A Fast Parametric Model for Contact-Substrate Coupling

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Abstract This paper presents a fast and yet accurate modeling method for substrate coupling between a device contact and a substrate backplane. Effects of the physical parameters and geometrical characteristics of the contact and the substrate on the model are reported. We have derived model expressions for extraction of circuit model elements of the substrate. The method is very efficient for speed, memory usage, and is relevant for implementing in CAD tools. We have validated the proposed model over a wide range of frequencies up to 20 GHz.

1. INTRODUCTION

The continuous trend toward a higher IC density and a greater speed introduces the serious problem of the substrate coupling especially in mixed-signal circuits. Substrate-coupled noise introduced by fast switching digital circuits may disturb other devices in sensitive analog circuits and cause functionality failures [Schmerbeck et al., 1991].

Recently, different modeling methods for substrate coupling have been published. In finite element methods (FEM) [Johnson et al., 1984; Stanislac et al., 1994; Vergheze et al., 1993] the entire bulk of the substrate is discretized to small volumes. Thus, FEMs can handle multi-layer substrates that contain different doping profiles, wells, etc. However, such methods are impractical for anything but simple problems [Stanislac et al., 1994]. This occurs because these methods encounter a huge model matrix produced by full discretization of the substrate. Boundary element methods (BEM) [Smedes et al., 1995; Gharpurey

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and Meyer, 1996] require discretization of only the surface of the contacts on the substrate and also the backplane contact, if it exists. Although the model matrix dimensions are smaller than those of the FEMs, it is still dense. Therefore, both approaches present difficulties because of long computational times.

Another problem is that in these methods the substrate is usually assumed to be purely resistive. The substrate coupling modeling approaches presented in [Su et al., 1993; Joardar, 1994] are good examples of this regard. Therefore, these techniques are valid only for low frequencies up to a few GHz.

In this work, we have investigated the details of modeling contact-to-substrate coupling and have proposed accurate and simple expressions for fast extraction of model elements. These expressions are functions of the contact geometry, as well as the thickness and physical parameters of the substrate. We are concerned with coupling from only a single contact to the substrate. As a matter of fact, the problem of substrate coupling in a multi-contact substrate with a ground backplane can be reduced to the problem of one-contact substrate when the coupling between contacts are negligible because of a large separation between them. We have used IE3D software (Zeland Software, Inc.) [Zeland Software, 1999] to verify the accuracy of our model.

In this paper, Section 2. presents general concepts of the substrate coupling. In section 3., the modeling technique for square-contact substrates is described. Section 4. addresses a general formulation method for rectangular contacts. We also expand on our modeling method, in section 5., to include 2-layer substrates.

2. GENERAL CONCEPTS

We consider a single-layer substrate as shown in Fig. 1. This is the case in most high-resistivity and low-resistivity substrates. In high-resistivity substrates, the high resistive bulk is the only common substrate for the devices. In low-resistivity substrates, the epitaxial layer is the only active medium, whereas the very low resistive bulk acts as a common node for the devices [Su et al., 1993].

In this paper, a contact between a typical device and the substrate is modeled by a conductive plate as shown in Fig. 1. *Contacts* (or ports) correspond to the areas where the designed circuit interacts with the substrate. Examples of these contacts include possible noise sources and receptors, such as contacts from a substrate or wells to supply lines, n^+ buried layer in BJTs, drain/source areas in MOS transistors, etc. The substrate circuit model consists of a parallel combination of a capacitor, C_{sub} , and a conductance, G_{sub} . These elements, in general, are nonlinear functions of a number of variables as shown below.

$$C_{sub} = f_1(a, b, h, \rho, \epsilon_r, f) \quad (1.1)$$

$$G_{sub} = f_2(a, b, h, \rho, \epsilon_r, f) \quad (1.2)$$

Figure 1 One-contact substrate and its equivalent circuit $B1$ for three different values of ρ_{sub}

where a and b are dimensions of the contact, and f stands for the frequency. Also, h , ρ , and ϵ_r represent thickness, resistivity, and relative permittivity of the substrate, respectively. In all simulations we have assumed an infinite substrate and the frequency range of 0.1 GHz to 20 GHz. Moreover, we have applied the input source along one edge of the contact.

