Influence of Chord Length on the Blocking Performance of Wavelength Routed Chordal Ring Networks

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Abstract: In this paper, we present an assessment of the blocking performance of wavelength routed optical networks with chordal ring topology. The performance is focused on chord lengths of 3, $N/4$, and $\sqrt{N} + 3$, being N the number of nodes. It is shown that, in a 100-node chordal ring network, the use of the chord length of $N/4$ instead of 3 clearly reduces the blocking probability. A reduction of the blocking probability is also observed when the chord length of $\sqrt{N} + 3$ is used, instead of $N/4$ or $w=3$. The increase of the node degree from 3 (chordal ring with 100 nodes and a chord length of $\sqrt{N} + 3$) to 4 (mesh torus with 100 nodes) has also revealed an important feature: very small blocking gains were observed due to the increase of the node degree from 3 to 4. Thus, the choice of a chordal ring with a chord length of $\sqrt{N} + 3$, instead of a mesh-torus, leads to a reduction in the number of network links, and hence in the total cable length, since the number of links in a N -node chordal ring is $3N$, and the number of links in a N -node mesh-torus is $4N$.

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1. INTRODUCTION

Internet traffic is increasing due to the rise of new communication services and due to the explosive growth of the number of hosts [1]-[2]. To satisfy the bandwidth requirements that enable those new services and a higher number of users, WDM systems are being deployed world wide, which is seen as the first phase of optical networking. After this, the introduction of optical add/drop multiplexers in a linear architecture and the use of WDM protection switches are expected. This architecture will rapidly evolve to a WDM ring architecture, and a further possible evolution scenario may be the interconnection of WDM rings and mesh networks [3]. Whereas the evolution from the point-to-point WDM transmission system to interconnected rings is clear, from the point of view of physical topology, the optimal topology to be used for the mesh network is a subject less studied. In [4], a study is presented of the influence of node degree on the fibre length, capacity utilisation, and average and maximum path lengths of wavelength routed mesh networks. It is shown that average node degrees varying between 3 and 4.5 are of particular interest.

In this paper, we consider chordal rings, which are a particular family of regular graphs of degree 3 [5]. Using an analytical model, we present an assessment of the blocking performance in wavelength routed chordal ring networks, considering chord lengths of 3, $N/4$ and $\sqrt{N}+3$, where N is the number of nodes.

The remainder of this paper is organized as follows. The analytical model used to compute the blocking probability in wavelength routed chordal ring networks is briefly described in section 2. The assessment of the blocking performance in chordal rings is presented in section 3 and main conclusions are presented in section 4.

2. COMPUTATION OF BLOCKING PROBABILITY IN CHORDAL RING NETWORKS

2.1 Chordal Ring Topology

A chordal ring is a ring structured network in which each node has an additional link, called a chord. The number of nodes in a chordal ring is assumed to be even and nodes are indexed 0, 1, 2, ..., $N-1$ around the N -node ring. It is also assumed that each odd-numbered node i ($i=1, 3, \dots, N-1$) is connected to a node $(i+w) \bmod N$, where w is the chord length, which is

assumed to be positive odd and, without loss of generality, we also assume that $w \leq N/2$, as in [5].

For a given number of nodes N , several chordal rings can be obtained for different values of the chord length. Figure 1 shows a chordal ring with $N=20$ and $w=5$.

Figure 1. Chordal ring network with $N=20$ nodes and $w=5$

According to the Theorem 2 presented in [5], the diameter of a chordal ring network with N nodes, where N is a square and $N \geq 8^2$, is greater than or equal to $\sqrt{N} + 3$. This equality holds when $\sqrt{N} + 3 \leq w \leq \sqrt{N} + 2h + 1$, where h is the largest integer that satisfies $\sqrt{N} \geq 2(2h^2 + h + 1)$. This Theorem is particularly important since it allows the identification of chord lengths that lead to the smallest network diameter.

2.2 Evaluation of Blocking Probability

To compute the blocking probability in optical networks with wavelength interchange, we have used the model given in [6], since it applies to ring topologies, has a moderate computational complexity, and takes into account dynamic traffic and the correlation between the wavelengths used on successive links of a multi-link path.

The following assumptions are used in the model [6]: 1) Call requests arrive at each node according to a Poisson process with rate λ , with each call equally likely to be destined to any of the remaining nodes; 2) Call holding

time is exponentially distributed with mean $1/\mu$, and the offered load per node is $\rho=\lambda/\mu$; 3) The path used by a call is chosen according to a pre-specified criterion (e.g. random selection of a shortest path), and does not depend on the state of the links that make up a path; the call is blocked if the chosen path can not accommodate it; alternate path routing is not allowed; 4) The number of wavelengths per link, F , is the same on all links; each node is capable of transmitting and receiving on any of the F wavelengths; each call requires a full wavelength on each link it traverses; 5) Wavelengths are assigned to a session randomly from the set of free wavelengths on the associated path.

