OBSERVATION EQUIVALENCES FOR THE SEMANTICS OF INHERITANCE

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Abstract. In the object-oriented languages, *inheritance* is a fundamental relation among classes, that originally indicated the *is-a* relation; however it has been often used for *code-reuse*. The integration between object-orientation and concurrency gives the possibility of using theoretical tools, such as the notions of action and state observation equivalences and preorders, developed within concurrency theories (Petri nets, CCS). In this paper we propose a semantic characterisation of two forms of inheritance in concurrent context: in particular, we propose a preorder based on action observation to give the semantics of inheritance used as *is-a* relation (type inheritance), and a preorder based on state observation for the semantics of inheritance as *code-reuse* (implementation inheritance).

Keywords: object-orientation, concurrency, inheritance, Petri nets, observation equivalences.

Introduction

The main entity of the object-oriented languages is the *object*, which is generally defined by a class. A fundamental relation between classes is *inheritance*, that originally indicated the *is-a* relation among classes. On the other hand, the success of object-oriented programming languages is due to the use of inheritance for *code-reuse*. Inheritance is often used in these two different ways. The aim of this paper is studying these two different faces of the inheritance relation in concurrent object-oriented languages. The unclear distinction between these two different interpretations of the notion of inheritance is misleading in object oriented software development and, as far as we know, does not find solution in the literature.

The integration of object-orientation with concurrency, yielding systems of active

objects modelled as concurrent processes, gives the possibility of using theoretical tools, such as the notions of observation preorders and equivalences, for the study of the inheritance relation. In particular, the place-transition duality in Petri nets [9] allows the definition of two types of preorders and equivalences between nets, based on action observation and on state observation [13]. A variety of observation preorders and equivalences have been already used for defining the semantics of the inheritance relation. Among the various proposals in the literature (see for a survey [12], [2]), we take into consideration those proposed by Nierstrasz [11] and van der Aalst and Basten [1] for what concerns action observability, while we consider the approach of CLOWN [5] for what concerns state observability.

Nierstrasz's hypothesis is that the sequences of requests that an object can accept constitute a regular language. Moreover, Nierstrasz defines a preorder based on *failures*, the *Request Substitutability* preorder (RS-preorder, $\leq_{RS}$), and considers this preorder as the semantics of the subtyping relation according to Wegner and Zdonik's substitutability principle [15]. Van der Aalst and Basten introduce preorders too, based on the *branching bisimulation* equivalence, and two operators, the *encapsulation* operator δ_H and the *abstraction* operator τ_1 , that removes and respectively labels as not observable, the transitions corresponding to the methods of the subclass not inherited from the superclass.

CLOWN is a concurrent object-oriented specification language whose basic building block is the elementary class, which specifies sequential objects through a set of instance data and methods, a typed interface and its (multiple) inheritance link(s). A labelled net associated with each elementary class describes the casual relationship between methods execution. The definition of the inheritance relation between classes is based on the State Transformation preorder between the corresponding nets. The State Transformation preorder (ST-preorder, $\leq_{ST}$) [14] compares systems with respect to their state space.

In this paper we develop the proposal sketched in [7] and propose a semantic characterisation of two forms of inheritance in concurrent contexts, based on two preorders: preorders are better suited than equivalence notions for modelling the behaviour extension.

According to the terminology used in [10], we use the term *type inheritance* to denote subtyping, i.e. the *is-a* relation among classes, and the term *implementation inheritance* to denote code-reuse. According to [3], *type inheritance* concerns and compares the external observable behaviour of objects; *implementation inheritance* does the same with respect to their internal structure. As a consequence, type inheritance will be based on a new action-based preorder resulting from an original integration of Nierstrasz's and van der Aalst and Basten's proposals. Implementation inheritance will be based on the State Transformation preorder.

We omit here to deal with instance data, (typed) interfaces, object creation and other topics which are typical issues for the design of a concurrent object-oriented specification language, and have been partially addressed in CLOWN. As usable formalisms require sound notions, here we make a step backward and, according to Nierstrasz, focus on basic behaviour.

The paper is organized as follows: in the section "*basic definitions*", we give some basic definitions about the notions based on action and state observation, in the section "*examples and open problems*" we discuss the inheritance relations proposed in the literature through examples, in the section "*the preorders based on action observability*" we define new preorders based on action observability, in the section

“soundness of the preorders based on action observability” we prove the soundness of these preorders, while in the section “inheritance as subtyping and code reuse” we give the definitions of type inheritance and implementation inheritance and show their use on the examples introduced in the section “examples and open problems”.

Basic definitions

We consider Elementary Net (EN) systems [9], the basic class of Petri Nets. Formally, a net is a 3-tuple (B, E, F) , where B is a finite set of places, E is a finite set of transitions such that $B \cap E = \emptyset$ and $B \cup E \neq \emptyset$, and $F \subseteq (B \times E) \cup (E \times B)$ is a transition function such that $\text{dom}(F) \cup \text{ran}(F) = B \cup E$. Given $x \in X = B \cup E$, the pre-set of x is the set $\bullet x = \{y \in X : (y, x) \in F\}$ and the post-set of x is the set $x \bullet = \{y \in X : (x, y) \in F\}$.

