Solution of FMS Scheduling Problems Using the Hybrid Dynamical Approach

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Abstract: Recently, one of the most promising approaches for the solution of FMS scheduling problems is the use of switching servers policies, or in other name the use of hybrid dynamical approach. In the literature the stability problem of the processes was formulated, different switching policies were analysed. Single-machine and multi-machine processing problems were drowning up.

In the present paper, new proposal for the determination of the demand rates, necessary for new approach, is given. Also, a novel proposal to use of the single-machine results for multi-machine case is given. This method is named: The controlled buffer technique.

Key words: Hybrid Dynamical Approach, FMS Scheduling, Demand Rate, Overlapping Production, Controlled Buffers.

1. INTRODUCTION

By the paper Perkins, Kumar [3], “ Stable Distributed, Real Time Scheduling of Flexible Manufacturing Assembly, Disassembly Systems”, published in 1989, a new direction of manufacturing scheduling, the hybrid dynamical approach has started. In 1993 Chase, Serrano and Ramage [4] formulated the Switched Arrival and Switched Server Problems. Showing, that periodic trajectories are characteristic for the behaviour of the second and chaos for the first.

In the book Matveev, Savkin [5], “ Qualitative Theory of Hybrid Dynamical Systems” research result for hybrid systems, that is, for system where network of digital and analogue devices figure, or a digital device

The original version of this chapter was revised: The copyright line was incorrect. This has been corrected. The Erratum to this chapter is available at DOI: [10.1007/978-0-387-35492-7_50](https://doi.org/10.1007/978-0-387-35492-7_50)

interact with continuous environment are presented. The theory of differential automats is used.

The given topic is very popular in literature and several hundred of papers, dealing with different aspects, were published.

In the present paper the demand rate determination and the multiple machine processing will be analysed and new proposals will be given.

2. HYBRID DYNAMICAL APPROACH

2.1 Production of High Numbers of parts (HNP)

Flexible Manufacturing Systems are dedicated to process one of a kind or small series in automated way. Nevertheless, some FMS-es are suitable to manufacture high number of parts (HNP). On high number we understand several hundred, several thousands or more parts (sometime less). Of course, the character of manufacturing depends not only on the number but on “size” of parts, too (the meaning of “size” here is the volume, the complication, etc.).

2.2 The Task and Data Base

The task of scheduling for an FMS is usually given by the MRP subsystem of PPS. (Formally, we use partly, the notation of French [1]):

One has:

n jobs ($J_1, J_2, \dots, J_n$), to be processed on m machines ($M_1, M_2, \dots, M_m$). On machines we understand an equivalent group of machines, which can be in practice one machine, too. Sometimes instead of word machine we use server. For all of the jobs the number of parts n_i the due date dd_i , the release date r_i (r_i - is the time at which J_i becomes available for processing) is given. T_{sch} is the scheduling period.

The CAPP subsystem determines for every job J_i (a job J_i is n_i parts of type i) the operation sequence o_{ij} , where $j=1, 2, \dots, j_l$ (j_l - is the number of operation sequences), o_{ij} -is an integer showing in which machine the given operation is performed. Very important information are the processing times q_{ij} . The processing times are determined by the operation planning subsystem of CAPP including the manufacturing data determination section. We note that the engineering database for scheduling is generated from CAPP results. But, it needs proper modification. In engineering database for

scheduling it is convenient to apply $\tau_{ij} = \frac{q_{ij}}{n_h}$, where n_h is the number of

machines in the equivalent group. Sometimes it is convenient to use: $\tau_{ih} = \tau_{ij}$, or $\tau_{ij}^h = \tau_{ij}$. Where h is the index of the machine, on which the j -th operation is realized for the i -th part type. In the following always the most convenient form will be used hoping that it will not cause misunderstanding.

In Perkins, Kumar [3] (unlike French [1]) revisiting of the same machine is also considered. In this case $\tau_{ih}^{(k)} = \tau_{ih}$ is used at the k -th visit to the same machine.

2.3 Hybrid Dynamical Approach to the Solution of FMS Scheduling Problems

Perkins, Kumar [3] proposed to use the hybrid dynamical approach to solve FMS Scheduling Problems covering assembling and disassembling tasks as well. In the present paragraph, only the job-shop type scheduling problem will be discussed. The development to rather complicated task (assembling disassembling) is possible, but not discussed here.

