PERFORMANCE OF MULTICAST SESSIONS IN WAVELENGTH-ROUTED WDM NETWORKS*

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Abstract We evaluate the blocking probabilities in optical networks which support multicast (or multipoint) service. Due to the difficulty of the problem, we restrict our attention to the case of the completely connected networks, for which the blocking probabilities act as a lower bound for other topologies. Nodes may, or may not be equipped with wavelength converters. In both cases, routing can be done either directly, or indirectly, and even in the latter case the number of hops can be either restricted to two hops, or unrestricted. We evaluate the blocking probability by first estimating the load offered to each link, and then use the iterative reduced load approximation method to refine this estimate. Using this estimate, we then obtain the probabilities of blocking and success. Numerical examples are presented, and are compared to simulation.

Keywords: Optical networks; wavelength division multiplexing; multicast service; circuit switching; blocking probability; analysis.

1. INTRODUCTION

The advent of Wavelength Division Multiplexing (WDM) [Brackett, 1990] provided a solution to the problem of speed mismatch between the fiber bandwidth and the electronics speed, which is better known as the electronic bottleneck problem [Kaminow, 1989]. As such, an aggregate bandwidth in the tens of Terabits per second range can be achieved by supporting multiple simultaneous transmissions, each utilizing a different wavelength. This can provide a means to serve those applications

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which individually or collectively require a large amount of bandwidth. Such applications include the class of services with multicast, or multipoint traffic which requires the delivery of data from the source to a group of destinations [Turner, 1987]. Such applications include video distribution, network news distribution, multi-party conferences, etc. Multicast service over optical networks can be implemented by either creating an independent channel from the source to each destination, which is quite expensive, or by having the destinations cooperate in relaying the traffic they receive to other destinations. In this paper we consider the second approach, and we assume that there is a provision by which a destination will relay the same traffic, after reading it, to another destination.

The purpose of this paper is to derive expressions for the call blocking probabilities in wavelength-routed optical networks using WDM and carrying multicast circuit-switched traffic. Several studies have derived the call blocking probabilities for unicast circuit-switched applications, but to the best of our knowledge, this is the first study to obtain such blocking probabilities for the multicast service.

A few studies dealt with multicast routing in optical networks, e.g., [Tridandapani and Meditch, 1995; Tridandapani and Mukherjee, 1996; Rouskas and Ammar, 1994; Modiano, 1998]. However, those studies were concerned with packet-switched communication, and were concerned with either the establishment of the routing tree, or the derivation of the packet delay. The recent study in [Sahin and Azizoglu, 1998] has also dealt with multicast service over wavelength routed networks, but from a different perspective. It found the upper bound on the number of multicast calls which can be supported on a given network using an integer linear program.

A few studies have dealt with the derivation of the blocking probabilities under unicast service and with circuit switched traffic. These include references [Barry and Humblet, 1996; Jeong and Ayanoglu, 1996; Kovacevic and Acampora, 1996; Birman, 1996] which considered similar network models. Most of those studies used Erlang's fixed point method and the reduced load approximation [Kelly, 1990; Chung et al., 1993] in order to arrive at the blocking probabilities.

The authors in [Subramaniam et al., 1996] derived the blocking probabilities when only a group of nodes are capable of wavelength conversion, and in [Subramaniam et al., 1997] they derived the blocking probabilities when the arriving traffic is non-Poisson. In [Yates et al., 1996] the blocking probabilities in all-optical networks employing limited wavelength translation were derived in terms of the link utilization, and reference [Sharma and Varvarigos, 1998] considered the same problem

in a mesh network. Reference [Harai et al., 1997] obtained the blocking probabilities in all optical switching networks when alternate routing is used.

In this paper we derive the blocking probabilities under multicast service when the topology forms a completely connected graph. We have chosen this topology for two reasons. First, it is a simple topology that avoids all the routing decisions which must be made by other special topologies which will affect (and in most cases complicate) the derivation of the blocking probabilities. Second, since a link exists between each pair of nodes in the completely connected topology, and since this number of routes from a source to any group of destinations is *at least* equal to the number of routes with any other topology, the blocking probabilities derived in this study serve as lower bounds on the blocking probabilities in other topologies.

We derive the blocking probabilities with and without wavelength conversion. We also consider two cases for the maximum number of hops allowed: a single hop, which correspond to direct routing, and a two-hop dynamic indirect routing. As expected, when the limit on the number of hops is increased, the blocking probabilities improve. However, increasing such a limit requires a more complicated control mechanism, but, as will be shown in the numerical examples section, this enhances the blocking probabilities only marginally over the two hop limit case. We also use simulation in order to assess the effectiveness of the unrestricted indirect routing strategy.