An EN systems is a 4-tuple (B, E, F, c_{in}) , where (B, E, F) is a net and $c_{in} \subseteq B$ is the initial case. Given an EN system $\Sigma = (B, E, F, c_{in})$, $e \in E$ is enabled at the case $c \subseteq B$, denoted $c|e>$, iff $\bullet e \subseteq c$; if e is enabled at c , then the occurrence of e leads from c to a case c' , denoted $c|e>c'$, iff $c|e>$ and $c' = (c - \bullet e) \cup e \bullet$. Given $w = e_1 e_2 \dots e_n \in E^*$, w is enabled at the case $c \subseteq B$, denoted $c|w>$, iff $\exists c_1, c_2, \dots, c_{n-1}$ such that $c|e_1>c_1|e_2>c_2 \dots c_{n-1}|e_n>$; the occurrence of w leads from c to c' , denoted $c|w>c'$, iff $c|e_1>c_1|e_2>c_2 \dots c_{n-1}|e_n>c'$. The set of cases, denoted C_Σ , is the smallest subset of 2^B such that $c_{in} \subseteq C_\Sigma$, and $\forall c \in C_\Sigma, \forall e \in E, \forall c' \subseteq B, \forall e \in E \mid c|e>c' \Rightarrow c' \in C_\Sigma$.

We denote as LEN systems the labelled EN systems in which the events represent actions which can be observable. Formally, a LEN system is a 5-tuple $\Sigma = (B, E, F, c_{in}, h)$, where (B, E, F, c_{in}) is a contact free EN system (i.e. such that $\forall e \in E, \forall c \in C_\Sigma \mid \bullet e \subseteq c \Rightarrow e \bullet \cap c = \emptyset$) whose underlying net is such that $E \subseteq \text{dom}(F) \cap \text{ran}(F)$, and $h: E \rightarrow L \cup \{\tau\}$ is the labelling function (L is the alphabet of observable actions and $\tau \notin L$ denotes the unobservable action).

Let ϵ be the empty word of E^* and λ the empty word of L^* . h is extended to an homomorphism $h^*: E^* \rightarrow L^*$ in the following way: $h^*(\epsilon) = \lambda$, and $\forall e \in E, \forall w \in E^* \mid h(e) \neq \tau \Rightarrow h^*(ew) = h(e)h^*(w) \wedge h(e) = \tau \Rightarrow h^*(ew) = h^*(w)$. In the following, we denote h^* by h .

Given a LEN system $\Sigma = (B, E, F, c_{in}, h)$, $v \in L^*$ is enabled at the case c , denoted $c|v>$, iff $\exists w \in E^* \mid h(w) = v \wedge c|w>$. If v is enabled at c , then the occurrence of v can lead from c to c' , denoted $c|v>c'$, iff $\exists w \in E^* \mid h(w) = v \wedge c|w>c'$.

According with the assumption of disregarding data and focusing on basic behaviour, we consider labelled EN systems, observing actions and states.

For what concern action observability, we associate with a class a LEN system $\Sigma = (B, E, F, c_{in}, h)$, where the labelling function h associates with each event an exported class method belonging to L . The internal methods are labelled by τ .

Now we give some basic definitions based on action observability.

Definition 1 (traces)

Let $\Sigma = (B, E, F, c_{in}, h)$ be a LEN system. Then $\text{traces}(\Sigma) = \{ w \in L^* \mid c_{in} |w> \}$ (i.e.: the traces are the sequences of requests that an object can accept).

Definition 2 (failures)

Let $\Sigma = (B, E, F, c_{in}, h)$ be a LEN system. Then $\text{failures}(\Sigma) = \{ (w, X), w \in L^*, X \subseteq L \mid \exists c \mid c_{in} |w>c \forall a \in X \text{ not}(c|a>) \}$.

Definition 3 (*relative failures*)

Let $\Sigma_1 = (B_1, E_1, F_1, c_{in1}, h_1)$ and $\Sigma_2 = (B_2, E_2, F_2, c_{in2}, h_2)$ be LEN systems. The set of the failures of Σ_1 relative to Σ_2 is defined as:

$$failures_{\Sigma_2}(\Sigma_1) = \{ (w, X) \in failures(\Sigma_1) \mid w \in traces(\Sigma_2) \}.$$

Now we give the definition of Nierstrasz's Request substitutability preorder [11].

Definition 4 (*Request Substitutability preorder*)

Let $\Sigma_1 = (B_1, E_1, F_1, c_{in1}, h_1)$ and $\Sigma_2 = (B_2, E_2, F_2, c_{in2}, h_2)$ be LEN systems. $\Sigma_1 \leq_{RS} \Sigma_2$ if and only if

- i) $traces(\Sigma_2) \subseteq traces(\Sigma_1)$
- ii) $failures_{\Sigma_2}(\Sigma_1) \subseteq failures(\Sigma_2)$.

The *encapsulation* operator δ_H [1] removes some transitions from a LEN system. Formally, given a LEN system $\Sigma = (B, E, F, c_{in}, h)$, for each $H \subseteq L$, $\delta_H(\Sigma) = \Sigma'$ such that $\Sigma' = (B, E', F', c_{in}, h')$, $E' = \{e \in E \mid h(e) \notin H\}$, $F' = F \cap ((B \times E') \cup (E' \times B))$ and $h' = h \cap (E' \times L)$.