When using the hybrid dynamical approach the parts are delivered to the machine groups with some constant demand rate d_i , $i=1,2,\dots,I$. Let $u_{ij}(t)$, $y_{ij}(t)$, $z_{ij}(t)$ the input, output and buffer content, respectively.

The hybrid dynamical approach, usually, is based on some switching policy. Let us consider one of the simplest (nevertheless effective) switching policy the so-called Clear the Largest Buffer Level Policy (CLB). According to that, always that buffer will be cleared (served) which has the largest buffer content when some other buffer has been emptied (see: Fig.1.). Let us consider a process beginning with point A where the buffer i was emptied ($z_{ij}(t_A) = u_{ij}(t_A) - y_{ij}(t_A) = 0$).

After t_A the server was occupied with some other (not i) process. Let us suppose that at t_{s1} one of the other buffers was emptied (for example: the one of the belonging to k -th flow). Let us also suppose that i had the largest buffer level. Then, after some inactivity period (due to set-up, transportation, and manipulation etc. times) the server begins to remove fluid from the i -th tank. As it can be recognized from Fig.1. in points A, B, C... the number of produced parts exactly coincide with the required number.

Figure 1. Clear the Largest Buffer Level Policy

Let us remark, that for manufacturing application only the time instants where z_{ij} and y_{ij} has integer value are meaningful.

2.3.1 Stability of Processes

In Perkins, Kumar [3] the definition of the stability of switching policies for a single machine (machine working in isolation) were given. According to that a switching policy is stable if

$$\sup_t z_i(t) \leq M_i < \infty, \quad (1)$$

for $i = 1, 2, \dots, n$, and for any t

That is, the buffers levels at the machine are bounded.

Considering Fig.1. it means that a line parallel to u_{ij} can be drawn, so that y_{ij} never goes below it. The smaller M_i is the better the output characteristics of the system are. The better the output follows the required input.

In Perkins, Kumar [3] results for the estimation of the buffer bounds were given.

2.3.2 Necessary Condition for Stability

The necessary condition of the stability for a single machine is

$$\rho_j = \sum_{i=1}^n \tau_{ij} d_i < 1, \quad (2)$$

j - is the index of the machine, ρ_j - is the so-called machine load.

Indeed, let during the period t_j on the machine $n_1, n_2, \dots, n_i, \dots, n_n$ part should be processed. The machine capacity should be higher, than the capacities needed for manufacturing the parts. That is

$$\sum_{i=1}^n \tau_{ij} n_i < t_j \quad (3)$$

$$\rho_j = \sum_{i=1}^n \tau_{ij} \frac{n_i}{t_j} = \sum_{i=1}^n \tau_{ij} d_i < 1 \quad (4)$$

If one chooses $d_i = \frac{n_i}{t_i}$ condition (2) is obtained, ρ_j - is the time needed for processing the part. It can be recognized that the time $(1 - \rho_j)$ is the reserve during which set-up, transportation, manipulation, etc., tasks can be solved.

When a switching policy is applied to clear a buffer (a tank) it is easy to compute the switching time instants. Indeed, let us suppose that at the time instant t_{sk} a switch to serving part type i occurs, then

$$t_{s(k+1)} = t_{sk} + \delta_{ki} + z(t_{sk})\tau_{ij} + (t_{s(k+1)} - t_{sk})d_{ij}\tau_{ij} \quad (5)$$

From equation (5) one gets

$$t_{s(k+1)} = t_{sk} + [1 - d_{ij}\tau_{ij}]^{-1}[\delta_{ki} + z_i(t_{sk})\tau_{ij}] \quad (6)$$

It is also clear from Fig.1., that the following condition is valid

$$d_{ij} < 1/\tau_{ij} \quad (7)$$

In Perkins, Kumar [3] a rather general switching policy then the Clear the Largest Buffer Level was analyzed. Results for Clear the Fraction (CAF) and results for Clear the Largest Work (CLW) switching policies were given, too.

In Matveev, Savkin [5], cyclic switching policies and special kind of Clear the Largest Buffer Level were analyzed.

