MULTI-AGENT BASED SUPPLY CHAIN MANAGEMENT WITH MARKET EMERGENCE PHENOMENON

Toshiya Kaihara

University of Marketing and Distribution Sciences, Japan

kaihara@umds.ac.jp

Supply chain management is categorised into a discrete resource allocation problem amongst several business units, and requires an optimal solution in terms of product distribution. Market-Oriented programming, which integrates market economics concept into multi-agent paradigm, solves product distribution problem by allocating the scheduled resources according to market prices. We formulate supply chain model as virtual market with the aggregation of agents based on the market-oriented programming, and demonstrate the applicability of economic analysis to this framework.

1. INTRODUCTION

During the last few years the focus has shifted from factory level to enterprise level due to the increasing global presence of the companies. Supply Chain Management (SCM) is now recognised as one of the best means by which enterprises can make instant improvements to their business strategies in virtual enterprise (VE) environment (Fisher, 1994). Sharing of information is central infrastructure to the optimisation of resource allocation (i.e. product distribution) in the supply chain. SCM is generally based on simple TOC (Theory Of Constraints) with throughput-based costing method, and would conduct effective strategies in the enterprise level by DBR(Drum, Buffer and Lope) concept (Goldratt, 1983).

The management of physical product flows amongst the nodes in the supply chain comes under the intensive study of effective operation in SCM. Resource allocation is a quite important operational criterion at workshop level in SCM. As the number of potential business units in the supply chain increases, an effective management on product distribution plays more important role in dynamic environment. Current SCM concept does not deal with the problem, because TOC is nothing to do with the combinatorial optimisation problem in resource allocation.

Recently the utilisation of multi-agent system in large-sized complex system increases (Deneubourg, 1991). Multi-agent paradigm has several characteristics, such as autonomy, pro-activeness, social ability, and emergence. In the paradigm a global goal of the whole system is achieved as the aggregation of their local objectives with their negotiation. In supply chain networks each business unit behaves independently and autonomously with simple goals of achieving myopic views of local optimum. The situation is quite similar to distributed decision making mechanism in multi-agent paradigm, and it is natural to model supply chain

networks in a multi-agent programming. In such an environment, each agent represents the independent business unit with conflicting and competing individual requirements, and may possess localised information relevant to its utilities. To recognise this independence, we treat the business units as agents, ascribing each of them autonomy to decide how to deploy resources under their control in service of their interests.

Within this model, a distributed SCM can be analysed according to how well it exhibits the following properties:

- Self-interest agents can make effective decisions with local information, without knowing the private information and strategies of other agents.
- The method requires minimal communication overhead.
- Solutions do not waste resources. If there is some way to make some agent better off without harming others, it should be done. A solution that cannot be improved in this way is called Pareto optimal.

Assuming that a resource allocation problem in SCM must be decentralised in considering a practical application, market concept can provide several advantages:

- Markets are naturally distributed and agents make their own decisions about how to bid based on the prices and their own utilities of the goods.
- Communication is limited to the exchange of bids and process between agents and the market mechanism.
- Since agent must back their representations with exchange offers, some mechanism can elicit the information necessary to achieve Pareto and system optima in some well-categorised situations.

Market-oriented programming is a multi-agent based concept to facilitate distributed problem solving. In market-oriented programming we take the metaphor of an economy computing multi-agent behaviour literally, and directly implement the distributed computation as a market price system. In a market-oriented programming approach to distributed problem solving, the resource allocation for a set of computational agents is derived by computing competitive market of an artificial economy (Wellman, 1996), (Kaihara *et al.*, 1999a), (Kaihara, 1999b).

In this paper we formulate supply chain model as a discrete resource allocation problem with supply/demand agents, and demonstrate the applicability of economic analysis to this framework by simulation experiments. Finally we prove the market mechanism can provide several advantages on resource allocation in SCM.

