IMPACT OF POLARIZATION MODE DISPERSION IN MULTI-HOP MULTI-RATE WDM RINGS *

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Abstract

Fiber Polarization Mode Dispersion (PMD) is perhaps the most critical transmission impairment in optical networks with transmission rates of 10 Gb/s and higher. Since the bandwidth-distance product, or transparency, of the optical channel is limited by PMD, the overall network design and cost may be significantly altered by the actual fiber PMD values.

The paper has two objectives. First, to quantify how PMD may increase the cost of WDM rings in a number of design scenarios, including first generation, single-hop, multi-hop, and multi-rate networking. Second, the polynomial-time algorithm proposed in [3] is modified to provide sub-optimal solutions for the above WDM rings, that take into account the limited bandwidth-distance product imposed by PMD. Presented results reveal that for high transmission rates, the cost of the multi-hop ring is less affected by PMD than the costs of first generation and single-hop rings.

Keywords: Polarization Mode Dispersion (PMD), Differential Group Delay (DGD), WDM Network Planning, Multi-Hop/Multi-Rate (M&M) Ring.

1. Introduction

The concept of *channel transparency* [12] has the potential to revolutionize the way wired network architectures are designed. Whether it is achieved using optical devices (the so called *optical transparency*), or electronic regeneration of the signal (the so called *opaque channel*), channel transparency allows two communicating end nodes to arbitrarily select a transmission rate, as well as a

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protocol, and a modulation technique, independently of the intermediate node hardware and software. This concept is absent in conventional networks (usually referred to as First Generation (FG) optical networks), i.e., SONET/SDH, requiring Optical/Electronic/Optical (O/E/O) conversion at every (intermediate) node. One of the main advantages of FG optical networks is their possibility to perform (electronic) traffic grooming at any node, potentially yielding high bandwidth-efficient solutions at the cost of a large number of Optical Terminals (OT)¹ being employed in the network.

Two possible network architectures that exploit the benefit of channel transparency are generally considered: Single-Hop (SH) optical networks [8] and Multi-Hop (MH) optical networks [9]. In SH optical networks, once transmitted, the optical signal propagates along the optical circuit, or *lightpath*, without requiring O/E/O conversion, until it is received at the destination node [3]. When compared to a FG optical network, the single hop network requires less optical terminals as they are required only at the end nodes of each lightpath. In MH optical networks, the tributary signal is transmitted from source to destination over a concatenation of optical channels and may undergo O/E/O conversion at some selected intermediate nodes. By allowing only some selected nodes to provide O/E/O conversion, MH optical networking requires less OTs when compared to FG architecture. By achieving traffic grooming at these selected nodes, MH optical networking yields more efficient bandwidth utilization, or alternatively it requires less wavelengths, when compared to SH architecture.

An additional feature of channel transparency is the possibility to transmit using multiple transmission rates, independently of the hardware at the intermediate nodes. The feature, called Multi-Rate (MR), provides the network designer with the flexibility to select the transmission rate on a per-lightpath basis, as opposed to Single-Rate (SR) solutions in which all lightpaths in the network must operate at the same rate. MR networking yields an even more efficient use of the wavelength bandwidth.

In [3], the authors showed that, in Wavelength Division Multiplexing (WDM) rings, the combined Multi-hop and Multi-rate (M&M) approach can potentially reduce the overall network cost significantly, when compared to FG, SH, and SR solutions. The study in [3] is carried out assuming a fully transparent optical channel. As it is well known, however, a number of physical impairments of the optical medium may affect and deteriorate the quality of the transmitted optical signal [1]. In the presence of all or some of these impairments, the above mentioned cost benefits, that originates from channel transparency, may be significantly reduced, if not completely lost. At transmission rates up to 10 Gb/s, for example, chromatic dispersion may partially limit the channel

¹Optical terminal is the interface between optical and electrical path layers.

transparency, and as a consequence the overall network cost may increase [2]. In optical systems operating at 10 Gb/s rate and above, Polarization Mode Dispersion (PMD) is proving to impose the most serious limitation on the signal propagation over long distances [13]. Although management and compensation of PMD is the subject of extensive study at the moment – with already some products being available on the market to compensate for such detrimental effects (e.g., [10]) – the PMD induced bandwidth-distance limitation may significantly affect the planning phase and cost of the entire network.