3. MODELING APPROACH FOR SQUARE CONTACT

3.1 SUBSTRATE RESISTIVITY AND SUBSTRATE HEIGHT DEPENDENCY

The plots in Fig. 2 illustrate simulation results for the substrate with different values of the resistivity. The values of resistivity, ρ_{sub} , between $10 \Omega\text{-cm}$ to $40 \Omega\text{-cm}$ are commonly used for both high-resistivity and low-resistivity substrates. For each substrate with a constant resistivity, the substrate height, h , has been varied over the range of $1 \mu\text{m}$ to $300 \mu\text{m}$. As this figure shows, the substrate susceptance, $B1$, varies linearly with the frequency. Thus, assuming $B1 = 2\pi f C_{sub}$, we consider that C_{sub} is independent of the frequency. Another notable result is that C_{sub} is a constant function of the substrate resistivity, ρ_{sub} , at least for all resistivity values greater than $10 \Omega\text{-cm}$, as Fig. 2 illustrates.

To extract the substrate model elements, we define the *capacitance-factor*, K_C , and the *conductance-factor*, K_G , as shown below.

$$K_{C, sim} = \frac{C_{sim}}{C_{PP}} \quad (1.3)$$

$$K_{G, sim} = \frac{G_{sim}}{G_{DC\Box}} \quad (1.4)$$

where C_{sim} and G_{sim} are the rigorous substrate capacitance and the substrate conductance, respectively, extracted from simulation results. Also C_{PP} stands

for the parallel plate capacitance given as, $C_{PP} = \epsilon_0 \epsilon_r \frac{A}{h}$, and $G_{DC\Box}$ denotes the conductance of the rectangular prism bulk under the contact given as, $G_{DC\Box} = \frac{1}{\rho} \cdot \frac{A}{h}$.

Figs. 3 and 4 show the variation of $K_{C,sim}$ and $K_{G,sim}$, respectively, versus the *shape-factor* ξ which is defined as $\xi = \frac{\sqrt{A}}{h}$. As indicated in these figures, the plots for the frequencies 0.1 GHz and 20 GHz are almost distinguishable. Finally, Fig. 2.5 illustrates a comparison between $K_{C,sim}$ and $K_{G,sim}$. As this figure indicates, we note that $K_{C,sim}$ and $K_{G,sim}$ are nearly equal (for h variable and A constant).

3.2 CONTACT SIZE DEPENDENCY

In the second stage, we investigate the silicon substrate structures with a constant height, h , while the contact area, A , is varied. Contact dimensions are varied from $10 \times 10 \mu m^2$ to $80 \times 80 \mu m^2$.

Figs. 2.6 and 2.7 illustrate values of the substrate capacitance C_{sim} compared with those of C_{PP} , and values of the substrate conductance G_{sim} compared with those of $G_{DC\Box}$, respectively. We have plotted both $K_{C,sim}$ and $K_{G,sim}$ in Fig. 2.8 for comparison. Considering this figure, we again observe that $K_{C,sim}$ and $K_{G,sim}$ are nearly equal (for h constant and A variable).

Fig. 2.9 compares values of $K_C(\xi)$ obtained from two different sets of substrate structures introduced in the previous sections. Clearly, the values of two plots are in very good agreement. Therefore, for any value of A and h (within the range we have studied) one can write: if $\xi_1 = \xi_2 \rightarrow K_C(\xi_1) \approx K_C(\xi_2)$. The effects of ϵ_r and ρ_{sub} are taken into account by C_{PP} and $G_{DC\Box}$, respectively. Finally, we have applied curve fitting technique to plots for $K_{C,sim}$. The following is the closed form relations obtained.

$$K_{C\Box} = 5.1675 + \frac{0.6342}{\xi}, \quad 0.01 \leq \xi < 0.15 \quad (1.5)$$

K_C and K_G

$$K_{C\Box} = 1.2538 + \frac{1.1783}{\xi}, \quad 0.15 \leq \xi \leq 1.5 \quad (1.6)$$

$$K_{C\Box} = 1.1658 + \frac{1.0604}{\xi}, \quad 1.50 < \xi \leq 10 \quad (1.7)$$

where $\Box$ implies a square contact. For simplicity, we call the above expressions $K_{C\Box}$ model expressions. Furthermore, We have derived single expression model, which approximates the above model expressions, as shown below.

$$K_{C\Box}^s = 1.7855 + \frac{0.85896}{\xi}, \quad 0.01 \leq \xi \leq 10 \quad (1.8)$$

Observing Fig. 2.10 we see that the plots for model expressions agree with those of simulations.