2. MARKET-ORIENTED PROGRAMMING

2.1 Basic Concept

In economics, the concept of a set of interrelated goods in balance is called general equilibrium. The general equilibrium theory guarantees a Pareto optimal solution at competitive equilibrium in perfect competitive market (Layard *et al.*, 1978). The connection between computation and general equilibrium is not all foreign to economists, who often appeal to the metaphor of market systems computing the activities of the agents involved (Shoven *et al.*, 1992).

The theory of general equilibrium provides the foundation for a general approach to the construction of distributed planning system based on price mechanism. In this

approach the constituent planning agents are regarded as suppliers and demanders in an artificial economy. Their individual activities are defined in terms of production and consumption of resources. Interactions amongst agents are cast as exchanges, the terms of which are mediated by the underlying economic mechanism, or protocol.

2.2 Bidding mechanism

Market-oriented programming is the general approach of deriving solutions to distributed resource allocation problems by computing the competitive equilibrium of an artificial economy (Wellman, 1996), (Kaihara *et al.*, 1999a). It involves an iterative adjustment of prices based on the reactions of the agent in the market. Bidding mechanism in market-oriented programming is shown in Figure 1.

Let $P_t(s)$ be the price of resource s at time t . ' f_{tms} ' and ' g_{tms} ' represent the supply function of supplier m on resource s at time t and the demand function of demander n on resource s at time t , respectively.

The bidding mechanism computes an equilibrium price in each separate market. It involves an iterative adjustment of prices based on reactions of agents in the market. Agent s submits supply and demand functions (f_{tms} and g_{tms}) and the auction adjusts individual prices to clear, rather than adjusting the entire price vector by some increment. The mechanism associates an auction with each distinct resource. Agents act in the market by submitting bids to auctions. In this paper bids specify a correspondence between prices and quantities of the resource that the agent offers to demand or supply as a basic study. Given bids from all interested agents, the auction derives a market-clearing price.

Each agent maintains an agenda of bid tasks, specifying in which it must update its bid or compute a new one. The bidding process is highly distributed, in that each agent need communicate directly only with the auctions for the resources of interest. Each of these interaction concerns only a single resource; the auctions never coordinate with each other. Agents need not negotiate directly with other agents, nor even know of each other's existence.

Figure 1 Product Distribution Basic Model

Agent activities in terms of products required and supplied are defined so as to reduce an agent's decision problem to evaluate the tradeoffs of acquiring different products in market-oriented programming. These tradeoffs are represented in market prices, which define common scale of value across the various products. The problem for designers of computational markets is to specify the strategy by which agent interactions determine prices (Wellman, 1996). Obviously supply chain model is well structured for market-oriented programming, and that means the proposed concept takes advantage of the theory. A Pareto optimal solution, which is conducted by microeconomics, is attainable in resource allocation problem in SCM.

3. AGENT DEFINITIONS

3.1 Preliminaries

Several variables to formulate agent utilities in this paper are defined as follows:

- x_{ki} : Input of resource i in agent k
- y_{kj} : Yield of resource j in agent k
- P_i : Purchase price of resource i per unit
- P_i : Sales price of resource i per unit
- E_k : Profit function of agent k
- C_k : Cost of agent k
- S_k : Sales of agent k
- C_k : Budget of agent k
- $l(i,j)(=n)$: Index of production function from resource i to resource j
- f_{kl} : l th production function in agent k
- x_{kl} : Input resource amount into production function f_{kl}
- y_{kl} : Output resource amount from production function f_{kl}
- E_{kl} : Profit by production function f_{kl}
- c_{kl} : Cost by production function f_{kl}
- s_{kl} : Sales by production function f_{kl}

3.2 Production function

Suppose supply agent k has a production function f_k described in equation (1).

$$Y_k = f_k(X_k) \quad (1)$$

where X_k and Y_k denote a set of input resources and yield resources in agent k described in equation (2) and (3), respectively.