The primary objective of this paper is to assess the impact of PMD on the overall network design process and cost for FG, SH, and MH WDM rings, considering both SR and MR approaches. The effect of PMD on the channel transparency is characterized using a model that estimates the channel maximum bandwidth-distance product, considering the time-variant behavior of PMD. The problem of optimally designing WDM ring in the presence of PMD – i.e., determining which optical channels are required in the ring to carry the given traffic while minimizing the network cost, defined here as the sum of wavelength and optical terminal cost – is solved using two complementary approaches. The first approach is based on an Integer Linear Programming (ILP) formulation of the problem, whose optimum solution is computationally intensive to find. The second approach is based on a modified version of the fast greedy algorithm proposed in [3], that takes into account the limited channel transparency due to PMD.

The study is carried out using a number of SONET-over-WDM ring benchmarks and various PMD values, ranging from ITU recommended values, to those of fibers manufactured and deployed a couple of years ago. Numerical results obtained using both ILP formulation and the proposed greedy algorithm, illustrate the network cost dependence on the fiber PMD values and reveal some interesting properties of the multi-hop multi-rate design.

2. Modeling PMD Effects

This section describes the PMD phenomenon and a model that can be used to characterize the signal degradation due to PMD.

PMD is a physical impairment that is primarily caused by fiber core eccentricity, ellipticity, and stresses introduced during manufacturing, cabling and installation processes. PMD causes temporal spreading of the signal due to the fiber birefringence. The birefringence makes the two polarization modes of the fiber propagate at different speeds. The time difference between the fractions of a pulse that are transmitted in the two principal states of polarization [11] of an optical signal is known as the Differential Group Delay (DGD). DGD is the fundamental measure of PMD and is conventionally measured in picosecond. In long fibers (longer than 1 km), there is a large mode coupling between

the polarization modes, i.e., energy is transferred randomly from one mode to the other as the signal propagates along the fiber. Assuming random (strong) polarization mode coupling, DGD in fiber can be statistically modeled with a Maxwellian distribution [6]. The time-variant nature of the fiber birefringence strongly hampers the PMD compensation.

The signal spreading due to PMD grows with the square root of the fiber length as follows:

$$\langle DGD \rangle = PMD_u \cdot \sqrt{L} \quad (1)$$

where $\langle DGD \rangle$ is the average DGD accumulated along the fiber, PMD_u is PMD per unit length, and L is the length of the fiber. The total accumulated PMD of concatenated fiber sections with each section i having its own PMD_u , can be calculated as follows:

$$\langle DGD \rangle = \sqrt{\sum_i \langle DGD_i \rangle^2}. \quad (2)$$

where $\langle DGD_i \rangle$ is the average $\langle DGD \rangle$ of section i . As data rates increase above 10 Gb/s, DGD can significantly degrade the digital signal quality. Practically speaking, the length of a lightpath that spans across a concatenation of multiple fibers (or lines)² with the same value of PMD_u is limited from above by the desired value of $\langle DGD \rangle$ as follows:

$$L_{max} = \left(\frac{\langle DGD \rangle}{PMD_u} \right)^2. \quad (3)$$

From Maxwellian statistics of the DGD, it is possible to characterize the relationship between maximum tolerable DGD and average DGD or $\langle DGD \rangle$ and to evaluate the probability of exceeding the maximum DGD. For example, the DGD will reach three times its mean value with a probability of $4.2 \cdot 10^{-5}$, which correspond to 21 minutes per year. Therefore, by selecting the desired probability of exceeding the maximum DGD, the ratio between average and maximum DGD can be extrapolated using the Maxwellian distribution of DGD, and the value of L_{max} can be obtained.