The analysis presented in [6] also assumes that the hop-length distribution is known, as well as the arrival rates of calls at a link that continue, and those that do not, to the next link of a path. The call arrival rates at links have been estimated from the arrival rates of calls to nodes, as in [6]. The hop-length distribution is a function of the network topology and the routing algorithm, and is easily determined for most regular topologies with the shortest-path algorithm. However, for a bi-directional N -node chordal ring with chord length w , it was not possible to obtain a general expression for the hop-length distribution. We have found analytical expressions for the hop-length distribution when the chord length is 3 ($w=3$), when the chord length is maximum ($w=N/2$ or $w=N/2-1$), when the chord length is as close as possible to the mean chord length ($w=N/4$), and for a chord length that leads to the smallest network diameter ($w=\sqrt{N}+3$).

2.3 Performance Metrics

The performance assessment presented in this paper is based on the evaluation of the path blocking probability. However, a direct comparison of blocking probabilities is sometimes preferred in order to put in evidence some features and to quantify benefits. Besides the blocking probability, two metrics are used in this paper to quantify and to put in evidence the benefits of using chordal ring networks. We consider the blocking gain due to a better choice of chord length (G_{cl}), and the blocking gain due to an increase in the node degree (G_{nd}). Since we consider only blocking gains along the paper, G_{cl} , and G_{nd} are also called here as chord length gain and node degree gain, respectively. These two metrics are defined as:

$$G_{cl} = \frac{P_{w3}}{P_{wl}} \quad (1)$$

where P_{w3} is the blocking probability in a chordal ring network with a chord length of 3, and P_{wl} is the blocking probability in a chordal ring network

with a chord length of wl (both obtained for the same number of nodes, wavelengths per link, and load per node);

$$G_{nd} = \frac{P_{(D-1)}}{P_D} \quad (2)$$

where $P_{(D-1)}$ is the blocking probability in a network with a node degree $D-1$, and P_D is the blocking probability in a network with a node degree D (both obtained for the same number of nodes, wavelengths per link, and load per node).

3. ASSESSMENT OF BLOCKING PERFORMANCE IN CHORDAL RING NETWORKS

In this section, we present an assessment of the blocking performance in chordal ring networks. The performance assessment is focused on chordal rings with $w=3$, $w=N/4$, and $w=\sqrt{N}+3$. The last one is very important since it leads to the smallest network diameter. If the number of nodes is larger or equal than 484, there are other chord lengths that also lead to the smallest network diameter, but here we concentrate only on the above referred chord lengths. We consider chordal ring networks with $N=100$ nodes. The blocking performance obtained with the maximum chord length ($w=N/2-1$) is similar to the performance obtained with $w=3$ and, therefore, it is not presented here.

Figure 2 shows the chord length gain for a 100-node chordal ring without wavelength interchange. As can be seen, the use of $w=N/4$, instead of $w=3$, clearly improves the blocking probability. The corresponding blocking probability gain, for 16 wavelengths per link, is of the order of 10^2 for a load per node of 1 Erlang, and of the order of 10^6 for a load of 0.1 Erlang, while, for 8 wavelengths per link, the blocking gain is about 2.7 for a load per node of 1 Erlang, and about 10^3 for a load of 0.1 Erlang (see Figure 2). A blocking probability improvement is also observed when the chord length of $\sqrt{N}+3$ is used, instead of $w=N/4$ or $w=3$. The blocking probability gain due to the choice of $w=\sqrt{N}+3$, instead of $w=3$, for 16 wavelengths per link, is of the order of 10^4 for a load per node of 1 Erlang, and of the order of 10^{10} for a load of 0.1 Erlang, while, for 8 wavelengths per link, the blocking gain is about 13.5 for a load per node of 1 Erlang, and of the order of 10^4 for a load of 0.1 Erlang (see Figure 2). This clearly shows that the chord length of $\sqrt{N}+3$ gives rise to very high chord length gains. From Figure 2, we may also see that similar chord length gains are obtained for $w=N/4$ with 12

wavelengths per link and for $w=\sqrt{N}+3$ with 8 wavelengths per link. Thus, for a similar blocking performance, the use of $w=\sqrt{N}+3$ reduces the number of wavelengths per link, which is important from the point of view of network dimensioning.

Figure 2. Blocking gain due to a better choice of chord length, as a function of load per node, for a chordal ring network with 100 nodes and without wavelength interchange.