Until now we considered the processes of the system in isolation. That is we did not consider that the inputs (e.g. d_{ij}) of one level can be the output of the other levels. When the levels are in isolation the single machine approach can be used. But, when these are not in isolation it cannot. Indeed, if we consider Fig.1., and suppose $u_{i(j-1)}$ and $y_{i(j-1)}$, values as are given on the graphics, even if there will be an ideal input ($u_{i(j-1)} = d_{i(j-1)}t$) the output $y_{i(j-1)}$ will highly different from the ideal input u_{ij} for the next machine. If $y_{i(j-1)}$ will form the input u_{ij} dynamic instability (see: [3]) of the system can occur. Perkins, Kumar [3] could construct example for that. In the above paper the authors proposed a technique, the so-called CAF policy with Backoff, which can result stable performance. Nevertheless, because this leaves machines idle at some conditions the effectiveness of this policy is doubtful.

In the 4-th paragraph of the present paper the so-called controlled buffer technique is proposed which can help to overcome this difficulty.

3. HYBRID DYNAMICAL APPROACH TO THE SOLUTION OF FMS SCHEDULING PROBLEMS

3.1 Demand Rate for Single Machine Processing

Now, let us try to determine the demand rates d_i ; $i = 1, 2, \dots, n$, for single machine processing. It is clear that the upper limit value of demand rate is

$$\overline{\lim} d_i = 1 / \tau_{ij}, i = 1, 2, \dots, n, \quad (8)$$

j – is the machine group identification number. This is due to the machining intensity constraint. As it was mentioned τ_{ij} is determined on CAPP subsystem level. In practice

$$d_i \leq (1 / \tau_{ij}) - (\Delta d_{up})_i \quad (9)$$

should be used, where $(\Delta d_{up})_i$ is the upper demand rate reserve value necessary for the realization. The lower limit of the demand rate is

$$\underline{\lim} d_i = n_i / (dd_i - r_i). \text{ Or} \quad (10)$$

$$\underline{\lim} d_i = n_i / T_{sch}, \quad (11)$$

which represent the production task requirement. These relation give the

$$d_i \geq n_i / (dd_i - r_i) + (\Delta d_{lo})_i \text{ or} \quad (12)$$

$$d_i \geq n_i / T_{sch} + (\Delta d_{lo})_i, \quad (13)$$

relation, where $(\Delta d_{lo})_i$ is the lower demand rate reserve value. In the followings, for the sake of simplicity, only (11) and (13) will be considered. To generalize the results for the other case is a single task.

Another upper limit is given by the machine capacity constraint. It can be derived from the necessary condition of stability formulated above. Let us consider this relation in form

$$\sum_p n_p \tau_{pj} \leq T_{sch}. \quad (14)$$

From that (see also relation (2))

$$\sum_p d_p \tau_{pj} \leq 1 \quad (15)$$

Now, let us determine the coefficient η which make (14) and (15) to turn into equality.

$$\eta = T_{sch} / (\sum_p n_p \tau_{pj}) = T_{sch} / t_{\Sigma j}, \quad (16)$$

where $t_{\Sigma j} = \sum_p n_p \tau_{pj}$ is the summary machining time requirement on the machine j .

From the above the other upper limit value is

$$\overline{\lim} d_i = n_i / t_{\Sigma j}. \quad (17)$$

Taking into attention (17) the upper limit should be

$$\overline{\lim} d_i = \text{Min}\{(1 / \tau_{ij}), (n_i / t_{\Sigma j})\}. \quad (18)$$

So, one gets the domain for demand rate

$$\frac{n_i}{T_{sch}} + (\Delta d_{lo})_i \leq d_i \leq \text{Min}\{(1 / \tau_{ij}), (n_i / t_{\Sigma j})\} - (\Delta d_{lo})_i. \quad (19)$$

Let us notice that the increase of d_i is desirable. But, the necessary set-up, etc., times requirements constraint this increase. (This should be reflected in the value of $(\Delta d_{up})_i$, too.) It seems to us that the best aids for final decisions are the simulation studies.

Now, let us turn to the case of multi-machine job-shop type manufacturing.