The selection of the maximum allowable value of DGD depends on various factors: statistical distribution of DGD, margin allocated to PMD, e.g., power penalty of 1 dB, tolerable probability of exceeding the chosen power penalty, correlation between the spatial fluctuations, of the principal states of polarization and the fluctuations of the DGD, and the orientation of the state

²In this study it is assumed that other optical components do not affect the total accumulated PMD significantly.

of polarization of the light relative to the principal states of polarization. Recommendation ITU-T G.691 defines the maximum DGD as the value of DGD that the system must tolerate with a maximum sensitivity degradation of 1 dB. In order to relate the sensitivity degradation to the maximum DGD, numerical simulations are required. Figure 1, obtained using the coarse step method [7], shows the power penalty suffered by the signal at $\text{BER} = 10^{-12}$, as a function of the DGD.

Figure 1. Average power penalty at $\text{BER} = 10^{-12}$, versus DGD at different transmission rates

Coefficients	2.5 Gb/s	10 Gb/s	40 Gb/s
c_1	2.05903e-04	-3.88453e-3	-1.14814e-2
c_2	2.00328e-05	5.97249e-4	5.06388e-3
c_3	4.51589e-08	0	2.89476e-4

Table 1. Values of coefficients of the polynomial in Equation 4

The curves represented in Figure 1 can be conveniently represented with a polynomial obtained through fitting:

$$\text{Power penalty [dB]} = c_1 \cdot (\text{DGD}) + c_2 \cdot (\text{DGD})^2 + c_3 \cdot (\text{DGD})^3. \quad (4)$$

For the curves of Figure 1, the obtained coefficients are given in Table 1. Using the curves in Figure 1, it is possible to obtain the DGD associated to a given power penalty and then to calculate the average DGD which can be tolerated to keep the outage probability below a selected value.

While typical values of PMD_u between 0.1 and 1.0 $\text{ps}/\sqrt{\text{km}}$ do not affect the transmission at lower rates such as 2.5 Gb/s (OC-48) and below, at higher bit rates (OC-192 and above) PMD becomes a key factor in planning optical networks. In old fibers with a PMD_u parameter of 0.5 $\text{ps}/\sqrt{\text{km}}$, the length of the transparent channel cannot exceed 876 km and 57.76 km for bit rates of 10 Gb/s and 40 Gb/s, respectively, giving a probability of $4.2 \cdot 10^{-5}$ to exceed 1 dB power penalty. Figure 2 shows the maximum lightpath length, L_{max} , as a

Fiber Classification	PMD_u (ps/ $\sqrt{km}$)
Fibers manufactured after 1998	~ 0.1
Fibers manufactured between 1995 and 1998	~ 0.2
Fibers manufactured before 1995	≥ 0.5
ITU-T recommendation	≤ 0.5

Table 2. Typical values of PMD_u

function of PMD_u , assuming a probability of $4.2 \cdot 10^{-5}$ to exceed 1 dB power penalty, for transmission rates of 2.5, 10, and 40 Gb/s.

Figure 2. Lightpath maximum length, L_{max} , versus PMD_u at different OC rates, under the assumption that the probability of exceeding 1 dB power penalty is set to $4.2 \cdot 10^{-5}$

2.1. Experimental Measurements and Results

The described PMD model is supported by experimental measurements and results.

Typical values of PMD_u measured in the field are presented in Table 2, sorted by the fiber manufactured year. In the first two rows, about 85% of the fibers have PMD value below 0.1 and 0.2 ps/ $\sqrt{km}$, respectively. Fibers manufactured before 1995 may have PMD_u as high as 5.0 ps/ $\sqrt{km}$, but most of them are between 0.5 and 1.0 ps/ $\sqrt{km}$. These numbers might vary, depending on the fiber manufacturer. As illustrated in the table, the fiber manufacturing process has improved over the years and is currently able to measure and control the PMD_u of newly produced fibers. However, a large number of old fibers is already deployed in the field and currently used.

