of the emerging role which martingales were to play in probability and statistics. The derivative of the log likelihood is a martingale. A general Central Limit Theorem for martingales emerged. This research was influenced by the first edition of Billingsley's then recent book (Billingsley (1999)).

Paper [M162] with Y. Yang was a serious attempt to bridge the gap between long range dependence as a property of processes and the inflexible and restricted formal definitions. "Long range dependence" is really a misnomer. The term persistence is better at catching the idea of large values following large values and small values following small values much more than under independence. Correlation plays no role in the concept, nor is there need for stationarity or finite variances, so mathematical definitions involving these restrictions are unhelpful. In this paper a self-normalised limit criterion is included to characterize the concept. None of the correlation related restrictions apply, but if they do happen to hold, the definitions of this paper are almost equivalent to the usual forms. The framework of the paper easily allows for treatment of the persistence of self-similar infinite variance processes such as fractional Lévy noises, which it had hitherto not been possible to integrate with that of fractional Brownian motion.

Paper [M169] introduces the fractal activity time geometric Brownian motion (FATGBM) risky asset model (also see [M180], [M181], [M189]). This is a minimal description model aimed at capturing the essentially empirical properties which the Geometric Brownian Motion (GBM) lacks. These are leptokurtic returns distributions (higher peaks and fatter tails than normal), persistent strong dependence of absolute values of functions of the returns and evident stochastic volatility effects. The efficient market hypothesis applies to the FATGBM model, the log returns being martingale differences. Empirical scales of the FATGBvM model strongly suggest fractal scaling of the activity time and empirical volatility, and provide considerable support for the need for power tailed return distributions rather than the somewhat more popular (tractable) exponentially tailed ones.

References

- [1] Billingsley, P. (1999) *Convergence of Probability Measures*, 2nd ed., Wiley, New York
- [2] Feller, W. (1971) *An Introduction to Probability Theory and its Applications*, Vol. II, 3rd ed., Wiley, New York

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¹Centre for Mathematical Analysis, Australian National University, Canberra, ACT