3.2 Demand Rate for Multi-Machine Processing. Overlapping Production

In the 1-st paragraph of the present paper, formulation of the classical problems of scheduling was given. Let us outline the basic features of the solution. The beginning of the first operation is at $t_{i1} \geq r_i$. The end of the last operation is at $t_{il} \leq dd_i$. At the input and output of the machines, buffers of size n_i are needed. At flexible manufacturing, overlapping production of parts is possible. This is due to the fact that buffers, transport and manipulation devices are automated. Set-up, transport, manipulation times are low. The overlapping production makes possible to decrease significantly the production times. In more details see: Somlo [7].

Now, let us deal with the determination of the demand rates for multi-machine processing. It is clear that the demand rate should satisfy the following upper limit

$$(d_i)_j < 1/\tau_{ij}, \text{ for all } j, \quad (20)$$

where j indicates the machines part-type i is processed on, and $(d_i)_j$ is the demand rate constrained by the given machine conditions.

It is supposed that the rate is constant for some part-type, independently of the machine it is processed on. So, from (20) one gets

$$d_i < 1/(\text{Max}_j \tau_{ij}). \quad (21)$$

Similarly as in the single-machine case the lower limit of demand rate is determined by

$$\underline{\text{lim}} d_i = n_i / T_{sch}. \quad (22)$$

Let us deal with the determination of the upper limit value given by the total time necessary for the production of parts.

The upper limit value of the demand rates can be determined based on the overall time necessary for overlapped production (see: Somlo [7]). It can be proved that this time

$$t_{\Sigma l}^a \approx n_i \underset{j}{Max} \tau_{ij}. \quad (23)$$

So,

$$\overline{\lim} d_i = (n_i / \tau_{\Sigma l}^a) \quad (24)$$

Let us simplify the above result. It is easy to recognize that

$$n_i \underset{j}{Max} \tau_{ij} < t_{\Sigma l}^a \leq (n_i - 1) \underset{j}{Max} \tau_{ij} + \sum_{j=1}^l \tau_{ij} \quad (25)$$

and so, for the demand rate one gets the upper limit given by relation (21). That is

$$d_i < 1 / \underset{j}{Max} \tau_{ij} \quad (26)$$

The important relation (26) reflects the fact, that the overall machining time for overlapping production gives the same upper limit value for demand rate as the machining intensity constraint.

Until now, the set-up, etc. times effects were neglected. For taking into attention these, reserves should be introduced. For those, decrease of demand rates from the upper limit value is necessary. To produce n_i parts of type J_i MRP allows to use $(dd_i - r_i)$ or in analyzed case T_{sch} time period. It is usually much higher than $t_{\Sigma l}^a$ at overlapped production. Of course, it should be kept in mind that in these cases several HNP series are processed in parallel.

For these cases the necessary condition for stability (2) should also be satisfied. To provide it let us determine the upper limit by

$$\overline{\lim} d_i = (1 / \underset{j}{Max} \tau_{ij}) (t_{\Sigma} / T_{sch}) \eta \quad (27)$$

Here η is properly chosen coefficient. In relation (27) instead of $t_{\Sigma l}^a$ simply t_{Σ} is used.

According to the necessary condition of stability (see relation (2)),

$$\delta_h = \sum_{i=1}^n d_i \tau_{ih} \leq 1, \quad h = 1, 2, \dots, m. \quad (28)$$

Because (see (23)),

$$t_{\Sigma} \approx n_i \underset{j}{Max} \tau_{ij} \quad (29)$$

$$d_i = (n_i / T_{sch}) \eta \quad (30)$$

Can be taken, and from (28)

$$\sum_{i=1}^n (n_i / T_{sch}) \tau_{ih} \eta \leq 1, \quad h = 1, 2, \dots, m \quad (31)$$

is obtained. In (31)

$$\sum_{i=1}^n n_i \tau_{ih} = t_h, \quad h = 1, 2, \dots, m \quad (32)$$

is the summary working time to perform the production tasks on the machine with index h .

