AN EXTENDED POSSIBILITY-BASED FUZZY NESTED RELATIONAL DATABASE MODEL AND ALGEBRA

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1. INTRODUCTION

In order to enhance the expressive power of relational database model, non-first normal (NF² or nested) relational database model has been introduced, where attribute values may be atomic or set-valued and even relations themselves [1], on one hand. On the other hand, Fuzzy values have been employed to model information imprecision in databases since Zadeh [4] proposed the theory of fuzzy sets. A NF² relational database model with fuzzy information accommodates imprecision and complexity of the real world objects simultaneously. Based on similarity relation, a fuzzy NF² relational database model is introduced in [3] and two restructuring operations: *Merge* and *Unmerge* are emphatically given. In this paper, an extended possibility-based fuzzy NF² relational database model is introduced, where the fuzziness of data comes from possibility distributions over universes of discourse and as resemblance relation [2]. We give database model and define fuzzy nested algebra.

2. DATABASE MODEL

A fuzzy NF² relational schema is a set of attributes (A1: D1, A2: D2, ..., An: Dn), where attribute domain Di ($1 \leq i \leq n$) can be one of the following: (1) The set of atomic values, denoted *dom*; (2) The set of null values, denoted *ndom*, where null values may be *unk*, *inap*, *nin*, and *onul*; (3) The set of fuzzy subset, denoted *fdom*; (4) The power set of the set in (1), denoted *sdom*; (5)

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The set of relation values. The corresponding attribute value, say a_i , is a tuple of the form $\langle a_{i1}, a_{i2}, \dots, a_{im} \rangle$ which is an element of $D_{i1} \times D_{i2} \times \dots \times D_{im}$ ($m > 1$ and $1 \leq i \leq n$), where each D_{ij} ($1 \leq j \leq m$) may be a domain in (1), (2), (3), and (4) and even the set of relation values. Formally, the attribute domain of A_i ($1 \leq i \leq n$) is represented as follows.

$$\tau_i = \text{dom} \mid \text{ndom} \mid \text{fdom} \mid \text{sdom} \mid \langle B_1 : \tau_{i1}, B_2 : \tau_{i2}, \dots, B_m : \tau_{im} \rangle$$

where $B_1, B_2, \dots, B_m$ are attributes.

A relation r over fuzzy NF² schema $(A_1 : \tau_1, A_2 : \tau_2, \dots, A_n : \tau_n)$ is a subset of Cartesian product $\tau_1 \times \tau_2 \times \dots \times \tau_n$. A tuple in r with the form of $\langle a_1, a_2, \dots, a_n \rangle$ consists of n components. Each component a_i ($1 \leq i \leq n$) may be an atomic value, null value, set value, fuzzy value, or another tuple.

The semantic relationships between two fuzzy data include equivalence, inclusion, intersection, and irrelevancy, which can be assessed with semantic inclusion degree proposed in [2].

Let π_A and π_B be two fuzzy data on $U = \{u_1, u_2, \dots, u_n\}$. Let Res be a resemblance relation on U and α ($0 \leq \alpha \leq 1$) be a threshold corresponding to Res . The semantic inclusion degree of π_A and π_B $SID_\alpha(\pi_A, \pi_B)$ based on α , which means π_A semantically includes π_B , is defined as

$$SID_\alpha(\pi_A, \pi_B) = \sum_{i=1}^n \min_{u_i, u_j \in D} (\pi_B(u_i), \pi_A(u_j)) / \sum_{i=1}^n \pi_B(u_i)$$

For a given threshold β , we have (1) π_A includes π_B iff $SID_\alpha(\pi_A, \pi_B) \geq \beta$, (2) π_A and π_B are equivalent to each other iff $SID_\alpha(\pi_A, \pi_B) \geq \beta$ and $SID_\alpha(\pi_B, \pi_A) \geq \beta$, and (3) π_A and π_B are irrelevant to each other iff $SID_\alpha(\pi_A, \pi_B) < \beta$ and $SID_\alpha(\pi_B, \pi_A) < \beta$. The notion of equivalence degree for two fuzzy data is hereby given as follows.