In order to validate the numerical simulations of the power penalty shown in Figure 1, experimental results were compared against theoretical and simulated results. Theoretical BER distributions induced by fading due to the presence of PMD [5], and of PMD and Polarization Dependent Loss (PDL) [4] were used. The experimental results showed BER fluctuations concurring with the theoretical predictions. Such BER fluctuations cause temporary channel outages and thus represent a major physical impairment in high bit rate WDM networks.

3. Designing WDM Rings in the Presence of PMD

To illustrate the impact of PMD on the design of WDM networks, a unidirectional WDM ring supporting SONET optical carriers is considered. (Similar studies can be carried out using other technologies, such as Gigabit Ethernet.) Each node of the SONET-over-WDM ring consists of an Optical Add-Drop Multiplexer (OADM) that demultiplexes (multiplexes) the incoming (outgoing) wavelengths and provides each wavelength with either optically transparent by-pass transmission or add-drop termination of the lightpath. Each dropped (added) wavelength is received (transmitted) by an Optical Terminal (OT) that feeds (is fed by) an electrical Add/Drop Multiplexer (ADM). At the ADM, various tributary signals are demultiplexed and handled individually. Each node i in the ring is assigned a minimum transmission rate $r_l(i)$ and a maximum transmission rate $r_u(i)$. These rates are represented, respectively, by the minimum and maximum SONET OC rates that the node OTs can handle, i.e., available rates are OC- m with $m \in M = \{3, 12, 48, 192, 768, \dots\}$. The transmission rate chosen for a lightpath must be available at both channel's end nodes. Thus for a lightpath between node i and node j only transmission rates between $r_u(i, j) = \min(r_u(i), r_u(j))$ and $r_l(i, j) = \max(r_l(i), r_l(j))$ are supported.

The selection of the actual transmission rate depends on design constraints, as explained in the following subsection.

3.1. Design Scenarios

All three architectures (FG, SH, and MH) are considered. For each architecture, the following two design scenarios are explored:

- Constant Bit - Single Rate (CBSR): OT interface cards operate at a constant bit rate and all the lightpaths make use of the same transmission rate. Thus, all the OTs in the network must operate at the same constant bit rate. The network designer can select the most cost-effective operating rate which must be between a given minimum and maximum bit rate determined by the node hardware.
- Constant Bit - Multi Rate (CBMR): OT interface cards operate at constant bit rates and the transmission rate chosen for each OT may be different

from the rates chosen for the other OTs. Thus, the network designer can optimize the selection of the transmission bit rate on a per-lightpath basis. The rate selected for the lightpath must be between the minimum and the maximum rates allowed at its end nodes.

Figure 3. MH (b), FG (c), and SH (d) CBMR WDM rings carrying the tributary traffic shown in (a)

Figure 3 illustrates how the same set of tributary signals (a) is transmitted using the multi-rate design option in the MH (b), FG (c), and SH (d) architectures, respectively.

3.2. Optimal Design

Once the desired architecture (FG, SH, MH) is chosen, the problem to solve is the optimal selection of the set of lightpaths required to carry the offered traffic. For each lightpath, the span (only in the case of MH architectures) and the transmission rate must be determined, bearing in mind that PMD may practically limit the lightpath maximum bandwidth-distance product. The objective is to minimize the overall network cost, defined as the sum of the *wavelength cost* and the *OT cost*.

While obtaining the optimal solution for both FG and SH architectures is straightforward, the same is not true for the MH architecture. MH optimal solution is hard to find, because for each tributary signal the optimal set of consecutive lightpaths and their respective span must be chosen. In both the FG and SH architectures this set is uniquely identified and does not pose a challenging problem.

Two complementary approaches are considered to find a solution for the MH ring. An Integer Linear Programming (ILP) formulation is described first, that can be applied to obtain the optimum solution. The complexity of the problem makes it difficult to reach the optimum solution with the use of currently available ILP solvers. Hence, an algorithmic approach is provided, which solves the problem sub-optimally, but in polynomial time.