Figure 3 shows the blocking probability, versus converter density, for 100-node chordal ring networks with chord lengths of 3, $N/4$, or $\sqrt{N}+3$, and with a load per node of 0.5 Erlang. The corresponding chord length gain is shown in Figure 4. As can be seen, there is a slight chord length gain when the number of wavelengths per link is small (4 or 8). If the number of wavelengths per link is large (12, 16), the chord length gain decreases up to a certain converter density, and then the chord length gain begins to increase slightly. Another observation that can be made from this figure is that the converter density, at which the gain starts increasing, increases with the number of wavelengths per link.

Figure 5 shows the blocking gain due to an increase of the node degree from 3 (chordal ring networks with 100 nodes) to 4 (mesh-torus networks with 100 nodes), as a function of the load per node. If $w=3$ is used with loads lower than 0.25 Erlang, the blocking gain is larger than 10^4 for 8 wavelengths per link, and larger than 10^{10} for 16 wavelengths per link.

For $w=N/4$, gains of the order of 10^2 and 10^5 were obtained for 8 and 16 wavelengths per link, respectively, with a load per node of 0.1 Erlang. For

the case of $w=\sqrt{N}+3$, very small node degree gains were observed. As the load per node decreases from 5 Erlang to 0.01 Erlang, the variation of the node degree gain remains within one order of magnitude for the numbers of wavelengths per link considered: 4, 8, 12 and 16. These small gains were also observed in networks with a larger size (1600 nodes).

These very small node degree gains, obtained for mesh-torus and chordal rings with $w=\sqrt{N}+3$, may be explained by the same order of magnitude of the average hop-length.

Figure 3. Blocking probability versus converter density for 100-node chordal ring networks with a load per node of 0.5 Erlang.

- $w=3, F=8$
- $w=3, F=12$
- . - . $w=3, F=16$
- $w=N/4, F=8$
- $w=N/4, F=12$
- . - . $w=N/4, F=16$
- $w=\sqrt{N}+3, F=8$
- $w=\sqrt{N}+3, F=12$
- $w=\sqrt{N}+3, F=16$

Figure 4. Blocking gain due to a better choice of chord length, as a function of the converter density, for 100-node chordal ring networks with a load per node of 0.5 Erlang.

Figure 5. Blocking gain due to an increase of the node degree from 3 to 4, as a function of the load per node, for chordal ring networks and mesh-torus networks, both with 100 nodes (without wavelength interchange).

If a small blocking performance degradation is allowed, the choice of chordal ring networks with $w=\sqrt{N}+3$, instead of mesh-torus networks, leads to a reduction in the number of network links, and hence in the total cable length, since the number of links in a N -node chordal ring is $3N$, and the number of links in a N -node mesh-torus is $4N$. However, there are some restrictions that limit the practical implementation of chordal rings with $w=\sqrt{N}+3$ (as well as mesh-torus), when compared with other chord lengths. In fact, as referred in section 2, the smallest network diameter is obtained with $w=\sqrt{N}+3$ for a N -node chordal ring, where N is a square ($N=m^2$) and $N \geq 64$. The restriction associated with the square is not imposed to other chord lengths such as $w=N/4$ or $w=3$.

4. CONCLUSIONS

Using an analytical model, we have presented an assessment of the blocking performance in wavelength routed chordal ring networks with chord lengths of 3, $N/4$, or $\sqrt{N}+3$. It was shown that, in a 100-node chordal ring, the use of $w=N/4$, instead of $w=3$, clearly improves the blocking probability. An improvement in blocking performance was also observed if the chord length of $\sqrt{N}+3$ is used, instead of $w=N/4$ or $w=3$. In the latter case, the blocking probability gain due to the choice of $w=\sqrt{N}+3$, instead of $w=3$, for 16 wavelengths per link, is of the order of 10^4 for a load per node of 1 Erlang, and of the order of 10^{10} for a load of 0.1 Erlang, while, for 8 wavelengths per link, the blocking gain is about 13.5 for a load per node of 1 Erlang, and of the order of 10^4 for a load of 0.1 Erlang.

Chordal rings have also been compared with mesh-torus networks. An important feature has been revealed by this comparison: very small blocking gains were observed due to the increase of the node degree from 3 (chordal ring with 100 nodes and $w=\sqrt{N}+3$) to 4 (mesh torus with 100 nodes). In this case, as the load per node decreases from 5 Erlang to 0.01 Erlang, the variation of the node degree gain remains within one order of magnitude for the numbers of wavelengths per link considered: 4, 8, 12 and 16. Thus, the choice of a chordal ring with $w=\sqrt{N}+3$, instead of a mesh-torus, leads to a reduction in the number of network links, and hence in the total cable length.

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