IMPROVED DYNAMIC PROPERTIES BY FEEDBACK FOR SYSTEMS WITH DELAY IN CONTROL

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Abstract It is considered a linear controlled system with delay in control and subject to constant exogeneous signals, for which the following problems are discussed: feedback stabilization and exact regulation by finite pole assignment using a linear control law deduced by applying finite assignment and Artstein transform for a system that is extended by integrators; the same extended system is then stabilized subject to a quadratic performance index. The theorems are illustrated by simulation results. In simulation there are considered other effects: saturation of the actuators and intelligent integrators.

Keywords: Systems with delay in control, Stabilization, Linear Quadratic Problem.

AMS (MOS) Subject Classification: 93D15, 34K06, 34K35, 49K25.

1. Motivation and state of the art

It has been pointed out that systems with input delays are of interest to control theorists and practitioners for various reasons. They originate from the simplest model of process control which assigns to the controlled plant a transfer function of the form $H(s)e^{-\tau s}$ with $H(s)$ a strictly proper rational function. Systems arising from such transfer functions are of special type: their state space is finite dimensional but either the input or the output operators are defined on infinite dimensional extensions and are unbounded. Throughout several decades it has been established that such systems have "a finite dimensional flavor". Indeed, there exists a remark of V.M. Popov (1960) which tells that problems as stability and optimality in the framework of hyperstability (or dissipativity/passivity as it is called now) may be solved for these

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systems like for systems without delay; nevertheless Popov did not follow this line in his research. We may add to this the results on feedback stabilization due to Olbrot (1978), Manitius and Olbrot (1979), Watanabe and Ito (1981). The techniques of these papers are essentially finite dimensional; the opinion of Pandolfi (1981, 1989, 1990, 1991) was that those results were not obtained in a standard way since they were not deduced from an abstract theory. In order to sustain his ideas, Pandolfi re-introduced in the model the *propagation effects* taken into account by the introduction of the delay and make use of the theory of the singular control. On the other hand, the finite dimensional results seem legitimate if the transform introduced by Artstein (1982) is used. The papers of Tadmor (e.g. 1995, 1998) also support the idea that finite dimension is the most adequate framework for systems with input delays. The authors of the present paper also followed the line opened by the papers of Olbrot and Manitius by using their approach in order to solve a LQ problem; also the hybrid piecewise constant control suggested by the paper of Halanay and Răşvan (1977) has been applied in the same framework and further completed by the continuous and discrete-time version of Artstein transforms (Popescu and Răşvan, 2001a, 2001b). In this paper we shall continue this line of research by considering the standard Linear Structurally Stable Regulator Problem (e.g. Francis, 1977; Francis and Wonham, 1975). For simplicity we shall consider here only the case of exogeneous constant (step) signals, a single input (control) signal and a single controlled (regulated) output, as in the paper of Ionescu and Răşvan (1991).

2. Problem statement and some preliminary results

The controlled system with input delay is the following

$$\begin{aligned} \dot{x}_1(t) &= A_1x_1(t) + A_3x_2(t) + b_{10}u(t) + b_{11}u(t - \tau) \\ \dot{x}_2(t) &= 0 \\ y(t) &= C_1x_1(t) + \int_{-\tau}^0 C_1e^{-A_1(\tau+\theta)}b_{11}u(t + \theta)d\theta + C_2x_2(t) \\ z(t) &= q^*y(t) \end{aligned} \tag{1}$$

Some explanations are necessary. Since we follow the line of the paper (Ionescu and Răşvan, 1991) the constant exogeneous signal vector (which incorporates both references and disturbances) is modelled by a suitably chosen autonomous system of differential equations of dimension n_2 while the state vector x_1 has dimension n_1 . Having in mind the transformation suggested by Artstein (1982) the measured output

$y(t)$ has been augmented by a term which takes into account the "history" of the control signal; if one takes into account that within the abstract model the past of the control signal belongs to the state and the outputs are always linear functionals defined on the state space, the structure of $y(t)$ turns to be legitimate. The form of $z(t)$ is given by the so-called *readability assumption*: the controlled output is readable from the measured output (e.g. Francis and Wonham, 1975).