3.2.1 ILP Formulation. For the CBMR case, the ILP problem formulation can be derived directly from the one presented in [3], by introducing an additional constraint that takes into account the limited lightpath transparency due to PMD as indicated in Equation 3. Let variable $l^{ij,m}$ denote the number of lightpaths from node i to node j , with transmission rate OC- m , that are chosen to carry the offered traffic. If the path length from node i to j is greater than the maximum length computed using Equation 3 for the given transmission rate, OC- m , then $l^{ij,m} = 0$.

For the CBSR case, the ILP formulation provided for the CBMR case must be modified to include two additional constraints. Let $b_m \in \{0, 1\}$ be a variable that indicates whether the network is designated to operate at bit rate OC- m or not. Let K be a constant whose value is larger than the possible number of lightpaths. The two additional constraints are given by:

$$\sum_{i,j} l^{ij,m} < K \cdot b_m \quad \forall m \in M, \quad \sum_{m \in M} b_m = 1 \quad (5)$$

Equation 5 ensures that only one bit rate is used in the planning. Regardless of the two additional constraints and new variables, the asymptotic model complexity and the asymptotic number of variables remain the same of the CBMR ILP formulation.

3.2.2 The Modified HRF-LLF Greedy Algorithm. This section describes the greedy algorithm designed to solve the MH optimization problem.

For the CBMR case, the High Rate First-Long Lightpath First (HRF-LLF) algorithm, originally proposed in [3], yields sub-optimal solutions for the MH and MR architecture under the assumption that the channel bandwidth-distance product is unlimited (the ideal medium case). The HRF-LLF algorithm presented in [3] can be modified to consider the limited bandwidth-distance product caused by PMD, as described next. In the presence of the PMD, all candidate lightpaths that exceed the maximum length constrained by PMD are removed from the set of potential lightpaths and are not considered during the optimization process.

To address the design of CBSR networks, the HRF-LLF algorithm is modified as follows. Let $m_u = \max_{(i,j)}(r_u(i, j))$ be the highest rate, and $m_l = \min_{(i,j)}(r_l(i, j))$

be the lowest rate allowed in the network. The optimal CBSR ring design is found through exhaustive search, by comparing the network cost obtained at every possible rate, from m_u down to m_l . The solution that yields the lowest cost is selected. When completed, the algorithm returns the chosen set of optical channels, $L_{MH-CBSR}$, and their total cost, $cost_{MH-CBSR}$.

For each possible transmission rate m let L_{temp} be the set of lightpaths that are chosen for the MH-CBSR ring at such rate. Initially, set L_{temp} is empty. Iteratively, the algorithm selects a set of tributary signals, and/or segments of tributary signals. The selected set is groomed together to be transmitted using a single lightpath. Grooming is first attempted for lightpaths that have the largest number of hops, i.e., lightpaths that span across the largest number of lines.

Let TS be a set of pairs, (tributary signal, ring line). A pair in set TS indicates which tributary signal has not been groomed yet on which ring line. Before beginning to groom the various tributary signals, set TS contains all the tributary signals, on all the lines the tributary signal is routed through. Every time a new candidate lightpath is selected to groom a certain number of tributary signals spanning across a segment of the ring, two actions are taken:

- 1) the pairs (tributary signal, ring line) corresponding to the groomed traffic are removed from the TS set
- 2) the selected candidate lightpath is added to L_{temp} .

Traffic grooming for the selected candidate lightpath is performed by applying two steps, sequentially: *homogeneous grooming*, i.e., only tributary signals with the same source-destination pair of the candidate lightpath, (s, d) , are groomed on the lightpath; *heterogeneous grooming*, i.e., any set of tributary signals may be groomed on the same lightpath.

Let $LP^m = \{lp^{ij} \forall i, j \in V : r_l(i, j) \leq m \leq r_u(i, j)\}$, be the set of candidate lightpaths, sorted by descending length, e.g., hop count. Only candidate lightpaths that do not exceed the maximum bandwidth-distance product introduced by PMD are considered.