The approximate approach we follow in this paper is to estimate the call arrival rate to a certain link given the number of available channels (wavelengths) on that link and on an adjacent link. This joint availability condition is necessary to capture the strong correlation that arises with multicast traffic. This estimated load is in fact the reduced load which is used to calculate the probabilities of wavelength availabilities. The procedure is iterative, and in most cases converge rapidly.

In section II we introduce the network model. In section III we derive the blocking probabilities when wavelength conversion is used, while the probabilities of blocking without wavelength conversion are obtained in section IV. Section V presents some numerical examples and discussions, and section VI concludes with some remarks.

2. NETWORK MODEL

We consider a completely connected network with N nodes such that each pair of nodes is connected by a single fibre link. Each link carries W wavelength channels. Calls are assumed to arrive at a node according to

a Poisson process with rate a , and the call holding time is exponentially distributed with a mean of one time unit. Each of the arriving calls are directed to k destinations with probability r_k , and the destination nodes are randomly chosen among all the nodes, excluding the source. We assume that all possible paths are exhaustively searched, and whenever several are found, only one is chosen at random.

It is assumed that each source node is able to transmit independently to different directly connected nodes using the same, or different wavelengths. It is also assumed that each of the links used by a session must lead to one of the destinations which have not been reached before, i.e., no node which is not a destination should be visited. The rationale behind this assumption is to minimize the amount of bandwidth required to carry out that call.

In order to obtain the probability of call blocking, we require the channel occupancy distribution on the different links. The reduced load approach¹ will be employed as an approximation [Kelly, 1990; Chung et al., 1993]. We develop a model which employs a generalization of Erlang's fixed point equation. Straightforward application of Erlang's fixed point method in which the channel occupancy is modeled as a one-dimensional Markov process is not appropriate under multicasting conditions. The reason is that strong correlations exist between the channel occupancies on links originating from the same node.

We capture the correlation between adjacent links by considering the behaviour of a target link in conjunction with the behaviour of an adjacent link, i.e., two links originating from the same node. We note that a similar correlation exists between this target link and the other adjacent links. Therefore, we can use this correlation when considering the other links.

We employ a 3-dimensional Markov process, where the dimensions are:

- i : the number of available channels on the target link.
- j : the number of available channels on the adjacent link.
- k : the number of channels on the target and the adjacent link, which are used for common calls.

According to the above, k should not exceed the minimum of $W - i$ and $W - j$, where W is the total number of channels per link. Although the above process is three-dimensional, the number of states is limited to a

¹The reduced load on a link will be that load which is carried on the link, which will be denoted by λ , and is assumed to be Poisson distributed.

moderate number since in practice the number of channels per link is small. Hence, the computational requirements imposed by this model are reasonable.

It should be noted that disjoint transitions in either of the i or j dimensions correspond to activities on the target or adjacent nodes, respectively, which does not result in any change in the third dimension, k . However, joint transitions on both dimensions correspond to using channels on both links, which also requires either increasing or decreasing k for channel use and release, respectively.

The transition rates are given by the elements of a matrix, $\mathcal{P}$. The matrix $\mathcal{P}$ depends on the routing method, and will be shown for each routing case separately. Once $\mathcal{P}$ is constructed, one can solve for the steady state probabilities, $\pi_{i,j,k}$, from which we obtain the marginal probability of having m available channels on the target link, $q(m)$.

In Appendix I we introduce the definitions which will be used in this paper, noting that all links will be treated as identical and therefore no link number or index will be used, except between the target and adjacent links. It should be noted that the terms *target* and *adjacent* links can be used interchangeably.

We will derive the probability of call blocking by first evaluating the probability of call success, $\text{Pr}(\text{success})$, and then using

$$\text{Pr}(\text{blocking}) = 1 - \text{Pr}(\text{success}).$$

$\text{Pr}(\text{success})$ depends on the routing strategy, and will be evaluated for each of the considered cases.

3. NETWORKS WITH WAVELENGTH CONVERSION

This section derives the session blocking probabilities under the assumption that nodes are capable of full wavelength conversion. We will consider the two cases in which routing is either direct, or indirect with a maximum of two hops.

3.1. DIRECT ROUTING

We first consider direct routing only in which the link between the source and each destination must have at least one available wavelength. Under these conditions, $\lambda^{(h)}(m, n)$ for $h > 1$ is equal to zero. Also, $\lambda^{(h)}(0, n)$ is equal to zero for all value of h .