Introducing the new state vector w_1 by

$$w_1(t) = x_1(t) + \int_{-\tau}^0 e^{-A_1(\tau+\theta)} b_{11} u(t+\theta) d\theta \quad (2)$$

the following controlled system is obtained

$$\begin{aligned} \dot{w}_1(t) &= A_1 w_1(t) + A_3 x_2(t) + (b_{10} + e^{-A_1 \tau} b_{11}) u(t) \\ \dot{x}_2(t) &= 0 \\ y(t) &= C_1 w_1(t) + C_2 x_2(t) \\ z(t) &= q^* y(t) \end{aligned} \quad (3)$$

We may state now the following basic assumptions:

i) the pair $(A_1, b_{10} + e^{-A_1 \tau} b_{11})$ is stabilizable and the pair (C_1, A_1) is detectable;

ii) the matrix $\begin{pmatrix} A_1 & b_{10} + e^{-A_1 \tau} b_{11} \\ q^* C_1 & 0 \end{pmatrix}$ is non-singular i.e. $\lambda = 0$ is not a transmission zero of the triple $(A_1, b_{10} + e^{-A_1 \tau} b_{11}, q^* C_1)$.

For system (3) we may state, as in (Ionescu and Răsvan, 1991) the Linear Structurally Stable Regulator Problem (LSSRP): find a measured-error-activated linear compensator

$$\begin{aligned} \dot{x}_c(t) &= A_c x_c(t) + B_c y(t) \\ u(t) &= f_c^* x_c(t) + g_c^* y(t) \end{aligned} \quad (4)$$

in order that the resulting closed loop linear system

$$\begin{aligned} \dot{w}_1(t) &= (A_1 + (b_{10} + e^{-A_1 \tau} b_{11}) g_c^* C_1) w_1(t) + (A_3 + \\ &\quad (b_{10} + e^{-A_1 \tau} b_{11}) g_c^* C_2) x_{20} + (b_{10} + e^{-A_1 \tau} b_{11}) f_c^* x_c(t) \\ \dot{x}_c(t) &= B_c C_1 w_1(t) + B_c C_2 x_{20} + A_c x_c(t) \end{aligned} \quad (5)$$

should be asymptotically (in fact exponentially) stable in the autonomous case (in the absence of the exogeneous signals i.e. for $x_{20} = 0$) and $\lim_{t \rightarrow \infty} z(t) = 0$ for any $x_{20} \neq 0$; also these properties should hold for any A_3 and for small parameter uncertainties of $A_1, b_{10}, b_{11}, \tau$. Assume that LSSRP has been solved for the transformed system. We want, by

applying the inverse transform to (2), to see the structure achieved for the compensator and the properties of the resulting closed loop system. The feedback system in the variables x_1, x_2, x_c is as follows

$$\begin{aligned}
 \dot{x}_1(t) &= A_1x_1(t) + A_3x_2(t) + b_{10}u(t) + b_{11}u(t - \tau) \\
 \dot{x}_2(t) &= 0 \\
 \dot{x}_c(t) &= B_cC_1x_1(t) + B_cC_2x_2(t) + A_cx_c(t) + \\
 &\quad B_cC_1 \int_{-\tau}^0 e^{-A_1(\tau+\theta)} b_{11}u(t + \theta) d\theta \\
 u(t) &= q_c^*(C_1x_1(t) + C_2x_2(t)) + f_c^*x_c(t) + \\
 &\quad g_c^*C_1 \int_{-\tau}^0 e^{-A_1(\tau+\theta)} b_{11}u(t + \theta) d\theta
 \end{aligned} \tag{6}$$