Let $g_{ho}(TS, lp^{ij}, m)$ be a function that returns a set of m tributary signals in TS that can be homogeneously groomed on lightpath lp^{ij} , whose transmission rate is m . Let $g_{he}(TS, lp^{ij}, m)$ ($g'_{he}(TS, lp^{ij}, m)$) be a function that returns a set of m (less than m) tributary signals in TS that can be heterogeneously groomed on a lightpath lp^{ij} , whose transmission rate is m . Let g be a set of tributary signals under consideration for grooming on a candidate lightpath. Let $cost(L_{temp})$ be the function that returns the total cost of the lightpaths in set L_{temp} , designated to carry the traffic.

When grooming on a lightpath that spans from node i to node j , tributary signals are selected using the following decreasing priorities:

- 1) $s = i$ and $d = j$;
- 2) $s = i$ and $d \neq j$, or, $s \neq i$ and $d = j$;
- 3) $s \neq i$ and $d \neq j$;

where s represents the tributary signal source node and d the tributary signal destination node.

Notice that for the same transmission rate, the PMD maximum bandwidth-distance product may be so stringent that the design may not be feasible at that rate. Under this circumstance, the algorithm skips the infeasible rate and continues with the next lower rate.

A pseudo-code description of the modified HRF-LLF algorithm is provided.

```

costMH-CBSR = ∞ ;
LMH-CBSR = empty;
Begin
  For  $m = m_u, \dots, m_l, m \in M$ 
    Ltemp = empty;
    For  $lp^{i,j}$  in  $LP^m$ 
      Repeat
         $g = g_{ho}(TS, lp^{i,j}, m)$ ;
        up-date Ltemp
      Endfor
    Until homogeneous grooming is completed
    For  $lp^{ij}$  in  $LP^m$ 
      Repeat
         $g = g_{he}(TS, lp^{ij}, m)$ ;
        up-date Ltemp
      Until heterogeneous grooming is completed
    Endfor
    For  $lp^{ij}$  in  $LP^m$ 
      Repeat
         $g = g'_{he}(TS, lp^{ij}, m)$ ;
        up-date Ltemp
      Until heterogeneous grooming is completed
    Endfor
    costMH-CBSR = min(costMH-CBSR, cost(Ltemp));
    If (costMH-CBSR == cost(Ltemp))
      LMH-CBSR = Ltemp;
    Endif
  Endfor
End

```

4. Results and Performance Comparison

Numerical results obtained using six-node unidirectional ring benchmarks are discussed in this section. Nodes are numbered from 1 to 6, with node 2 being the first downstream from node 1, etc. Line lengths are: $d_{(1,2)} = d_{(4,5)} = 40$ km, $d_{(2,3)} = d_{(5,6)} = 80$ km, and $d_{(3,4)} = d_{(6,1)} = 160$ km. Two scenarios are considered: the OC-192 ring in which all six nodes can transmit at any rate

between OC-3 and OC-192; and the OC-768 ring, in which all six nodes can transmit at any rate between OC-3 and OC-768.

The OT cost c_{OT}^m is assumed to double for each 4-fold growth of the working bit rate OC- m , i.e., $c_{OT}^{4m} = 2 \cdot c_{OT}^m$. The wavelength cost, c_w , is assumed to be unitary for each km of wavelength [2].

To explore different cost ratios between the optical bandwidth and OT cost, the following parameter is defined:

$$\gamma = \frac{c_w \cdot L_{av}}{c_{OT}^3 + c_w \cdot L_{av}}, \quad \gamma \in (0, 1] \quad (6)$$

where L_{av} is the average line length. Given c_w and γ , it is possible to evaluate c_{OT}^3 and consequently evaluate $c_{OT}^m \forall m \in M$. When $\gamma \rightarrow 0$, the OT cost is dominant. When $\gamma = 1$, the wavelength cost is dominant. Other values of γ represent possible intermediate wavelength-to-OT cost ratios.