Considering a target link, we further divide $\lambda^{(1)}(m, n)$ into two components: $\lambda_d^{(1)}(m, n)$ which is carried on the target link only, i.e., it is disjoint from the adjacent considered link; and $\lambda_s^{(1)}(m, n)$ which is car-

ried on both the target and adjacent link, i.e., it is shared with the adjacent link. As such, these can be expressed as

$$\lambda_d^{(1)}(m, n) = \sum_{k=1}^{N-1} a_k d_k \times (1 - t_1(k))(1 - B(m))^{k-1} \quad , \text{ and} \quad (1)$$

$$\lambda_s^{(1)}(m, n) = \sum_{k=2}^{N-1} a_k d_k \times t_1(k)(1 - B(m))^{k-2}, \quad \text{for } n > 0 \quad (2)$$

In equation (1) the reduced load on a certain *target* link, given that the call is destined to k stations, is the fraction of offered load that does not use the adjacent link (the probability of which is equal to $1 - t_1(k)$). The last term in the equation is the probability that the other $k - 1$ destinations are reachable directly, given that the target link has m available channels. Equation (2) considers the case in which the node at the end of the adjacent link is a destination (with probability $t_1(k)$).

The elements of the transition rate matrix $\mathcal{P}$ are now given by:

$$\begin{aligned} P_{(i,j,k),(i,j-1,k)} &= \lambda_d^{(1)}(j, i) \\ P_{(i,j,k),(i-1,j,k)} &= \lambda_d^{(1)}(i, j) \\ P_{(i,j,k),(i-1,j-1,k+1)} &= \frac{\lambda_s^{(1)}(i, j) + \lambda_s^{(1)}(j, i)}{2} \\ P_{(i,j,k),(i,j+1,k)} &= W - j - k \\ P_{(i,j,k),(i+1,j,k)} &= W - i - k \\ P_{(i,j,k),(i+1,j+1,k-1)} &= k \end{aligned}$$

We note that the amount of traffic common to two adjacent links is not in general known. In an exact characterization, since call arrivals are independent of the link states, then $\lambda_s^{(1)}(i, j)$ and $\lambda_s^{(1)}(j, i)$ should be equal. However, since our approach is approximate they may not be exactly the same, but were found to be very close in the numerical examples. We therefore average $\lambda_s^{(1)}(j, i)$ and $\lambda_s^{(1)}(i, j)$ in order to find that fraction of traffic corresponding to the downward diagonal transition rate.

Once $q(m)$ is found, the probability of success is given by

$$\text{Pr}(\text{success}) = \sum_{k=1}^{N-1} r_k \sum_{m=1}^W q(m)(1 - B(m))^{k-1} \quad .$$

3.2. INDIRECT ROUTING WITH TWO LEVELS

We now allow indirect routing with a maximum of two hops. To facilitate the derivation of the reduced loads for this case, we introduce the

probability $U(k)$ whose definition is given in Appendix I. This probability applies only if the target link is used on the second hop (or farther).

We can now write

$$\lambda^{(h)}(0, n) = 0, \quad \forall h, \text{ and}$$

$$\lambda^{(h)}(m, n) = 0, \quad \text{for } h > 2, \forall m, n.$$

The expressions for the reduced load carried on a target link, which is not shared with the adjacent link when the adjacent link is not blocked ($n > 0$) are now given by:

$$\lambda_d^{(1)}(m, n) = \sum_{k=1}^{N-1} a_k d_k (1 - t_1(k)) \sum_{i=0}^{k-1} \chi(k-1, i, B(m)) H(i|k-i) \quad \text{for } n > 0, \quad (3)$$

and

$$\begin{aligned} \lambda_d^{(2)}(m, n) = & \sum_{k=2}^{N-1} a_k e_k (N - k) P_{a\&b} \\ & \times \left\{ (1 - t_2(k)) \sum_{i=0}^{k-2} \chi(k-2, i, B) U(k-i-2) (1 - B^{k-i-2} B(m))^i \right. \\ & + t_2(k) \left[(1 - B) \sum_{i=0}^{k-3} \chi(k-3, i, B) U(k-i-2) (1 - B^{k-i-2} B(m))^i \right. \\ & + B \sum_{i=0}^{k-3} \chi(k-3, i, B) (1 - B^{k-3-i} B(m))^i \\ & \left. \left. \cdot U(k-i-3) (1 - U(k-i-3)) \right] \right\} \quad (4) \end{aligned}$$

In deriving equation (3), we condition on having i destinations being blocked, and obtain the probability that they can be reached through the $k-i$ unblocked nodes, including the node at the end of the target link. The node at the end of the adjacent link in this case is not a destination. In equation(4), since we are concentrating on the second hop, there are $N-k$ ways of choosing the source. Also, the probability of reaching the first node directly and not reaching the node at the end of the adjacent link directly is $P_{a\&b}$. The two terms inside the curly bracket correspond to the cases in which the node at the end of the adjacent link is not a destination, and is a destination, respectively.