As a mathematical object system (6) is legitimate: let us substitute $u(t)$ from its integral equation in the first equation of (6) thus obtaining

$$\begin{aligned}
 \dot{x}_1(t) &= (A_1 + b_{10}g_c^*C_1)x_1(t) + (A_3 + b_{10}g_c^*C_2)x_2(t) + b_{10}f_c^*x_c(t) + \\
 &\quad b_{11}u(t - \tau) + b_{10}g_c^*C_1 \int_{-\tau}^0 e^{-A_1(\tau+\theta)} b_{11}u(t + \theta) d\theta \\
 \dot{x}_2(t) &= 0 \\
 \dot{x}_c(t) &= B_c(C_1x_1(t) + C_2x_2(t)) + A_cx_c(t) + \\
 &\quad B_cC_1 \int_{-\tau}^0 e^{-A_1(\tau+\theta)} b_{11}u(t + \theta) d\theta \\
 u(t) &= q_c^*(C_1x_1(t) + C_2x_2(t)) + f_c^*x_c(t) + \\
 &\quad g_c^*C_1 \int_{-\tau}^0 e^{-A_1(\tau+\theta)} b_{11}u(t + \theta) d\theta
 \end{aligned} \tag{7}$$

The solution of (7) may be constructed by steps, each step having two substeps, as follows: given $x_1(0) = x_{10}, x_2(0) = x_{20}, x_c(0) = x_{c0}$ and $u(\theta) = u^0(\theta), -\tau \leq \theta < 0$ we can construct from the first three equations $x_1(t), x_2(t) (\equiv x_{20})$ and $x_c(t)$ on $[0, \tau)$; then we use the last equation to obtain $u(t)$ on the same interval $[0, \tau)$ and the first step is accomplished. We repeat this procedure on intervals $[k\tau, (k+1)\tau)$ with k a positive integer.

In a similar way we can discuss the internal stability property. Let $x_{20} = 0$ what gives $x_2(t) \equiv 0$. We know that (5) is exponentially stable hence there exist $\beta_1 > 0, \alpha > 0$ such that

$$|w_1(t)| + |x_c(t)| \leq \beta_1 e^{-\alpha t} (|w_1(0)| + |x_c(0)|) \tag{8}$$

We deduce from (2)

$$|x_1(t)| \leq |w_1(t)| + \int_{-\tau}^0 e^{\gamma(\tau+\theta)} |b_{11}| |u(t + \theta)| d\theta$$

On the other hand $w_1(0) = x_{10} + \int_{-\tau}^0 e^{-A_1(\tau+\theta)} b_{11} u^0(\theta) d\theta$ and, therefore

$$|w_1(0)| \leq |x_{10}| + \beta_2 \|u^0\|, \quad \|u^0\| = \left(\int_{-\tau}^0 |u^0(\theta)|^2 d\theta \right)^{1/2}$$

From (3) and (4) it follows that

$$|x_1(t)| \leq \left[\beta_1 + \beta_3 e^{\gamma\tau} \int_{-\tau}^0 e^{(\gamma-\alpha)\theta} d\theta \right] e^{-\alpha t} (|x_{10}| + \beta_2 \|u^0\| + |x_{c0}|)$$

what shows exponential stability of (6) (or (7)) in the autonomous case.

The regulator property $\lim_{t \rightarrow \infty} z(t) = 0$ for (6) - with $z(t)$ defined in (1) - follows at once from the regulator property of the finite dimensional closed loop system (5) with respect to the transformed controlled output $z(t)$. In this way we have proved the following preliminary result.

Theorem 1. *Consider the controlled system (1) and the transformed system (3) via the change of coordinates (2). If (4) defines a regulating compensator for (3), solving LSSRP (Linear Structurally Stable Regulator Problem) in the sense that the autonomous system (with $x_2(t) \equiv 0$) is exponentially stable and $\lim_{t \rightarrow \infty} z(t) = 0$ for any $x_2(0) \neq 0$, the properties holding for any A_3 and for small parameter uncertainties of $A_1, b_{10}, b_{11}, \tau > 0$, then the compensator defined by the last two equations of (6) solves LSSRP for (1).*