Values of $PMD_u=0, 0.5, 0.8$, and $1.0 \text{ ps}/\sqrt{\text{km}}$ are considered. Case $PMD_u=0$ represents an ideal fiber with negligible PMD. In accordance with the power tolerance recommended by ITU-T G.691, the maximum probability to exceed a 1 dB power penalty due to PMD is set to $4.2 \cdot 10^{-5}$. With such outage probability, the allowed maximum accumulated DGD is three times its mean value. The corresponding $\langle DGD \rangle$ values are then 62 ps at 2.5 Gb/s, 14.8 ps at 10 Gb/s, and 3.8 ps at 40 Gb/s transmission rate.

Figures 4(a) and 5(a) show the network cost of the OC-192 ring for the CBSR and CBMR cases, respectively. Figures 4(b) and 5(b) show the network cost of the OC-768 ring for the CBSR and CBMR cases, respectively. These plots are obtained provisioning the network resources necessary to carry a set of uniform traffic demands, that consist of 400 STS-1 tributary signals from any node to any other node. All plots are normalized to the network cost of the MH ring with $PMD_u = 0$, i.e., the ideal medium case. In the OC-192 ring, the case with $PMD_u = 0.5 \text{ ps}/\sqrt{\text{km}}$ yields the same network cost of the ideal medium.

The FG OC-192 ring cost is not affected by PMD, as shown in Figures 4(a) and 5(a). This is due to the fact that at OC-192, the longest line in the ring is shorter than the lightpath maximum length for the chosen values of PMD_u . At OC-768, however, the cost of the FG architecture is affected by PMD too.

In general, the FG ring is more expensive than the MH ring, topping almost 3 times when the PMD_u value does not exceed $0.5 \text{ ps}/\sqrt{\text{km}}$. However, when $\gamma = 1$, the FG and MH rings have the same cost, indicating that under this cost assumption (OT cost is negligible) optical transparency does not provide any cost benefit.

The cost of the SH ring is significantly affected (up to 16 times in the CBSR case) by PMD_u . This large cost increase is due to the fact that the length of the high speed lightpaths between the tributary source and destination cannot be too

Figure 4. CBSR OC-192 ring (Figure 4(a)) and OC-768 ring (Figure 4(b)), with uniform traffic demands consisting of 400 STS-1 per node pair

Figure 5. CBMR OC-192 ring (Figure 5(a)) and OC-768 ring (Figure 5(b)), with uniform traffic demands consisting of 400 STS-1 per node pair

long, thus forcing a large number of lightpaths to operate at lower transmission rates. Such cost penalty is exacerbated when γ approaches 1. In the CBSR case, the cost of the SH ring is the same for fibers whose PMD_u value is between 0.8 and 1.0 ps/ $\sqrt{\text{km}}$.

All figures show that the MH ring is the most cost-efficient architecture, even in the presence of a physical impairment such as PMD. Quantitatively, the effect of PMD on the MH ring cost is a function of γ . When $PMD_u = 0.8$ ps/ $\sqrt{\text{km}}$ and $\gamma = 0.1$, the MH ring cost is approximately twice (or more) than the MH

cost in the ideal medium case. No cost increase is incurred when $\gamma = 1$ in the CBSR case (Figures 4(a) and 4(b)). Interestingly, the MH ring design seems to well adapt to varying values of PMD_u , whereas the FG and SH ring designs do not reveal significant changes. From these observations, one could conclude that the MH ring copes well with PMD, and more in general with other physical impairments. This feature of the MH ring is well documented by the plots of the CBMR case: the curves show that in the presence of PMD the MH cost follows the same behavior of the SH ring cost when γ is less than 0.5, whereas it follows the FG ring cost when γ is greater than 0.5.

When comparing the CBSR case against the CBMR, it is worth noticing that the later copes better with PMD than the former does. A clear indication of this trend is visible in the SH ring, for example. The cost increase determined by PMD in the CBMR case is less than the one experienced by the CBSR. Similar conclusions can be reached for the MH ring.