The *abstraction* operator τ_I labels as not observable some transitions of a LEN system. Formally, given a LEN system $\Sigma = (B, E, F, c_{in}, h)$, for each $I \subseteq L$ $\tau_I(\Sigma) = \Sigma'$ such that $\Sigma' = (B, E, F, c_{in}, h')$, and, for each $e \in E$, $h(e) \in I \Rightarrow h'(e) = \tau$ and $h(e) \notin I \Rightarrow h'(e) = h(e)$.

Now we give some basic definitions based on state observability [14].

A pair (Σ, O) (also denoted with Σ) is an S-labelled system iff $\Sigma = (B, E, F, c_{in})$ is an EN system, $O \subseteq B$ is such that $\forall s \in O \exists c$ belonging to the set of cases C_Σ such that $s \in c$ and $c \subseteq O$ (O is the set of the observable places, or conditions), and $c_{in} \subseteq O$. $C_{obs}(\Sigma) = \{c \in C_\Sigma : c \subseteq O\}$ is the set of observable cases. For all $w \in E^*$, the set of firable permutation, denoted $F-Perm(w)$, is the set of permutations w' of w such that $\exists c, c' \in C_\Sigma : c[w] > c'$. Let Σ be an S-labelled system, $w \in E^*$ and $c, c' \in C_{obs}(\Sigma)$ such that $c[w] > c'$; then w is an *elementary observable path* from c to c' iff $\forall u_1, \dots, u_k \in E \mid u_1 \dots u_k \in F-Perm(w) \wedge c[u_1] > c_1 \dots c_{k-1}[u_k] > c' \Rightarrow \forall i = 1, \dots, k-1 \ c_i \notin C_{obs}(\Sigma)$.

W_Σ denotes the set of elementary observable paths.

Given an S-labelled system (Σ, O) , its *observable local state transformation algebra* (OLST algebra), denoted I , is the tuple $\langle I, -, \cap, \cup, c_{in}, \rightarrow \rangle$, where $\langle I, -, \cap, \cup \rangle$ is the observable local state algebra, where I is the smallest set contained in the power set of O that contains $C_{obs}(\Sigma)$ and that is closed under the operation of set difference $-$; $\rightarrow \subseteq I \times I$ is an unreflexive relation called *observable local state transformation relation* and defined as: $\forall x, y \in I \ x \rightarrow y \Leftrightarrow \exists w \in W_\Sigma \mid \bullet w \subseteq x \wedge y = (x - \bullet w) \cup w \bullet$, where $\bullet w$ and $w \bullet$ are respectively the pre-set and the post-set of w , i.e.: the minimal set of conditions giving concession to the word w and the set of conditions which become true after the occurrence of the word w .

Now we give the definition of the State Transformation preorder ($\leq_{ST}$) [14], which compares systems by observing local states and is such that $\Sigma_1 \leq_{ST} \Sigma_2$ if and only if the state space of Σ_1 is a substructure of the state space of Σ_2 such that for any observable local state transformation in Σ_1 there is a corresponding observable local state transformation in Σ_2 .

Definition 5 (*State Transformation preorder*)

Let Σ_1 and Σ_2 be two S-labelled systems, $I_1 = \langle I_1, -, \cap, \cup, c_{in1}, \rightarrow_1 \rangle$ and $I_2 = \langle I_2, -, \cap, \cup, c_{in2}, \rightarrow_2 \rangle$ be the respective OLST algebras. Then $\Sigma_1 \leq_{ST} \Sigma_2$ if and

only if there exists an injective morphism $h: \langle I_1, -, \cap, \cup \rangle \rightarrow \langle I_2, -, \cap, \cup \rangle$ such that:

- i) $h(c_{in1}) \subseteq c_{in2}$
- ii) $\forall x, y \in I_1 [x \rightarrow_1 y \Rightarrow [h(x) \rightarrow_2 h(y) \vee \exists n \geq 1, \exists i_1, i_2, \dots, i_n \in (I_2 - h(I_1)) h(x) \rightarrow_2 i_1 \rightarrow_2 \dots i_n \rightarrow_2 h(y)]]$.

When the condition $[x \rightarrow_1 y \Rightarrow h(x) \rightarrow_2 h(y)]$ is satisfied, then the S-labelled system Σ_2 is an *extension* of Σ_1 , while when the condition $[x \rightarrow_1 y \Rightarrow \exists n \geq 1, \exists i_1, i_2, \dots, i_n \in (I_2 - h(I_1)) h(x) \rightarrow_2 i_1 \rightarrow_2 \dots i_n \rightarrow_2 h(y)]$ is satisfied, then the S-labelled system Σ_2 is an *expansion* of Σ_1 .

Example 1

Let us consider the systems given in figure 1, where shaded places are the observable ones, and let us consider mappings of observable places which are represented by using the same name for corresponding places. Then, the net systems Σ_1 and Σ_2 are ST-equivalent, since $\Sigma_1 \leq_{ST} \Sigma_2$ and $\Sigma_2 \leq_{ST} \Sigma_1$. Analogously, the systems Σ_4 and Σ_5 are equivalent.

$\Sigma_2 \leq_{ST} \Sigma_3$, in fact Σ_3 performs the analogous state-transformation from p_1 to p_2 as Σ_2 does, and, moreover, performs a further state transformation from p_2 to p_3 .

Analogously, $\Sigma_3 \leq_{ST} \Sigma_4$ as Σ_4 extends the state transformations of Σ_3 by adding a branch.