3. Compensator synthesis

We shall recall here some results from (Ionescu and Răsvan, 1991) on LSSRP for the case of constant exogeneous signals. The structure of compensator (4) is as follows

$$A_c = \begin{pmatrix} A_w & a_a \\ 0 & 0 \end{pmatrix}, \quad B_c = \begin{pmatrix} B_w \\ q^* \end{pmatrix}, \quad f_c^* = (f_w^* \quad f_a), \quad g_c^* \quad (9)$$

where $(A_w, a_a, B_w, f_w^*, f_a, g_c^*)$ defines a stabilizing compensator for the extended system

$$\begin{aligned} \dot{w}_e(t) &= A_e w_e(t) + b_e u(t) \\ y_e(t) &= C_e w_e(t) \end{aligned} \quad (10)$$

where

$$\begin{aligned} A_e &= \begin{pmatrix} A_1 & 0 \\ q^* C_1 & 0 \end{pmatrix}, \quad b_e = \begin{pmatrix} b_{10} + e^{-A_1 \tau} b_{11} \\ 0 \end{pmatrix}, \quad C_e = \begin{pmatrix} C_1 & 0 \\ 0 & 1 \end{pmatrix}, \\ w_e &= \begin{pmatrix} w_1 \\ x_a \end{pmatrix}, \quad y_e = \begin{pmatrix} y \\ x_a \end{pmatrix} \end{aligned} \quad (11)$$

and the compensator is given by

$$\begin{aligned}\dot{w}(t) &= A_w w(t) + B_w y(t) + a_a x_a(t) \\ u(t) &= f_w^* w(t) + g_c^* y(t) + f_a x_a(t)\end{aligned}\tag{12}$$

Worth mentioning that due to the basic assumptions *i)* and *ii)* such a stabilizing compensator can always be constructed while the structure resulting from the separation theorem - state feedback + observer - is not compulsory. On the contrary the integrator occurs in a necessary way and must be always present in the structure of the compensator; moreover, for a structurally stable design with respect to A_1 , b_{10} , b_{11} , A_3 , τ , this integrator must be contained in the compensator even if the initial system contains an integrator (i.e. has a zero eigenvalue). From (9) and (12) the following compensator equations can be written

$$\begin{aligned}\dot{w}(t) &= A_w w(t) + a_a x_a(t) + B_w y(t) \\ \dot{x}_a(t) &= q^* y(t) \\ u(t) &= f_w^* w(t) + f_a x_a(t) + g_c^* y(t)\end{aligned}\tag{13}$$

This compensator is implemented in (1) - with y defined as in (1) - in order to define the feedback system which is exponentially stable, has the regulator property for constant exogeneous signals with respect to the regulated output $z(t)$ and both these properties are robust i.e. structurally stable.

4. Optimal stabilization with respect to a quadratic performance index

We shall turn back to system (1) and associate it the simplest quadratic performance index

$$J(u) = \int_0^\infty [z^2(t) + \kappa u^2(t)] dt\tag{14}$$

where $\kappa > 0$, the index being defined on admissible pairs. Remark the overall system (1) has zero eigenvalues and is not completely controllable. We shall use some early results of V.M. Popov (1967) in order to obtain a state feedback law that ensures stabilization of the resulting closed loop system and a minimal value of (14).

Performing again the transformation (2) we obtain (3) which is finite dimensional and associate it to (14). In the following we shall check the assumptions required by the positivity theory of Popov for the uncontrollable case. In order to simplify the exposition we shall strengthen assumption *i)* of Section 2 as follows:

i') the pair $(A_1, b_{10} + e^{-A_1\tau}b_{11})$ is controllable and the pair (q^*C_1, A_1) is observable.

The characteristic function of the system, which coincides with the one of the controllable part (with the state vector w_1) is as follows

$$\chi(\lambda, \sigma) = \kappa + b_1^*(\lambda I - A_1^*)^{-1}C_1^*qq^*C_1(\sigma I - A_1)^{-1}b_1 \quad (15)$$

where $b_1 = b_{10} + e^{-A_1\tau}b_{11}$; the characteristic polynomial is

$$\pi(\lambda, \sigma) = \det(\lambda I - A_1^*) \det(\sigma I - A_1) \chi(\lambda, \sigma) \quad (16)$$

We shall discuss first the assumptions $P_1 - P_3$ from the cited reference (Popov, 1967) since they may be considered as *non-degeneracy conditions*. We shall have

$$\pi(-i\omega, i\omega) = |\det(i\omega I - A_1)|^2 \left(\kappa + |q^*C_1(i\omega I - A_1)^{-1}b_1|^2 \right)$$