	$PMD_u = 0.5$			$PMD_u = 0.8$			$PMD_u = 1.0$		
	Rate	N.	Mileage	Rate	N.	Mileage	Rate	N.	Mileage
MH	OC-192	68	17760	OC-192	87	17920	OC-48	134	17920
SH	OC-192	90	24960	OC-48	270	74880	OC-48	270	74880
FG	OC-192	192	17920	OC-192	192	17920	OC-192	192	17920

Table 3. Distribution of number of lightpaths and wavelength mileage for the CBSR OC-768 ring, assuming $\gamma = 0.5$

	Number of lightpaths			Tot. Mileage [km]		
	MH	SH	FG	MH	SH	FG
$\gamma = 0.1$	(87, 62)	(190, 20)	(6, 186)	(36400, 8240)	(65280, 2400)	(560, 17360)
$\gamma = 0.5$	(78, 68)	(190, 20)	(6, 186)	(32880, 9120)	(65280, 2400)	(560, 17360)
$\gamma = 0.9$	(2, 129)	(190, 20)	(6, 186)	(520, 17360)	(65280, 2400)	(560, 17360)

Table 4. Distribution of number of lightpaths and wavelength mileage at OC-48 and OC-192 transmission rate for the CBMR OC-192 ring, assuming $PMD_u = 1.0 \text{ ps}/\sqrt{\text{km}}$

The above observations are confirmed by the results presented in Tables 3 and 4. For each transmission rate, Table 3 reports the total number of lightpaths, and the total wavelength mileage in km for the CBSR OC-768 ring, under the assumption that $\gamma = 0.5$. Table 4 presents tuples that indicate, on the left, the number of lightpaths at OC-48 and OC-192, on the right, the total wavelength mileage of the lightpaths at OC-48 and OC-192, that are necessary in the CBMR OC-192 ring. Value of PMD_u is set to $1.0 \text{ ps}/\sqrt{\text{km}}$. Both tables reveal that the

MH ring requires less lightpaths and lower wavelength mileage when compared to both the FG and SH rings. The increase of both the number of lightpaths and total wavelength mileage in the MH ring due to increasing values of PMD is relatively less when compared to the FG and SH rings. Table 3 confirms the earlier observations that in the presence of PMD, the transmission rate is forced to decrease and more low-rate lightpaths are required. Table 4 demonstrates the flexibility of the CBMR MH ring design. In the FG and SH rings, both the number of lightpaths and the total wavelength mileage remain unchanged for a large range of γ values. The MH architecture seems to adapt to the changes of γ and make use of the network resources more efficiently.

Figure 6. CCSR OC-192 ring (Figure 6(a)) and CBMR OC-768 ring (Figure 6(b)), with uniform traffic demands consisting of 400 STS-1 per node pair

Figures 6(a) and 6(b) compare the ILP against the HRF-LLF algorithm solutions. Figure 6(a) shows the network cost of the CCSR OC-192 ring. Figure 6(b) shows the network cost of the CBMR OC-768 ring. The solutions found by the greedy algorithm are within 10% (in the CCSR case) and 25% (in the CBMR case) of the optimum solutions found by the ILP solver. The computational time required to find a solution for the six-node ring is measured in hours in the case of the ILP solver, and in seconds, or even fractions of a second, in the case of the greedy algorithm.

5. Summary

The paper assessed the impact of PMD in a number of high transmission rate WDM rings, i.e., first generation, single-hop, multi-hop, and multi-rate architectures. It was assumed that PMD compensation is not available, thus requiring a network design that takes into account the limited optical channel transparency — or bandwidth-distance product — introduced by PMD.

The network cost increase due to the limited channel transparency caused by PMD, was numerically estimated, leading to some interesting conclusions.

The benefits of optical transparency still hold in the presence of PMD, although the network design cost increases. The multi-hop architecture appears to be the most robust solution in the presence of PMD transmission impairment, since its network cost increase caused by increasing PMD values is moderate. In addition, the possibility to select the optimal transmission rate for each optical channel, the so called multi-rate design, gives more flexibility to the network designer, further reducing the overall network cost in the presence of PMD.

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