APPLICATION OF UNCERTAIN VARIABLES TO DECISION MAKING IN A CLASS OF DISTRIBUTED COMPUTER SYSTEMS

Z. Bubnicki

Institute of Control and Systems Engineering

Wroclaw University of Technology

Wyb. Wyspianskiego 27, 50-370 Wroclaw, POLAND

email: bubnicki@ists.pwr.wroc.pl

Abstract: The paper is concerned with a class of uncertain distributed systems described by a set of relations containing unknown parameters. The purpose of the paper is to present an application of so called uncertain variables to decision making in the systems under consideration. In the first part of the paper, a problem of the decision making in the distributed system in which the knowledge of the system as a whole is decomposed into two levels is considered. In the second part it is shown how to apply the uncertain variables in the determination of computational tasks allocation in a set of parallel processors.

Key words: distributed systems, parallel processing, uncertain systems

1. INTRODUCTION

The paper concerns a class of distributed systems described by a set of relations between the variables characterizing the decisions and their effects (a relational knowledge representation). For the decision making in such a class of systems the algorithms of learning have been proposed [1]. The purpose of this paper is to present an application of so called *uncertain variables* to the decision making in the same class of distributed systems. The uncertain variable $\bar{x}$ is defined by a certainty distribution $h_x(x) = v(\bar{x} \in x)$, i.e. the certainty index v that “ $\bar{x}$ is approximately equal to x ”. The certainty distribution is given by an expert and describing his/her knowledge of the approximate values of the variables [2, 3, 4].

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2. DECISION MAKING PROBLEM

Let us consider a distributed system with a structure presented in Fig.1 where $u_j \in U_j$, $x_j \in X_j$ ($j \in \overline{1, k}$), $y \in Y$ are real number vectors. The lower and upper level subsystems are described by relations

$$R_j(u_j, x_j; c_j) \subseteq U_j \times X_j, \quad j \in \overline{1, k} \quad (1)$$

$$R(x_1, \dots, x_k, y; b) \subseteq X_1 \times \dots \times X_k \times Y, \quad (2)$$

respectively. The set of relations forms the knowledge representation of our system with the input $u = (u_1, \dots, u_k)$ (**relational knowledge representation**). The unknown parameters b and c_j are assumed to be values of uncertain variables $\bar{b}$ and $\bar{c}_j$ described by the certainty distributions $h_b(b)$ and $h_{c_j}(c_j)$, respectively. The decision problem consists in finding u^* maximizing the certainty index that the set of all possible y approximately belongs to the set $D_y \subset Y$ given by a user. The problem may be decomposed into two levels.

Figure 1. Two-level system

1. Upper level problem: Given $R(x_1, \dots, x_k, y; b)$, $h_b(b)$ and D_y , find

$$x^* = \arg \max_x v[D_y(x; \bar{b}) \subseteq D_y] \stackrel{\Delta}{=} v(x)$$

where $x = (x_1, \dots, x_k)$ and $D_y(x; \bar{b})$ is the set of all possible y for the fixed

x. It may be shown [1, 2] that

$$v(x) = \max_{b \in D_b(x)} h_b(b)$$

where $D_b(x) = \{b \in B : (x, y) \in R(x, y; b) \rightarrow y \in D_y\}$.

2. Lower level problem: Given $R_j(u_j, x_j; c_j)$, $h_{c_j}(c_j)$ and D_{x_j} , find u_j^* maximizing $v(\bar{x}_j \tilde{\in} D_{x_j})$; $j \in \overline{1, k}$. The sets D_{x_j} are such that $D_{x_1} \times D_{x_2} \times \dots \times D_{x_k} \subseteq D_x$ where D_x is the set of solutions x^* of the upper level problem. The solution is analogous to that for the upper level.

3. TASK ALLOCATION

The approach presented in the previous section may be applied to a problem of the tasks allocation in the group of parallel processors with uncertain execution times. We assume that the global computational task to be distributed may be decomposed into N separate parts (programs or parts of programs) which may be executed simultaneously by the separate processors. Each partial task is characterized by an upper bound of the execution time τ_j for j -th processor ($j = 1, 2, \dots, k$), and τ_j is assumed to be the same for each partial task. The decision problem consists in the determination of the numbers of the partial tasks $n_1, n_2, \dots, n_k$ assigned to the processors taking into account the execution time $T = \max\{T_1, T_2, \dots, T_k\}$ where T_j is the execution time for j -th processor; $n_1 + \dots + n_k = N$. If N is sufficiently large, we can determine the decisions $u_j \in R^+$ (any positive numbers) and then obtain n_j by rounding off u_j to the nearest integer. To apply the notation in the previous section let us denote $T_j \triangleq x_j$, $T \triangleq y$, $\tau_j \triangleq c_j$. Then the knowledge representation of our distributed system is as follows: $x_j \leq c_j u_j$, $y = \max\{x_1, x_2, \dots, x_k\}$. The unknown parameters c_j are assumed to be values of uncertain variables $\bar{c}_j$ described by the certainty distributions $h_{c_j}(c_j)$ given by an expert estimating the execution times for the partial tasks. The **decision problem** may be formulated as follows: Given α and $h_{c_j}(c_j)$, $j \in \overline{1, k}$, find the

allocation $u^* = (u_1^*, u_2^*, \dots, u_k^*)$ maximizing the certainty index of the property: "the user's requirement is approximately satisfied", i.e. "the approximate value of the execution time $T = y$ is less than α ". Then

$$u^* = \arg \max_u v(T \lesssim \alpha)$$

where " $T \lesssim \alpha$ " = " $D_T(u) \subseteq [0, \alpha]$ ", $D_T(u)$ is the set of possible values T for the fixed u and $v[T \lesssim \alpha] = v[(T_1 \lesssim \alpha) \wedge \dots \wedge (T_k \lesssim \alpha)] = \min v_j(u_j)$ where

$$v_j(u_j) = v[T_j \lesssim \alpha] = \max_{c_j \in D_{c_j}(u_j)} h_{c_j}(c_j) \quad (3)$$

and $D_{c_j}(u_j)$ is described by the inequality $c_j \leq \alpha u_j^{-1}$.

The procedure of the determination of u^* is then the following:

1. To determine $v_j(u_j)$ according to (3). 2. To determine $u_1^*, u_2^*, \dots, u_k^*$ by solving the maximization problem

$$\max_{u_1, \dots, u_k} \min\{v_1(u_1), \dots, v_k(u_k)\}$$

subject to constraints $u_1 + \dots + u_k = N$. 3. To round off the values u_j^* and to obtain the number n_j^* .

Numerical examples and simulations showed a significant influence of the parameters in the certainty distributions on the final results.

NOTES

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