Since $\kappa > 0$, $\pi(-i\omega, i\omega)$ is not identically 0 hence $\pi(-\sigma, \sigma)$ is not identically 0 at least on $i\Re$; therefore P_1 holds. Moreover $\pi(-i\omega, i\omega) > 0$ for all ω hence P'_1 holds. Since A_{22} - the matrix defining the dynamics of x_2 - is the zero matrix, its only significant eigenvalue is 0 (with some multiplicity) P_2 holds because $\pi(0, 0) > 0$ (from P'_2 , in fact); obviously P_3 does not hold and we have to take this into account. In any case, since $\pi(-i\omega, i\omega) > 0$ and $\kappa > 0$ we deduce that $\chi(-i\omega, i\omega) > 0$. Using Theorem 1 of (Popov, 1967) we deduce existence of the polynomial $\psi(\sigma)$ with no roots in the closed RHP such that

$$\pi(-\sigma, \sigma) = \overline{\psi(-\bar{\sigma})} \psi(\sigma)$$

and also of the scalar γ , vectors p_1, p_2 and matrix N such that

$$\chi(-\sigma, \sigma) = \overline{\varphi(-\bar{\sigma})} \varphi(\sigma)$$

$$\varphi(\sigma) = \gamma + p_1^*(\sigma I - A_1)^{-1}b_1, \quad \varphi(\sigma) \det(\sigma I - A_1) \equiv \psi(\sigma)$$

$$\sqrt{\kappa} = \gamma, \quad N_{11}b_1 = p_1\gamma$$

$$C_1^*qq^*C_1 + N_{11}A_1 + A_1^*N_{11} = p_1p_1^*$$

$$N_{12}^*b_1 = p_2\gamma, \quad A_3^*N_{11} + N_{12}^*A_1 + C_2^*qq^*C_1 = p_2p_1^*$$

$$N_{12}^*A_3 + A_3^*N_{12} + C_2^*qq^*C_2 = p_2p_2^*$$

$$\text{where } N = \begin{pmatrix} N_{11} & N_{12} \\ N_{12}^* & N_{22} \end{pmatrix}.$$

Here γ is computed in a straightforward way, p_1 follows from the factorization and N_{11} from a Liapunov equation. It may be shown by

following the lines of (Popov, 1967) that N_{12}^* and p_2 can be obtained from the above equations and that under these circumstances the last equation is automatically satisfied. Since N_{22} does not enter in these equations it might be chosen arbitrarily e.g. $N_{22} = 0$.

If we consider the integral

$$\eta(0, t_1) = \int_0^{t_1} [z^2(t) + \kappa u^2(t)] dt \quad (17)$$

then we may write, following (Popov, 1967)

$$\begin{aligned} \eta(0, t_1) = & -(w_1^* N_{11} w_1)|_0^{t_1} - (w_1^* N_{12} x_2)|_0^{t_1} - (x_2^* N_{12}^* w_1)|_0^{t_1} + \\ & \int_0^{t_1} |\gamma u(t) + p_1^* w_1(t) + p_2^* x_2(t)|^2 dt \end{aligned} \quad (18)$$

We choose now

$$u(t) = -\frac{1}{\gamma}(p_1^* w_1(t) + p_2^* x_2(t)) \quad (19)$$

and system (3) becomes

$$\begin{aligned} \dot{w}_1(t) &= \left(A_1 - \frac{1}{\gamma} b_1 p_1^*\right) w_1(t) + \left(A_3 - \frac{1}{\gamma} b_1 p_2^*\right) x_2(t) \\ \dot{x}_2(t) &= 0 \end{aligned} \quad (20)$$

having bounded solutions. With the arguments of (Popov, 1967) convergence of the integral (14) along the solution defined by (19) is obtained. If we come back to the initial variables, the following dynamic feedback law is obtained:

$$u(t) + \frac{1}{\gamma} p_1^* \int_{-\tau}^0 e^{-A_1(\tau+\theta)} b_{11} u(t+\theta) d\theta = -\frac{1}{\gamma}(p_1^* x_1(t) + p_2^* x_2(t)) \quad (21)$$

and the optimal value of the index will depend on the initial history of u on $(-\tau, 0)$ - see also (Răsvan and Popescu, 2001a, 2001b).