In this figure, considering the S-labelled systems Σ_1 and Σ_3 , where $\Sigma_1 \leq_{ST} \Sigma_3$, if we construct the morphism h that associates p_1 (of Σ_1) with p_1 (of Σ_3) and p_2 (of Σ_1) with p_2 (of Σ_3), then Σ_3 is an extension of Σ_1 ; while, if we construct the morphism k that associates p_1 (of Σ_1) with p_1 (of Σ_3) and p_2 (of Σ_1) with p_3 (of Σ_3), then Σ_3 is an expansion of Σ_1 .

Figure 1 (Example 1). State transformation preorders and equivalences.

Examples and open problems

Our aim here is to stress, through examples, the two different aspects and uses of inheritance relation, i.e.: the *is-a* relation as subtyping and the *implementation* inheritance as code reuse. In the literature inheritance has been formalized without a clear distinction between these two forms.

In the first example, we consider Nierstrasz's RS-preorder, based on action observability, where a class B is subclass of a class A iff each sequence of requests accepted by A is also accepted by B, and, if the class A, after having received a sequence s , can receive another request r , then also B, after the sequence s , can accept the request r .

In the second example, we consider the ST-preorder, based on state observability, where a class B is subclass of a class A iff the S-labelled system associated with B is an extension and/or an expansion of the S-labelled system associated with A.

In both cases, the examples exhibit some counter-intuitive feature, that we point out.

Example 2

This example is taken from Nierstrasz's paper [11]. In figure 2, the class VAR, that represents variables, is subclass of the class BUF, that represents one place buffers. Rephrasing Nierstrasz from CCS to Petri nets, the RS-preorder between the LEN system associated with the class VAR and the LEN system associated with the class BUF is satisfied, i.e. $\Sigma_{VAR} \leq_{RS} \Sigma_{BUF}$.

However, we believe that this class hierarchy is somehow misleading: while, according to Nierstrasz, a variable *is* seen as *a* (specialization of a) buffer, programmers intuition is that a buffer can be *implemented* through a variable. Then we want to relate the class BUF and the class VAR through a notion of *implementation inheritance* rather than through an *is-a* relation.

Figure 2 (Example 2). Nierstrasz's example.

Figure 3 (Example 3). CLOWN's multiple inheritance hierarchy between printers.

Example 3

This example is taken from [5]. Figure 3 shows a multiple inheritance hierarchy between printers. In CLOWN, this hierarchy is studied through the ST-preorder, which considers the state (place) observation. However, observing states is somehow counter-intuitive: in fact, e.g., the class LOCKED PRINTER is subclass of PRINTER since it inherits the methods “load”, “print” and “reset”, and is subclass of LOCKER as it inherits the methods “lock” and “unlock”. This is to say that one could wish to build that hierarchy observing methods (actions) rather than states. By observing states, it is possible to capture and compare the internal structure of objects, while in the hierarchy in the figure behaviour comparison requires action observation.

Indeed, the state labelling policy defined in [5] considers the state observability with respect to the exportability of methods associated with transitions. Then, the transition observability is indirectly considered.

Unfortunately, using Nierstasz's RS-preorder one can't get the previous hierarchy, since the class LOCKED PRINTER isn't subclass of LOCKER.

Also according to van der Aalst and Bastens's approach, that combines the *branching bisimulation equivalence* with the *encapsulation* and *abstraction* operator (see [1]), the class PRINTER can't be a subclass of the class MONO PRINTER. The *life-cycle inheritance* defined by van der Aalst and Basten is in fact a too strong relation, that can't be applied in many cases.

The preorders based on action observability

As said in the introduction, *type inheritance* concerns and compare the external observable behaviour of objects; *implementation inheritance* does the same with respect to their internal structure. Moreover, we believe that preorders are better suited than equivalence notions for comparing behaviour.

Therefore, we base the *type inheritance* on an action observation preorder, while we base *implementation inheritance* on a state observation preorder. In particular, the first preorder is based on *failure*, since *bisimulation* distinguishes systems also with respect to some aspect of their internal structure; in the second case we consider ST-preorder.

While this latter does not require modifications w.r.t. the original definition, we need to integrate the two approaches by Nierstrasz and van der Aalst and Basten, since no one of them is fully satisfactory, as shown by the examples in the previous section. To this purpose we define a further operator, namely the *renaming* operator $\rho_{R,S}$, and new preorders based on action observation.

Formally, for each $R \subseteq L$ and for each set of labels S such that $S \cap L = \emptyset$ and $|S| = |R|$, we define a bijective function $s : R \rightarrow S$, that maps each element of R into a new label. Let L' be a superset of S containing $L - R$, i.e. $S \subseteq L'$ and $L - R \subseteq L'$. On the basis of the function s we define the *renaming* function:

$r : L \rightarrow L'$ that maps each label into a new label in the following way:

- (1) $r(x) = x$ if $x \notin R$
- (2) $r(x) = y$ if $x \in R$, $y \in S$ and $s(x) = y$.

This function r is injective, since the function s is bijective.

We define the *renaming* operator $\rho_{R,S}$ in the following way:

$\rho_{R,S}(\Sigma) = \Sigma'$ such that $\Sigma' = (B, E, F, c_{in}, h')$, where $h' = r \circ h$.