The semantic equivalent degree of π_A and π_B $SE_\alpha(\pi_A, \pi_B)$, denoting the degree that π_A and π_B are equivalent to each other, is hereby defined.

$$SE_\alpha(\pi_A, \pi_B) = \min(SID_\alpha(\pi_A, \pi_B), SID_\alpha(\pi_B, \pi_A))$$

Two fuzzy data π_A and π_B are considered α - β -redundant iff $SE_\alpha(\pi_A, \pi_B) \geq \beta$. The equivalent degree of two crisp data (atomic or set-valued) is 1.0 if they are equal to each other. The elimination of duplicate could be achieved by merging π_A and π_B and producing a new fuzzy data π_C . We define the following merging operation according to Zadeh's extension principle [5].

$$\begin{aligned} \pi_C = \pi_A \cup_j \pi_B = \{ \pi_C(w) / w \mid (\exists \pi_A(u_i) / u_i) (\exists \pi_B(v_j) / v_j) (\pi_C(w) = \text{Max}(\pi_A(u_i), \\ \pi_B(v_j)) \wedge (w = u_i \mid \pi_C(w) = \pi_A(u_i) \vee w = v_j \mid \pi_C(w) = \pi_B(v_j)) \wedge \text{Res}(u_i, v_j) \geq \alpha \wedge u_i, v_j \\ \in U \wedge 1 \leq i, j \leq n) \vee (\exists \pi_A(u_i) / u_i) (\forall \pi_B(v_j) / v_j) (\pi_C(w) = \pi_A(u_i) \wedge w = u_i \wedge \\ \text{Res}(u_i, v_j) < \alpha \wedge u_i, v_j \in U \wedge 1 \leq i, j \leq n) \vee (\exists \pi_B(v_j) / v_j) (\forall \pi_A(u_i) / u_i) (\pi_C \\ (w) = \pi_B(v_j) \wedge w = v_j \wedge \text{Res}(u_i, v_j) < \alpha \wedge u_i, v_j \in U \wedge 1 \leq i, j \leq n) \} \end{aligned}$$

Informally, any two tuples in a nested relation are redundant, if, for pair of the corresponding attribute values, the equivalence degree is greater than or equal to the threshold value. If the pair of the corresponding attribute values is simple, the equivalence degree is one for two values. For two values

of structured attributes, however, the equivalence degree is one for structured attributes. Two redundant tuples t and t' are written $t \equiv t'$.

3. RELATIONAL ALGEBRA OPERATIONS

Union and Difference: Let r and s be two union-compatible fuzzy nested relations. Then

$$r \cup s = \min (\{t \mid t \in r \vee t \in s\}) \text{ and } r - s = \{t \mid t \in r \wedge (\forall v \in s) (t \neq v)\}$$

Here, the operation $\min ()$ means to remove the fuzzy redundant tuples in r and s . Of course, the threshold value should be provided for the purpose.

Cartesian Product. Let $r(R)$ and $s(S)$ be two fuzzy nested relations. Then $r \times s$ is a fuzzy nested relation with the schema $R \cup S$.

$$r \times s = \{t \mid t(R) \in r \wedge t(S) \in s\}$$

Projection. Let $r(R)$ be a fuzzy nested relation and $S \subset R$.

$$\Pi_S(r) = \min (\{t \mid (\forall v \in r) (t = v(S))\})$$

Here, an attribute in S may be of the form $B.C$, in which B is a structured attribute and C is its component attribute.

Selection. In classical relational databases, the selection condition is of the form $X \theta Y$, where X is an attribute, Y is an attribute or a constant value, and $\theta \in \{=, \neq, >, \geq, <, \leq\}$. In order to implement fuzzy query for fuzzy relational databases, “ θ ” should be fuzzy, denoting $\approx, \neq, >, <, \geq$, and $\leq$. In addition, X is only a simple attribute or the simple attribute of a structured attribute but Y may be (a) a constant, crisp or fuzzy one, (b) a simple attribute, or (c) a simple component attribute of a structured attribute, having the form $A.B$, where A is a structured attribute and B is its simple component attribute.