5. An example. Simulation results

Let consider a system with input delay and constant disturbance described by

$$\dot{x}(t) = Ax(t) + B_1 u(t - \tau) + \nu_0 \quad (22)$$

where

$$A = \begin{pmatrix} 0 & 1 \\ -\omega_0^2 & 0 \end{pmatrix}, \quad B_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The controlled variable is

$$z(t) = Dx(t) \quad (23)$$

where $D = \begin{pmatrix} 1 & 0 \end{pmatrix}$.

In the design of high-accuracy regulators and tracking systems, it is necessary to eliminate completely the effect of offset errors caused by constant disturbances. This can be done by the application of integral control action. The destabilizing effect of the integral control, can be counteracted by the appropriate state feedback action, so that one can eventually achieve a satisfactory transient response, as well as the desired zero steady-state error.

Adjoin to the system variables the integral state" v , defined by

$$\dot{v}(t) = z(t) \quad (24)$$

with $v(0)$ given. Denoting the extended state vector by $\bar{x}$

$$\bar{x} = \begin{pmatrix} x \\ v \end{pmatrix} \quad (25)$$

the equations (22)–(24) can be written as follows

$$\begin{aligned} \dot{\bar{x}}(t) &= \bar{A}\bar{x}(t) + \bar{B}_1 u(t - \tau) + \bar{E}\nu_0 \\ z(t) &= \bar{D}\bar{x}(t) \end{aligned} \quad (26)$$

where

$$\bar{A} = \begin{pmatrix} 0 & 1 & 0 \\ -\omega_0^2 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \bar{B}_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \bar{E} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}, \bar{D} = (1 \ 0 \ 0)$$

It is easy to show that the $(\bar{A}, \bar{B}_1)$ pair is controllable. Choosing a performance index of the form

$$J(u) = \int_0^\infty [\bar{x}^T(t)Q\bar{x}(t) + u^T(t)u(t)] dt \quad (27)$$

the solution of the optimization problem is determined

$$u(t) = -\bar{B}_1^T P e^{\bar{A}\tau} \left[\bar{x}(t) + \int_{-\tau}^0 e^{-\bar{A}(\tau+\theta)} \bar{B}_1 u(t+\theta) d\theta \right] \quad (28)$$

where P is the positive-definite solution of the matrix Riccati equation

$$Q + \bar{A}^T P + P\bar{A} - P\bar{B}_1 \bar{B}_1^T P = 0 \quad (29)$$

Then, the following piecewise constant control is applied to the system (22)

$$u(t) = -\bar{B}_1^T P e^{\bar{A}\tau} \left[\bar{x}_k + \sum_{-N}^{-1} \bar{A}(\delta)^{-(N+j+1)} \bar{B}_1(\delta) u_{k+j} \right], \quad (30)$$

$k\delta \leq t < (k+1)\delta$, where

$$\bar{\mathbf{A}}(\delta) = \begin{pmatrix} \cos(\omega_0\delta) & \frac{\sin(\omega_0\delta)}{\omega_0} & 0 \\ -\omega_0\sin(\omega_0\delta) & \cos(\omega_0\delta) & 0 \\ \frac{\sin(\omega_0\delta)}{\omega_0} & \frac{1-\cos(\omega_0\delta)}{\omega_0^2} & 1 \end{pmatrix}, \quad \bar{\mathbf{B}}_1(\delta) = \begin{pmatrix} \frac{1-\cos(\omega_0\delta)}{\omega_0^2} \\ \frac{\sin(\omega_0\delta)}{\omega_0} \\ \frac{\omega_0\delta - \sin(\omega_0\delta)}{\omega_0^3} \end{pmatrix},$$

$$\delta = \tau/N$$

For simulation purposes, we choose $\omega_0 = \pi/2$ (rad/s), $\tau = 1$ (s), $N = 10$, $Q = \rho I$, $\rho = 10$. The simulation results obtained for a step disturbance ν_0 , considering the actuator saturation for $|u(t)| > 2$ and "intelligent" integrators confirm the performance and robustness of the designed controller.