The renaming function r is extended in the following way: $r^* : L^* \rightarrow L'^*$, such that, for each $w \in L^*$ and for each $w' \in L'^*$, $w = v_1 v_2 \dots v_n$, $w' = v'_1 v'_2 \dots v'_n$, $r^*(w) = w'$ iff $r(v_1) = v'_1$, $r(v_2) = v'_2$, $\dots$, $r(v_n) = v'_n$.

Now we can give the definitions of four preorders based on Nierstrasz's RS-preorder and on the *encapsulation*, *abstraction* and *renaming* operators. In the following, Σ_A and Σ_B are LEN systems associated with a class A and a class B, while L_A and L_B are the sets of event labels associated respectively with Σ_A and Σ_B and corresponding to the methods of the classes A and B.

The first preorder, the *strong substitutability*, is equivalent to the RS-preorder.

Definition 6

Σ_B is less or equal to Σ_A w.r.t. *strong substitutability*, denoted $\Sigma_B \leq_{SS} \Sigma_A$, if and only if there exists a set $H \subseteq L_B$ such that $\delta_H(\Sigma_B) \leq_{RS} \Sigma_A$.

If $\Sigma_B \leq_{SS} \Sigma_A$ then an object of class A can be substituted by an object of class B and the environment will not be able to notice the difference, i.e.: if an object of class

A may accept after a request sequence w another request a , then an object of class B must accept the request a after the request sequence w , whatever is the reached state.

The new methods added in class B are considered as not available through the encapsulation operator δ_H .

Example 4

In figure 3, $\Sigma_{\text{MONO PRINTER}} \leq_{SS} \Sigma_{\text{ROOT}}$, $\Sigma_{\text{LOCKER}} \leq_{SS} \Sigma_{\text{ROOT}}$, $\Sigma_{\text{PRINTER}} \leq_{SS} \Sigma_{\text{MONO PRINTER}}$, $\Sigma_{\text{LOCED PRINTER}} \leq_{SS} \Sigma_{\text{PRINTER}}$, but not ($\Sigma_{\text{LOCKED PRINTER}} \leq_{SS} \Sigma_{\text{LOCKER}}$). In fact, the methods "load" and "reset" are not inherited from the class LOCKER, and if we apply the encapsulation operator to the corresponding transitions, the transitions corresponding to the methods "suspend" and "resume", inherited (and renamed) from LOCKER, become unreachable.

The second preorder, the strong substitutability with renaming, is less restrictive than strong substitutability.

Definition 7

Σ_B is less or equal to Σ_A w.r.t. *strong substitutability with renaming*, denoted $\Sigma_B \leq_{SSR} \Sigma_A$, if and only if there exist a set $H \subseteq L_B$, a set $R \subseteq L_A$ and a set $S \subseteq L_B$ such that $S \cap L_A = \emptyset$ and $\delta_H(\Sigma_B) \leq_{RS} \rho_{R,S}(\Sigma_A)$.

If $\Sigma_B \leq_{SSR} \Sigma_A$, then an object of class A can be substituted by an object of class B but the environment must make allowance for class A methods that are renamed in class B by the renaming operator $\rho_{R,S}$.

The third preorder, the weak substitutability, is another extension of the strong substitutability.

Definition 8

Σ_B is less or equal to Σ_A w.r.t. *weak substitutability*, denoted $\Sigma_B \leq_{WS} \Sigma_A$, if and only if there exist a set $H \subseteq L_B$ and a set $I \subseteq L_B$ such that :

$$I \cap H = \emptyset \text{ and } \tau_I \circ \delta_H(\Sigma_B) \leq_{RS} \Sigma_A.$$

If $\Sigma_B \leq_{WS} \Sigma_A$, then an object of class A can be substituted by an object of class B and the environment will not be able to notice the difference since the new methods added in class B are either considered unobservable, through the abstraction operator τ_I , or considered unavailable, through the encapsulation operator δ_H .

The fourth and last preorder, the weak substitutability with renaming, is the weaker preorder.

Definition 9

Σ_B is less or equal to Σ_A w.r.t. *weak substitutability with renaming*, denoted $\Sigma_B \leq_{WSR} \Sigma_A$, if and only if there exist a set $H \subseteq L_B$, a set $I \subseteq L_B$, a set $R \subseteq L_A$ and a set $S \subseteq L_B$ such that $I \cap H = \emptyset$, $S \cap L_A = \emptyset$ and $\tau_I \circ \delta_H(\Sigma_B) \leq_{RS} \rho_{R,S}(\Sigma_A)$.

If $\Sigma_B \leq_{WSR} \Sigma_A$, then an object of class A can be substituted by an object of class B and the environment must consider the renamed methods, while the new methods are either considered unobservable, through the abstraction operator τ_I , or considered unavailable, through the encapsulation operator δ_H .

Example 5

In figure 3, $\Sigma_{\text{LOCKED PRINTER}} \leq_{\text{WSR}} \Sigma_{\text{LOCKER}}$. In fact, $I = \{\text{load, reset}\}$, $R = \{\text{lock, unlock}\}$, $S = \{\text{suspend, resume}\}$, $r(\text{create}) = \text{create}$, $r(\text{leave}) = \text{leave}$, $r(\text{lock}) = \text{suspend}$, $r(\text{unlock}) = \text{resume}$.

First of all, we show some properties of the RS-preorder. The RS-preorder is a congruence for the *encapsulation* operator δ_H , as demonstrated in the following lemma 1, and for the renaming operator $\rho_{R,S}$ (see lemma 2) while it isn't a congruence for the *abstraction* operator τ_I , as shown in the example 6. The proofs are in [4] or in the internal report [8].