From relation (31) one gets the efficiency coefficient η , which turns the relation into equality for every machine group

$$(\eta)_h = T_{sch} / t_h, \quad h = 1, 2, \dots, m \quad (33)$$

Accordingly,

$$\eta \leq T_{sch} (1 / \underset{h}{Max} t_h), \quad h = 1, 2, \dots, m. \quad (34)$$

From (27), (29) and (33)

$$\overline{\lim} d_i = n_i / \underset{h}{Max} t_h \quad (35)$$

Having (21) and (34) the upper limit value of demand rate is

$$\overline{\lim} d_i = \underset{j}{Min} \left\{ (1 / \underset{j}{Max} \tau_{ij}), (n_i / \underset{h}{Max} t_h) \right\} \quad (36)$$

The lower limit value is given by (22). So

$$\frac{n_i}{T_{sch}} + (\Delta d_{lo})_i \leq d_i \leq \underset{j}{Min} \left\{ \frac{1}{\underset{j}{Max} \tau_{ij}}, \frac{n_i}{\underset{h}{Max} t_h} \right\} - (\Delta d_{up})_i \quad (37)$$

Now, let us consider the demand rates for the consecutive machines. The first machine gets a d_i ramp-input signal. The consecutive machines get the same with τ_{ij} time delay as it is shown on Fig.2.

Figure 2. Demand rates at consecutive machine group

It is an interesting opportunity to use the secondary optimization method (see: Somló J., Nagy J. [2])) to get time gain in FMS scheduling (in more details see: Somló J. [7])

4. THE CONTROLLED BUFFER TECHNIQUE

As it was mentioned, Perkins, Kumar [3] proposed to use distributed CAF (in particular Clear- the- Largest Buffer Level) policy with Backoff to provide stability of processes at general job shop type FMS scheduling. Matveev, Savkin [5] analyzed multiple server flow networks which also cover this field. They proposed to construct regularizable switched multiple server flow networks which exhibit a regular behavior.

Perkins, Humes, Kumar [6] proposed to use the so-called regulated buffer stabilizing techniques to reach stable performance for FMS scheduling problems.

Here we propose a different technique to be able to use the results of hybrid dynamical approach for single machines in the case of multiple-machine, job shop type FMS scheduling.

The following, so called controlled buffer technique is proposed here. Let us introduce into the system structure the, so called auxiliary buffers, as it is shown on Figure 3. These, together with the input buffers are controlled. The controlled buffers act as follows. At the input buffer of j -th machine always the ideal input (d_i demand rate) and the corresponding number of parts already machined on $(j-1)$ machine should be introduced. If the output buffer of machine $(j-1)$ is unable to cover demands then the auxiliary buffer comes to its place and delivers the proper number of parts (not finished production). The proper content of the auxiliary buffer is provided before the regular system actions. (It is also possible to include service sections during regular actions to fulfill this task.) There is always surplus which goes to fill back the auxiliary buffer. When the part type i production on machine $(j-1)$ begins the d_i demand rate on machine j is covered by this production. It can be recognized that the same number of parts that was taken of the auxiliary buffer, later, is returned from the output of the $(j-1)$ -th machine. Indeed, because in an “activity” period the $(j-1)$ machine produces exactly the number of parts between two empty buffer position it will return the number of parts which were taken from the auxiliary buffer. In more details see: Somlo [7].

Figure 3. Controlled buffer technique

5. CONCLUSIONS

The idea of hybrid dynamical approach to the solution of FMS scheduling problem give an opportunity to increase significantly the effectiveness of utilization of the equipment of these high value systems. So, the economic effect of the use of this may be very high.

The theoretical results obtained, recently concerning hybrid dynamical systems give strong basis for the solution of problems. Perkins, Kumar [3], Matveev, Savkin [5] can be considered as basic works.

In the present paper an attempt is made to clear the relation of classical scheduling problems formulation and hybrid dynamical approach. It has been shown that the essence of the effect of the last is the opportunity of overlapping production. The use of hybrid dynamical approach to a single-machine problem is well established with nice results. Not so clear is the situation when dealing with job-shop type multi-machine problems in FMS-es (with possible assembling and disassembling processes included). In the present paper the buffer control is proposed as a solution to reduce the problems to the use of the single-machine case.

The modern simulation methods (Taylor, Simple ++, etc.: and as well the continuos system simulation languages Matlab, Matrix-x, etc.) give excellent opportunity to analyze the application problems details concerning the use of hybrid- dynamical approach. These studies can lead to new results and better understanding of the problems.

ACKNOWLEDGEMENT

The authors express thanks to OTKA of the Hungarian Academy of Sciences (OTKA T 026407 and 0229072) and to the Australian Research Council for the support of the research.

Special thanks for the advices and exceptionally high help to Dr. F. Erdélyi.

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