$$X_k = \{x_{k1}, \Lambda, x_{kn}\}, \quad Y_k = \{y_{k1}, \Lambda, y_{kn}\} \quad (2),(3)$$

In this paper we adopt Cobb-Douglas function (Layard *et al.*, 1978) as a production function described in equation (4). Since Cobb-Douglas function handles economical scale in the market by index constant b , and in $0 < b < 1$ the production function is defined as a concave down function, in other words, a diminishing returns function. If production function is defined as concave down, market prices are established at a predictable level in the general equilibrium theory.

$$y = ax^b \quad (\text{where } 0 < a, 0 < b < 1) \quad (4)$$

Then the production function f_{kl} of agent k for input-output resource set $l(i, j) = l$ is given by

$$y_{kl} = f_{kl}(x_{kl}) = a_{kl} x_{kl}^{b_{kl}} \quad (5)$$

where x_{kl} and y_{kl} denote the amount of input resource i for f_{kl} and the amount of yield resource j for f_{kl} , respectively. Then x_{ki} and y_{kj} are defined as

$$x_{ki} = \sum_j x_{kl(i,j)}, \quad y_{kj} = \sum_i y_{kl(i,j)} \quad (6),(7)$$

3.3 Profit function

Suppose a set of single unit purchase prices for a resource set $\{x_{k1}, \dots, x_{km}\}$ is $\{p_1, \dots, p_m\}$, and a set of single unit sales prices for a resource set $\{y_{k1}, \dots, y_{kn}\}$ is $\{P_1, \dots, P_n\}$, then the total production cost C_k of agent k is defined as

$$C_{kl(i,j)} = p_i x_{kl} \quad , \quad C_k = \sum_i c_{kl} \quad (8),(9)$$

and the total sales of agent k is defined as

$$S_{kl(i,j)} = P_j y_{kl} \quad , \quad S_k = \sum_j s_{kl} \quad (10),(11)$$

Then the profit function E_k of agent k is finally acquired as

$$E_{kl} = S_{kl} - C_{kl} \quad , \quad E_k = \sum_l E_{kl} \quad (12),(13)$$

3.4 Budget constraint

In this paper budget constraint of each agent is considered so as to realise our market model. Suppose the maximum budget of agent k is $\max C_k$, then we have

$$C_k = \sum_l c_{kl} \leq \max C_k \quad (14)$$

and agent k should behave to maximise its profit E_k autonomously.

The basic principle of agents is to maximise their profits under the budget constraints. Their activities should follow the newly proposed theorem, named Profit Maximise Theorem, shown below.

[Theorem] Profit function E_k of agent k is maximised by minimised r_k , which satisfies the following conditions:

$$\forall l: \frac{\partial E_k}{\partial c_{kl}} = r_k \quad (r_k \geq 0) \cap \quad C_k \leq \max C_k \quad (15)$$

subject to

- f_k is differentiable in any $x \in X_k$

$$- \forall l: \left. \frac{\partial f_{kl}}{\partial x_{kl}} \right|_{x_{kl}=x} > \left. \frac{\partial f_{kl}}{\partial x_{kl}} \right|_{x_{kl}=x+\Delta} \quad (16)$$

We have the following equation (17) by equation (8), (10), (12), (13).

$$\begin{aligned} \frac{\partial E_k}{\partial c_{kl(i,j)}} &= \frac{\partial E_{kl}}{\partial c_{kl}} = \frac{\partial}{\partial c_{kl}} (s_{kl} - c_{kl}) \\ &= \frac{\partial}{\partial c_{kl}} [P_j f_{kl}(c_{kl}/p_i) - c_{kl}] \\ &= \frac{P_j}{p_i} f'_{kl}(c_{kl}/p_i) - 1 \end{aligned} \quad (17)$$

The proof of the theorem is given in Appendix.