Lemma 1

Let Σ_1 and Σ_2 be two LEN systems such that $\Sigma_1 \leq_{\text{RS}} \Sigma_2$. Let L_1 and L_2 be the set of labels associated respectively with Σ_1 and Σ_2 . Let $L = L_1 \cup L_2$. Then, for each set $H \subseteq L$, it holds $\delta_H(\Sigma_1) \leq_{\text{RS}} \delta_H(\Sigma_2)$.

Lemma 2

Let Σ_1 and Σ_2 be LEN systems such that $\Sigma_1 \leq_{\text{RS}} \Sigma_2$. Let L_1 be the set of labels corresponding to Σ_1 and L_2 be the set of labels corresponding to Σ_2 . Let $L = L_1 \cup L_2$. Then, for each set $R \subseteq L$ and for each set S such that $S \cap L = \emptyset$, it holds $\rho_{R,S}(\Sigma_1) \leq_{\text{RS}} \rho_{R,S}(\Sigma_2)$.

Example 6

The RS-preorder isn't a congruence for the abstraction operator τ_I . In figure 4, $\Sigma_1 \leq_{\text{RS}} \Sigma_2$, but, if we choose $I = \{c\}$, then $\text{not}(\tau_I(\Sigma_1) \leq_{\text{RS}} \tau_I(\Sigma_2))$. In fact, the failure $(a, \{a, b\}) \in \text{failures}_{\tau_I(\Sigma_2)}(\tau_I(\Sigma_1))$, but $(a, \{a, b\}) \notin \text{failures}_{\tau_I(\Sigma_2)}$.

Figure 4 (Example6). RS-preorder isn't a congruence for τ_I .

As a consequence, the *abstraction* operator τ_I must be applied to a particular set of labels I that is constructed in the following way. Let the class B be a subclass of the

class A. Let L_B and L_A be the sets of labels corresponding to class B and class A methods. The *abstraction* operator τ_1 is applied to a subset B of class methods that are not inherited from the class A, i.e. $I \subseteq L_B - L_A - S$. The methods whose name is a label belonging to S are inherited from the class A, then the *abstraction* operator can't be applied to them. The set I is constructed in the following way:

- (1) $I = \emptyset$, i.e. the set I is initialized to the empty set;
- (2) for each $w \in \text{traces}(\Sigma_A)$, if $w \in \text{traces}(\Sigma_B)$ then I doesn't change, otherwise, let $w' \in \text{traces}(\Sigma_B)$ be the shorter string such that $w' = u_1 z_1 u_2 z_2 \dots u_{n-1} z_{n-1} u_n$ and $w = v_1 v_2 \dots v_{n-1} v_n$, where $v_i \in L_A^*$, $u_i \in L_B^*$, $u_i = r^*(v_i)$, $i = 1, \dots, n$, $z_k \in (L_B - L_A - S)^*$, $k = 1, \dots, n-1$. Then the elements of $z_1, z_2, \dots, z_{n-1}$ are added to I.

Of course, if no methods of the superclass is renamed in the subclass, $S = \emptyset$ and $u_i = v_i$ for each $i = 1, \dots, n$.

On the basis of the *renaming* operator and of the *failure equivalence* we can define another equivalence relation, the *failure equivalence with renaming*. This definition will be useful in order to prove the soundness of the strong substitutability with renaming and of the weak substitutability with renaming.

Definition 10 (*failure equivalence with renaming*)

Let Σ_1 and Σ_2 be LEN systems. Σ_1 is equivalent to Σ_2 , denoted $\Sigma_1 \approx_{FR} \Sigma_2$, iff there exist a set $R \subseteq L_2$ and a set $S \subseteq L_1$ such that $S \cap L_2 = \emptyset$ and $\Sigma_1 \approx_F \rho_{R,S}(\Sigma_2)$ (or $\Sigma_2 \approx_F \rho_{S,R}(\Sigma_1)$).

The proof that this definition is sound is given in [4] or in [8].

The following theorem compares the four preorders.

Theorem 1

Let Σ_A and Σ_B be two LEN systems associated respectively with a class A and a class B, and let L_A and L_B be the respective sets of labels.

- i) If $\Sigma_B \leq_{SS} \Sigma_A$, then $\Sigma_B \leq_{SSR} \Sigma_A$
- ii) If $\Sigma_B \leq_{SS} \Sigma_A$, then $\Sigma_B \leq_{WS} \Sigma_A$
- iii) If $\Sigma_B \leq_{SSR} \Sigma_A$, then $\Sigma_B \leq_{WSR} \Sigma_A$
- iv) If $\Sigma_B \leq_{WS} \Sigma_A$, then $\Sigma_B \leq_{WSR} \Sigma_A$.

Figure 5. The relations between preorders.

Soundness of the preorders based on action observability

In the following we demonstrate that the definitions 6, 7, 8, 9 are sound, i.e.: that the relationships so defined are indeed preorders. The extended proofs are in [4] or in the internal report [8]; in the following we give only the main results.

Theorem 2 (*strong substitutability*)

The strong substitutability $\leq_{SS}$ is a preorder.

The following theorem shows that the strong substitutability is equivalent to the RS-prorder.