6. Concluding remarks

The application of the Artstein transform which reduces systems with delay in control to delayless systems allows to construct a finite dimensional theory for such systems. Nevertheless this is possible provided suitable output signals and integral indices are chosen.

References

- [1] Artstein, Z.(1982). Linear Systems with Delayed Controls: a Reduction, *IEEE Trans. Aut. Control*, vol.27, no.4, pp.869-879.
- [2] Drăgan, V. and A. Halanay(1999). *Stabilization of Linear Systems*, Birkhäuser Verlag, Boston.
- [3] Francis, B.(1977). The linear multivariable regulator problem, *SIAM J. Contr. Optim.*, vol.15, pp.486-505.
- [4] Francis, B. and W.M. Wonham(1975). The internal model principle for linear multivariable regulators, *Appl. Math. Optim.*, vol.2, no.2, pp.170-194.
- [5] Halanay, A. and Vl. Răsvan(1977). General Theory of Linear Hybrid Control, *Int. J. Control*, vol.20, no.4, pp.621-634.
- [6] Ionescu, V. and Vl. Răsvan(1991). The nonlinear regulator problem for constant signals, *Kybernetika*, vol.27, no.1, pp.12-22.
- [7] Krikelis, N.J.(1980). State feedback integral control with "intelligent" integrators, *Int. J. Control*, vol.32, no.3, pp.465-473.
- [8] Manitius, A. and A.W. Olbrot(1979). Finite spectrum assignment for systems with delays, *IEEE Trans. Aut. Control*, vol.AC-24, pp.541-553.
- [9] Olbrot, A.W.(1978). Stabilizability, detectability and spectrum assignment for linear systems with general time delays, *IEEE Trans. Aut. Control*, vol.AC-23, pp.887-890.
- [10] Pandolfi, L.(1981). A state space approach to control systems with delayed controls, In: *Proceedings of Conference on Functional Differential Equations and Related Topics (M. Kisielewicz, Ed.)*, pp.256-266, Zielona Gora, Poland.

- [11] Pandolfi, L.(1989). Dynamic stabilization of systems with input delays, In: *System Structure and Control: State Space and Polynomial Methods*, Prague.
- [12] Pandolfi, L.(1990). Generalized control systems, boundary control systems and delayed control systems, *MCSS*, vol.3, pp.165-181.
- [13] Pandolfi, L.(1991). Dynamic stabilization of systems with input delays, *Automatica*, vol.27, no.6, pp.1047-1050.
- [14] Popescu, D. and Vl. Răşvan(2001). Stabilization and Suboptimal Control of Systems with Input Delay, *The 11th Int. Symp. on Modeling, Simulation and Systems' Identification*, pp.259-266, Galati.
- [15] Popov, V.M.(1960). Stability criteria for systems containing non-univocal elements (in Romanian). In: *Probleme de Automatizare*, III, pp.143-151, Ed.Academiei, Bucharest.
- [16] Popov, V.M.(1967). Incompletely controllable positive systems and applications to optimisation and stability of automatic control systems. *Rev. Roum. Sci. Techn., Serie Electrot. et Energ*, tome 12, no.3, pp.337-357.
- [17] Răşvan, Vl. and D. Popescu(2001a). Feedback Stabilization of Systems with Delays in Control, *Control Engineering and Applied Informatics*, vol.3, no.2, pp.62-66.
- [18] Răşvan, Vl. and D. Popescu(2001b). Control of systems with input delay by piecewise constant signals, *9th Medit. Conf. on Control and Automation*, Paper WM1-B/122, Dubrovnik, Croatia.
- [19] Tadmor, G.(1995). The Nehari problem in systems with distributed input delays is inherently finite dimensional, *Syst. and Contr. Let.*, vol.26, no.1, pp.11-16.
- [20] Tadmor, G.(1998). Robust control of systems with a single input lag, In: *Stability and Control of Time Lag Systems-LNCIS*, no.228, pp.259-282, Springer Verlag.
- [21] Watanabe, K. and M. Ito(1981). An observer for linear feedback control laws of multivariable systems with multiple delays in controls and output, *Syst. and Contr. Let.*, vol.1, no.1, pp.54-59.