3.4 Supply / demand functions

Since Cobb-Douglass function shown in (4) is differentiable and

$$\frac{\partial f_{kl}}{\partial x_{kl}} = a_{kl} b_{kl} x_{kl}^{b_{kl}-1} > a_{kl} b_{kl} (x + \Delta)^{b_{kl}-1} > 0 \quad (18)$$

then the proposed product function (5) perfectly satisfies the conditions (16). Demand function x_{kl} , which maximises the agent's profit, is obtained by the Profit Maximise Theorem as follows:

$$\begin{aligned} \frac{\partial E_k}{\partial c_{kl(i,j)}} &= \frac{\partial E_{kl}}{\partial c_{kl}} = \frac{\partial}{\partial c_{kl}} [P_j a_{kl} (c_{kl} / p_i)^{b_{kl}} - c_{kl}] \\ &= a_{kl} b_{kl} P_j c_{kl}^{b_{kl}-1} p_i^{-b_{kl}} - 1 = a_{kl} b_{kl} P_j p_i^{-1} x_{kl}^{b_{kl}-1} - 1 = r_k \end{aligned} \quad (19)$$

Then we have

$$x_{kl(i,j)} = [p_i (r_k + 1) / a_{kl} b_{kl} P_j]^{1/b_{kl}-1} \quad (20)$$

Supply function y_{kl} , which maximises the profit, is also obtained by equation (5), (20) as follows:

$$y_{kl(i,j)} = a_{kl} [p_i (r_k + 1) / a_{kl} b_{kl} P_j]^{b_{kl}/b_{kl}-1} \quad (21)$$

We denote a concrete meaning of the Profit Maximise Theorem. It is obvious to maximise the production function f_{kl} at $r_k = 0$ by (19), because the function is defined as concave down type. Then agent k has the maximum profit at $r_k = 0$, if it satisfies the budget constraint ($C_k \leq_{\max} C_k$). However if agent k breaks the budget constraint ($C_k >_{\max} C_k$), then it has to reduce some amount of input resource x_{kl} to satisfy the constraint. The theorem conducts it should adjust the amount of input resources to have the equivalent value of $\partial E_{kl} / \partial c_{kl}$ in all the production function f_{kl} . If the value r_k increases, the amount of the demanded resources decreases, and that leads to reduce the cost C_k . Then the minimised r_k in $C_k \leq_{\max} C_k$ leads to maximise the profit function E_k .

4. EXPERIMENTAL RESULTS

4.1 Experimental model

A basic SCM model shown in Figure 2 is prepared to investigate a validity of the proposed approach by computer simulation. The model has a series of three-layered market structure with two-layered agent groups. This model comprises the three types of agent in each layer and three types of good. The interconnectedness of agents and goods defines the market configuration. Comparative analysis of the three market structures reveals the qualitatively distinct economic and computational behaviours realised by the proposed configurations.

Each agent has production functions to transform the resource from market ($M[i][j]$) to market ($M[i+1][j]$), and the parameters are defined as $a[j]$, $b[j]$ in equation (4). The parameters in each agent group are described in Table 1. The parameter b is set in common to each type of the goods, because this parameter is deeply concerned with the price elasticity of the goods. In this figure the outside

demand function and the outside supply function, which are correspond to sink and source in the experimental model, are defined respectively as

$$x_i = \alpha_i p_i^{\beta_i} \quad (\alpha_i > 0, \beta_i < -1) \quad , \quad y_j = \alpha_j p_j^{\beta_j} \quad (\alpha_j > 0, \beta_j > 0) \quad (22), (23)$$

and each parameter in (22), (23) is described in Table 2.