Theorem 3

Let Σ_1 and Σ_2 be LEN systems. Then $\Sigma_1 \leq_{RS} \Sigma_2$ if and only if $\delta_H(\Sigma_1) \leq_{RS} \Sigma_2$, where $H = L_1 - L_2$.

In order to demonstrate that the definition 7 (*strong substitutability with renaming*) is sound, we must present two lemmas. The second one shows that the strong substitutability ($\leq_{SS}$) is a congruence for the renaming operator $\rho_{R,S}$.

Lemma 3

Let Σ be a LEN system and let L be the set of labels associated with Σ . Then for each set $H \subseteq L$, for each set $R \subseteq L$ and for each set S such that $S \cap L = \emptyset$, let $\rho_{R,S}$ a *renaming* operator associated with R and S , and let $r: L \rightarrow L'$ be the corresponding *renaming* function. If $H' = r(H)$, then $\rho_{R,S} \circ \delta_H(\Sigma) = \delta_{H'} \circ \rho_{R,S}(\Sigma)$.

Lemma 4

Let Σ_1 and Σ_2 be LEN systems such that $\Sigma_1 \leq_{SS} \Sigma_2$. Let L_1 and L_2 be the set of labels associated with Σ_1 and Σ_2 and let $L = L_1 \cup L_2$. Then, for each set $R \subseteq L$ and for each set S such that $S \cap L = \emptyset$, it holds $\rho_{R,S}(\Sigma_1) \leq_{SS} \rho_{R,S}(\Sigma_2)$.

Now we can show the soundness of definition 7.

Theorem 4 (*strong substitutability with renaming*)

The strong substitutability with renaming $\leq_{SSR}$ is a preorder.

Theorem 5 (*weak substitutability*)

The weak substitutability $\leq_{WS}$ is a preorder.

In order to demonstrate that the definition 9 (*weak substitutability with renaming*) is sound, we must present two lemmas. The second one shows that the weak substitutability ($\leq_{WS}$) is a congruence for the renaming operator $\rho_{R,S}$.

Lemma 5

Let Σ be a LEN system and let L be the set of labels associated with Σ .

Then, for each set $H \subseteq L$, for each set $I \subseteq L$ such that $H \cap I = \emptyset$, for each set $R \subseteq L$ and for each set S such that $S \cap L = \emptyset$, let $\rho_{R,S}$ be a *renaming* operator associated with R and S , and let

$r: L \rightarrow L'$ be the corresponding *renaming* function.

If $H' = r(H)$ and $I' = r(I)$, it holds: $\rho_{R,S} \circ \tau_I \circ \delta_H(\Sigma) = \tau_{I'} \circ \delta_{H'} \circ \rho_{R,S}(\Sigma)$.

Lemma 6

Let Σ_1 and Σ_2 be LEN system such that $\Sigma_1 \leq_{WS} \Sigma_2$. Let L_1 and L_2 be the sets of labels associated with Σ_1 and Σ_2 and let $L = L_1 \cup L_2$. Then, for each set $R \subseteq L$ and for each set S such that $S \cap L = \emptyset$, it holds $\rho_{R,S}(\Sigma_1) \leq_{WS} \rho_{R,S}(\Sigma_2)$.

Theorem 6 (*weak substitutability with renaming*)

The weak substitutability with renaming $\leq_{WSR}$ is a preorder.

Inheritance as subtyping and code reuse

We now formalise the notion of type inheritance and implementation inheritance, and illustrate them on the hierarchies considered in Examples 2 and 3.

Type inheritance is based on the weak substitutability with renaming defined in the section “*the preorders based on action observability*” (see def. 9). A class B is a subclass of a class A if there is an *is-a* relation between the subclass and the superclass. An object of class A can be substituted by an object of class B, but the environment must take into account the renamed methods and the new methods added in the subclass.

Definition 11

Let Σ_A and Σ_B be LEN systems associated with a class A and a class B. Then B is *subclass* of A with respect to *type inheritance* if and only if $\Sigma_B \leq_{WSR} \Sigma_A$.

Figure 6 (Examples 7 and 8). Inheritance as subtyping and code reuse.

Example 7

In figure 6, the net associated with the class BUF represents the behaviour of one place buffers, while the net of the class VAR represents the behaviour of variables. The net of class V-BUF represents the behaviour of one place buffers implemented through a variable, which inherits from both classes BUF and VAR. This captures programmers intuition who use variables to implement one cell buffers preserving the exhibited behaviour.

Let's verify that the corresponding definitions hold. The class V-BUF is subclass of BUF with respect to *type inheritance*, as it preserves the requests that can be accepted from clients. In fact, between the subclass and the superclass the strong substitutability holds: $\Sigma_{V-BUF} \leq_{SS} \Sigma_{BUF}$ (and then, for theorem 1, also the weak substitutability with renaming holds: $\Sigma_{V-BUF} \leq_{WSR} \Sigma_{BUF}$). In particular, $H = \emptyset$, since in the subclass every method is inherited from the superclass, then $\Sigma_{V-BUF} \leq_{RS} \Sigma_{BUF}$. This is true, since $traces(\Sigma_{BUF}) \subseteq traces(\Sigma_{V-BUF})$ and $failures_{\Sigma_{BUF}}(\Sigma_{V-BUF}) \subseteq failures(\Sigma_{BUF})$.

Implementation inheritance is based on the State Transformation preorder (see def. 5). If a class B is a subclass of a class A, then the system associated with B must be an extension or an expansion of the system associated with class A, i.e. the internal structure of the class B is an extension or an expansion of the internal structure of the class A.