The fuzzy comparison operations are defined as follows. (a) $X \approx Y$ iff $SE_\alpha(X, Y) \geq \beta$, where β is a selected cut (the followings are the same); (b) $X \neq Y$ iff $SE_\alpha(X, Y) < \beta$; (c) $X > Y$ iff $X \neq Y$ and $\min(\text{Supp}(X)) > \min(\text{Supp}(Y))$; (d) $X \geq Y$ iff $X \approx Y$ or $X > Y$; (e) $X < Y$ iff $X \neq Y$ and $\min(\text{Supp}(X)) < \min(\text{Supp}(Y))$; (f) $X \leq Y$ iff $X \approx Y$ or $X < Y$.

Depending on Y , the following situations can be identified for $X \theta Y$. Let X be the attribute A_i ; τ_i in a fuzzy nested relation.

(1) $A_i \theta c$, where c is a crisp constant. According to τ_i , $A_i \theta c$ is as follows: if τ_i is *dom*, then $\theta \in \{=, \neq, >, <, \geq, \leq\}$; if τ_i is *fdom*, then $\theta \in \{\approx, \neq, >, <, \geq, \leq\}$; if τ_i is *ndom*, $A_i \theta c$ is a null comparison and regarded as the special fuzzy comparison; if τ_i is *sdom*, $A_i \theta c$ is a element-set comparison. Then $A_i \theta c$ if c and any element in the value of A_i of a tuple satisfy the “ θ ”.

(2) $A_i \theta f$, where f is a fuzzy value: if τ_i is *dom*, *fdom*, or *ndom*, $A_i \theta f$ is a fuzzy comparison and $\theta \in \{\approx, \neq, >, <, \geq, \leq\}$; if τ_i is *sdom*, $A_i \theta f$ is a fuzzy set comparison. Then $A_i \theta f$ if f and any element in the value of A_i of a tuple satisfy the fuzzy “ θ ”, where $\theta \in \{\approx, \neq, >, <, \geq, \leq\}$.

(3) $A_i \theta A_j$, where A_j : τ_j is a simple attribute and $i \neq j$: if τ_i and τ_j are all *dom*, $A_i \theta A_j$ is a traditional comparison; if τ_i and τ_j are *dom* and *fdom*, *fdom* and *fdom*, or *ndom* and *fdom*, $A_i \theta A_j$ is a fuzzy comparison; if τ_i and τ_j are *dom* and *ndom*, $A_i \theta A_j$ is a null comparison; if τ_i and τ_j are *dom* and *sdom*, $A_i \theta A_j$ is a element-set comparison; if τ_i and τ_j are *fdom* and *sdom*, $A_i \theta A_j$ is a fuzzy set comparison; if τ_i and τ_j are all *ndom*, $A_i \theta A_j$ is a null-null comparison. Then $A_i \theta A_j$ if they have the same null values on the same universe of discourse; if τ_i and τ_j are *ndom* and *sdom*, $A_i \theta A_j$ is a null-set comparison and regarded as the special element-set comparison; if τ_i and τ_j are *sdom* and *sdom*, $A_i \theta A_j$ is a set-set comparison and regarded as the special element-set comparison.

(4) $A_i \theta A_j$. B , where A_j is a structured attribute ($i \neq j$) and B is a simple attribute. The situations are the same as those in above case (3).

Let Q be a predicate denoting the selection condition. The selection operation for a fuzzy nested relation r is defined as $\sigma_Q(r) = \{t \mid t \in r \wedge Q(t)\}$

We do not give the definitions of two restructuring *Nest* and *Unnest* in the database model because of the space limitation.

4. CONCLUSION

In this paper, we introduced fuzzy data into nested relational databases to model complex objects and imprecise and uncertain information. The fuzzy data here is extended possibility-based fuzzy data expressed by possibility distribution over a universe of discourse and meanwhile a resemblance relation on the universe is in effect. We gave the structure of fuzzy nested relational model and the approach for assessing fuzzy data redundancies. On the basis, the algebra is defined for manipulating fuzzy and complex data.

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