Table 1. Production function parameters

—	a[0]	b[0]	a[1]	b[1]	a[2]	b[2]	Budget
A[0][0]	5	0.7	5	0.5	5	0.3	10
A[0][1]	3	0.7	8	0.5	3	0.3	10
A[0][2]	7	0.7	7	0.5	7	0.3	4
A[1][0]	10	0.3	10	0.6	10	0.4	50
A[1][1]	4	0.3	4	0.6	4	0.4	100
A[1][2]	8	0.3	4	0.6	4	0.4	6

Table 2. Outside production function

	$\alpha[0]$	$\beta[0]$	$\alpha[1]$	$\beta[1]$	$\alpha[2]$	$\beta[2]$
Supply function	100	1.5	100	1.5	100	1.5
Demand function	100	-2	100	-2	100	-2

Figure 2 Supply chain model

4.2 Market dynamism

Dynamical changes of i) the amount of dealing goods in supply and demand, ii) the prices of the goods, at each layer in the market structure are shown in figure 3, figure 4, respectively.

First of all, it is obvious that both the amount of dealings and the prices are converged into equilibrium in these figures. Since our methodology is perfectly endorsed by 'general equilibrium theory' in the competitive market, we can get a Pareto optimal solution in the equilibrium. That means the goods distribution policy followed by the acquired solutions, i.e. the amounts and the prices, are Pareto optimal in the entire market. Efficient SCM with market mechanism are attainable by the proposed approach.

Secondly it is observed that the number of iterations required to reach equilibrium seems to rise with the price elasticity. For example, Market [0][0] with 0.7 in price elasticity takes longer time to converge than Market [0][2] with 0.3 in price elasticity in figure 3(a). We attribute this to the natural characteristic of price elasticity, and the experimental values completely agree with the general theory in market economics.

Finally it takes longer time to converge into the equilibrium at the market in the middle layer, Market [1][..], compared with the other markets, Market [0][..] and Market [2][..]. This observation is explained by the following reason. We applied the functions described in (22) and (23) as outside functions in Market [2][..] and

Market [0][..], and they are defined as static functions in the experimental model. On the other hand, Market [1][..] is operated by supply and demand agents with dynamic utility functions. As a result, Market [1][..] behaves dynamically and is more sensitive to trading situation in the competitive market.

(a) Market[0][0] □ ₪

(b) Market[1][0] □ ₪

(c) Market[2][0] □ ₪

(a) Market[0][0] □ ₪

(b) Market[1][0] □ ₪

(c) Market[2][0] □ ₪

Figure 3 Supply and demand oscillation

Figure 4 Price oscillation

4.3 Market equilibrium

The market equilibrium dynamism should be explained by our market definition. The comparison between figure 3(a) and figure 4(a) shows us that the good with larger trading amount has higher price in the equilibrium in Market [0][..]. That is because we defined the function shown in (23) as outside supply function, which characterises positive correlation between the dealing amount and the price. On the

other hand, the good with larger trading amount has lower price in the equilibrium in Market[2][..] shown in figure 3(c) and figure 4(c). The outside demand function defined in (22), which has negative correlation between the dealing amount and the price, influences the equilibrium. These experimental values are perfectly explicable by our market formulation. The middle layered market, Market [1][..] has more complex dynamism in figure 3(b) and figure 4(b). The dynamism is emerged and explained by the agent utility parameters shown in Table 1. In this case, Market [1][1] is high both on the dealing amount and on the price in the equilibrium. A set of Agent [0][..] has to offer higher sales price to increase their profit, because $b[1]$ in Agent [1][..] ($=0.6$) is the highest amongst $b[.]$ in the second layer, but $b[1]$ in Agent [0][..] ($=0.5$) is lower than $b[0]$ in the first layer.

It has been clarified that all the experimental values in the complex SCM model are perfectly explicable by our market formulation. The experimental results agree with the theoretical trends of perfect competitive market in microeconomics. It is obvious that each market is perfectly competitive and holds market mechanism in general equilibrium.