Definition 12

Let Σ_A and Σ_B be EN systems, associated with a class A and a class B, in which some appropriate places are observable. Then B is *subclass* of A with respect to *implementation inheritance* if and only if $\Sigma_A \leq_{ST} \Sigma_B$.

Example 8

In figure 6, we must now verify that the class V-BUF is subclass of VAR with respect to *implementation inheritance*: $\Sigma_{VAR} \leq_{ST} \Sigma_{V-BUF}$, as the VAR methods "assign" and "get_item" are used for implementing the buffer's "put" and "get" methods.

To prove that the preorder holds, all places are considered observable. Then, the sets of observable cases are:

$$I_{VAR} = \{\emptyset, \{\text{unborn}_{VAR}\}, \{\text{alive}_{VAR}\}, \{\text{idle}_{VAR}\}, \{\text{dead}_{VAR}\}\},$$

$$I_{V-BUF} = \{\emptyset, \{\text{unborn}_{V-BUF}\}, \{\text{full}_{V-BUF}\}, \{\text{empty}_{V-BUF}\}, \{\text{idle}_{V-BUF}\}, \{\text{dead}_{V-BUF}\}\}.$$

The morphism h is constructed in the following way:

$$h(\emptyset) = \emptyset, h(\{\text{unborn}_{VAR}\}) = \{\text{unborn}_{V-BUF}\}, h(\{\text{alive}_{VAR}\}) = \{\text{alive}_{V-BUF}\},$$

$$h(\{\text{idle}_{VAR}\}) = \{\text{full}_{V-BUF}\} \text{ (or } h(\{\text{idle}_{VAR}\}) = \{\text{empty}_{V-BUF}\}),$$

$$h(\{\text{dead}_{VAR}\}) = \{\text{dead}_{V-BUF}\}.$$

The property of ST-preorder are satisfied:

$$(1) h(\{\text{unborn}_{VAR}\}) = \{\text{unborn}_{V-BUF}\};$$

$$(2) \{\text{unborn}_{VAR}\} \rightarrow_{VAR} \{\text{alive}_{VAR}\} \Rightarrow \{\text{unborn}_{V-BUF}\} \rightarrow_{V-BUF} \{\text{alive}_{V-BUF}\},$$

$$\{\text{alive}_{VAR}\} \rightarrow_{VAR} \{\text{idle}_{VAR}\} \Rightarrow \{\text{alive}_{V-BUF}\} \rightarrow_{V-BUF} \{\text{full}_{V-BUF}\},$$

$$\{\text{idle}_{VAR}\} \rightarrow_{VAR} \{\text{idle}_{VAR}\} \Rightarrow \exists \{\text{empty}_{V-BUF}\} \in (I_{V-BUF} - h(I_{VAR}))$$

$$\text{such that } \{\text{full}_{V-BUF}\} \rightarrow_{V-BUF} \{\text{empty}_{V-BUF}\} \rightarrow_{V-BUF} \{\text{full}_{V-BUF}\},$$

$$\{\text{idle}_{VAR}\} \rightarrow_{VAR} \{\text{dead}_{VAR}\} \Rightarrow \exists \{\text{empty}_{V-BUF}\} \in (I_{V-BUF} - h(I_{VAR})) \text{ such that}$$

$$\{\text{full}_{V-BUF}\} \rightarrow_{V-BUF} \{\text{empty}_{V-BUF}\} \rightarrow_{V-BUF} \{\text{dead}_{V-BUF}\}.$$

Example 9

The classes of the printer hierarchy in figure 3 satisfy the given definition. That is, the hierarchy is now captured observing methods instead of states.

In fact, as shown in the example 4, the $\leq_{SS}$ relation holds for the ordered pairs (MONO PRINTER, ROOT), (LOCKER, ROOT), (PRINTER, MONO PRINTER), (LOCKED PRINTER, PRINTER). Therefore, because of theorem 1 (ii and iv), for these pairs the $\leq_{WSR}$ relation holds too. Moreover, as shown in the example 5, $\Sigma_{LOCKED\ PRINTER} \leq_{WSR} \Sigma_{LOCKER}$.

Conclusions

In this paper we propose preorders based on action observation for characterising type inheritance, and based on state observation for characterising implementation inheritance. To this purpose, we defined a new preorder based on action observation, namely the *weak substitutability with renaming* preorder, assuming Nierstrasz's hypothesis that the sequences of requests that an object can accept constitute a regular language. The result is promising as the two inheritance relations so defined support class hierarchies which look natural and appropriate to solve critical examples in the literature.

The (major) open problem is the characterisation of an object behaviour and, consequently, the definition of the inheritance relations, when the Nierstrasz's constraint is removed, i.e., when the sequences of requests that an object can accept constitute a non-regular language. In this case, the behaviour would be described by an high-level net, with guards associated with transitions. (In CLOWN, e.g., the class semantics, i.e. the object behaviour, is described by modular algebraic high level nets [6]). The problem is not at all trivial since the notions of equivalence and preorder between nets are defined in terms of EN or PT systems, i.e. low level systems, while there aren't equivalences defined for high level models. Moreover, the proposed notions of type and implementation inheritance would be embedded into a concurrent object oriented specification language: we intend to reuse CLOWN as the framework to achieve this goal.

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