4. RECTANGULAR CONTACTS

Based on an extensive study of simulated data we have derived the *general parametric model expression*, F_C , which can be applied to any rectangular contact. The model expression F_C is defined as

$$F_C = \frac{C_{sub}}{C_{PP}} \quad (1.9)$$

Assuming $a \leq b$, for the contact excited from the edge, a , the capacitance C_{sub} is approximated as

$$C_{sub} = K_{C\Box} C_{PPa} + \frac{2\tilde{a}}{3a} (K_{C\Box} - 1) C_{PPa} \kappa_{edge} + C_{PP\tilde{a}} \quad (1.10)$$

where $\tilde{a} = b - a$, $C_{PPa} = (a/b)C_{PP}$, and $C_{PP\tilde{a}} = (\tilde{a}/b)C_{PP}$. Also κ_{edge} is the factor that accounts for the variations of the edge effects of the contact and

Figure 2.10: K_C for two different set of structures

Figure 2.11: K_C extracted from IE3D simulation results, our model expressions (2.5), (2.6), (2.7), and the single model expression (2.8)

is given as

$$\kappa_{edge} = 1.2 + 0.4 e^{-0.5 \frac{\sqrt{a \cdot \tilde{a}}}{a}} \quad (1.11)$$

Eventually, the parametric model F_C is given as

$$F_C = \frac{K_{C\Box}}{b} \left(a + \frac{2}{3} \tilde{a} \kappa_{edge} \right) + \frac{\tilde{a}}{b} \left(1 - \frac{2}{3} \kappa_{edge} \right) \quad (1.12)$$

We have compared the results and accuracy of our method with those of IE3D simulation results in Fig. 2.11. This figure indicates that our model agree with simulation results.

4.1 ADVANTAGES AND LIMITATIONS OF THE METHOD

Table 2.1 summarizes the results of the computation times, and the memory requirements for a single-contact substrate by the three methods. The major advantage of our method is its higher speed over other published methods in [Gharpurey and Meyer, 1995] and [Costa et al., 1999]. Another significant feature of the proposed model is that the memory requirements for storing the input data and computations is negligible compared to other methods.

One limitation of the proposed model is the assumption regarding the infinite substrate. However, this matter doesn't affect the adaptability of the method. The rough guideline reported in [Smedes et al., 1995] suggests that the sidewall effects may be neglected, if the contact is farther than twice the epi-layer thickness from the edges of the chip. The other limitation with the proposed model is that it doesn't extract the coupling between the contacts. As mentioned, the focus of this paper is to thoroughly examine single-contact substrates.

Table 1.1 Comparison of the computation times required for extraction of substrate coupling elements for a single-contact substrate.

Modeling Method	Comput. Time (sec)	Speedup Factor	Mem. Usage
The Model F_C (M1)	353×10^{-6}	$\frac{M1}{M3} = 17785$ $\frac{M1}{M2} = 1185$	Negligible
Eigendecomposition (M2) [Costa et al., 1999]	Not Reported	$\frac{M2}{M3} = 15$	Considerable
Green's Function (M3) [Gharpurey and Meyer, 1995]	6.28	—	263k Long Word

5. EXTENSION TO TWO-LAYER SUBSTRATES

We have extended our modeling method to a two-layer substrate, as illustrated in Fig. 12. In this approach, we consider that the total substrate capacitor is a series combination of the oxide capacitor, C_{ox} , and the substrate bulk contribution, C_{sub} . Thus, the total two-layer substrate capacitance, C_{2layer} , is given as

$$C_{2layer} = \frac{C_{ox}C_{sub}}{C_{ox} + C_{sub}} \quad (1.13)$$

The plots in Fig. 13 illustrate a comparison of the total capacitance, C_{2layer} , for several methods. The results of the model shown in this figure are accurate over a wide range of frequencies. The rigorous results obtained from the numerical solution by IE3D agree with our results, whereas the results of the other approximate method based on the uniform field assumption presented in [Valkodai and Manku, 1997] deviate substantially from the rigorous data. This significant and accurate result of our modeling method indicates that it is a very suitable application for CAD tools.

Figure 12 A two layer substrate with fringing fields C_{2layer} obtained by using 3 approaches F_C with simulation results

6. CONCLUSIONS

We have investigated the effects of frequency variations up to 20 GHz, as well as the effects of the substrate, and the contact physical and geometrical parameters on the model elements of the substrate coupling.

Based on extensive simulations, we have obtained the general formula, F_C , for rectangular contacts. This parametric model is accurate and applicable within a wide range of substrate resistivities, substrate permittivities, and contact sizes.

The parametric model facilitates a significant speedup factor over the published methods in [Gharpurey and Meyer, 1995; Costa et al., 1999] for single-contact substrates, and an accuracy that corresponds the simulation data of IE3D. We have applied our model to a 2-layer $Si - SiO_2$ structure.

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