5. CONCLUSIONS

In this paper we proposed a SCM with market economics. We formulated SCM as distributed resource allocation systems, based on general equilibrium theory and competitive mechanism. The approach works by deriving the competitive equilibrium corresponding to a particular configuration of agents and markets. After defined production functions, we introduced budget constraint for practical use and newly proposed Profit Maximise Theorem as an agent strategy. It has been confirmed by simulation experiments that careful constructions of the decision process according to economic principles can lead to efficient distributed resource allocation in SCM, and the behaviour of the system can be analysed in economic terms.

The contribution of the paper lies in the idea of SCM based on market-oriented programming, an algorithm for distributed computation of competitive equilibria of computational economics, and an initial illustration of the approach on a simple SCM. Effective SCM in global environment is expected by this research.

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APPENDIX

[Theorem] Profit function E_k of agent k is maximised by minimised r_k , which satisfies the following conditions:

$$\forall l : \frac{\partial E_k}{\partial c_{kl}} = r_k \quad (r_k \geq 0) \quad \cap \quad C_k \leq_{\max} C_k \quad (A1)$$

[Proof]

Note that
$$\frac{\partial f_{kl}}{\partial x_{kl}} = \frac{\partial f_{kl}}{\partial c_{kl}} \cdot \frac{\partial c_{kl}}{\partial x_{kl}} = p_i \frac{\partial f_{kl}}{\partial c_{kl}} \quad (A2)$$

then
$$\left. \frac{\partial f_{kl}}{\partial c_{kl}} \right|_{c_{kl}=c} > \left. \frac{\partial f_{kl}}{\partial c_{kl}} \right|_{c_{kl}=c+\Delta} \quad \text{in any positive value } \Delta \quad (A3)$$

Also, note that
$$E_{kl} = P_j f_{kl}(x_{kl}) - p_i x_{kl} = P_j f_{kl}(c_{kl}/p_i) - c_{kl} \quad (A4)$$

then E_{kl} is regarded as concave down by (A3). E_{kl} is maximised with the condition (A5).

$$\forall l : \frac{\partial E_k}{\partial c_{kl}} = 0 \quad (A5)$$

Let C'_k denote total expense in (A4) and if $C'_k \leq_{\max} C_k$, then the maximum profit is given with (A5). Otherwise the maximum profit is not given with (A5) due to the budget constraint. If $C'_k >_{\max} C_k$, agent k must consider to increase C_{kl} by Δc , and reduce $C_{kl'}$ by Δc shown in (A6), (A7).

$$\left. \frac{\partial E_k}{\partial c_{kl}} \right|_{c_{kl}=c+\Delta c} = \frac{P_j}{p_i} f'_{kl}[(c_{kl} + \Delta c)] - 1 = r_{kl} \quad (A6)$$

$$\left. \frac{\partial E_k}{\partial c_{kl'}} \right|_{c_{kl'}=c-\Delta c} = \frac{P_h}{p_g} f'_{kl'}[(c_{kl'} - \Delta c)] - 1 = r_{kl'} \quad (A7)$$

then we obtain that $r_{kl} < r_k < r_{kl'}$ from (A3).

Let ΔE_{kl} and $\Delta E_{kl'}$ denote the increased profit in f_{kl} and the diminished profit in $f_{kl'}$, respectively, then

$$\Delta E_{kl} = \alpha \Delta c \quad (r_{kl} < \alpha < r_k) \quad , \quad \Delta E_{kl'} = \beta \Delta c \quad (r_k < \beta < r_{kl'}) \quad (A8)$$

It is obtained that $\Delta E_{kl} < \Delta E_{kl'}$ from (A8), and that means the diminished profit is greater than the increased profit in any Δc .

Therefore the profit function E_k of agent k is maximised with the condition

$$\forall l : \frac{\partial E_k}{\partial c_{kl}} = r_k \quad (r_k \geq 0) \quad \text{in} \quad C_k \leq_{\max} C_k \quad (A9)$$

Finally, r_k is minimised with the condition (A9), since the profit function E_k is concave down. This completes the proof.