The expressions for the reduced load on the target link when the adjacent link is blocked are given by:

$$\lambda_d^{(1)}(m, 0) = \lambda_d^{(1)}(m, 1)$$

$$+ \sum_{k=2}^{N-1} a_k d_k t_1(k) \sum_{i=0}^{k-2} \chi(k-2, i, B(m)) H(i+1|k-i-1), \quad (5)$$

$$\begin{aligned} \lambda_d^{(2)}(m, 0) &= \sum_{i=0}^{k-2} a_k e_k (N-k) P_{a \& b} \\ &\left\{ (1 - t_2(k)) \sum_{i=0}^{k-2} \chi(k-2, i, B) U(k-i-2) (1 - B^{k-2-i} B(m))^i \right. \\ &+ t_2(k) \left[(1 - B) \sum_{i=0}^{k-3} \chi(k-3, i, B) U(k-i-2) (1 - B^{k-2-i} B(m))^i \right. \\ &\left. \left. + B \sum_{i=0}^{k-3} \chi(k-3, i, B) (1 - B^{k-3-i} B(m))^i U(k-i-3) (1 - B^{k-3-i}) \right] \right\} \end{aligned} \quad (6)$$

The above two equations are similar to equations (3) and (4), except that the node at the end of the adjacent link must be reached through other links if it is a destination. This is accounted for by the second term in equation (5), and $(1 - B^{k-3-i})$ in equation (6). The load shared between the target and the adjacent links, when both m and n are greater than zero is now expressed as:

$$\lambda_s^{(1)}(m, n) = \sum_{k=2}^{N-1} a_k d_k t_1(k) \sum_{i=0}^{k-2} \chi(k-2, i, B(m)) H(i|k-i) \quad , \quad (7)$$

$$\begin{aligned} \lambda_s^{(2)}(m, n) &= \sum_{k=3}^{N-1} a_k e_k (N-k) t_2(k) P_{ab} \\ &\cdot B(0) \sum_{i=0}^{k-3} \chi(k-3, i, B) (1 - B^{k-3-i} B(m))^i U(k-i-3)^2 \end{aligned} \quad (8)$$

In both of the above equations, the node at the end of the adjacent link must be a destination. However, in equation (8) that node must be unreachable on the *first* hop, and reachable through the adjacent link on the second hop. At the same time, the target link must be used to reach the node at its end. The last term in the equation accounts for these last two probabilities.

The probability $H(i|l)$ in this case is equal to $(1 - B^l)^i$

The elements of the transition matrix are similar to those in the direct routing case, but take into account the fact that two hops may be used by using $\lambda^{(1)} + \lambda^{(2)}$ instead of $\lambda^{(1)}$.

Finally, after solving for the steady state probabilities, and after obtaining $q(m)$, the probability of call success can be expressed as:

$$P(\text{success}) = \sum_{k=1}^{N-1} r_k \left\{ B \sum_{i=0}^{k-2} \chi(k-1, i, B(0)) H(i+1|k-i-1) + \sum_{m=1}^W q(m) \sum_{i=0}^{k-1} \chi(k-1, i, B(m)) H(i|k-i) \right\}. \quad (9)$$

In equation (9) the first term between the curly bracket corresponds to the case in which a chosen target link is blocked, but is reachable through any of the $k-i-1$ other nodes which have been reached on the first hop. The second term corresponds to the case in which the chosen target link is not blocked, and has m available channels.

4. NETWORKS WITHOUT WAVELENGTH CONVERSION

In this section we assume that the switches do not implement wavelength conversion. The derivation of the reduced load equations in this section is more involved since one has to keep track of the number of commonly available wavelength channels on a given path.

4.1. DIRECT ROUTING

This case is similar to networks with wavelength conversion.

4.2. INDIRECT ROUTING WITH A LIMIT OF TWO LEVELS

With a limit of two hops, a call can be routed from the source to the destination either:

- on the direct link, in which case any available channel can be used on the direct link; or
- if the direct link is blocked, the call can be routed via a destination that has been reached via a direct link. This requires that the wavelength used on the first link be available on the second link.

In this section, we require a few more probabilities, which are also defined in Appendix I.

We can now obtain the reduced load on the target link. Similar to subsection III.2, we have

$$\lambda^{(h)}(0, n) = 0, \quad \forall h,$$

$$\lambda^{(h)}(m, n) = 0, \text{ for } h > 2, \forall m, n.$$

The reduced load on the target link with m available channels, when it is not shared with the adjacent link that has $n, n > 0$, channels can be derived by considering the same situations considered under wavelength conversion. However, we have to involve the probabilities of wavelength continuity. The reduced load is given by

$$\lambda_d^{(1)}(m, n) = \sum_{k=1}^{N-1} a_k d_k (1 - t_1(k)) \sum_{i=0}^{k-1} \chi(k-1, i, B(m)) \cdot \sum_{j=0}^i F(j|i, m) H(i-j|k-i-1) \quad \text{for } n > 0, \quad (10)$$

$$\begin{aligned} \lambda_d^{(2)}(m, n) = & \sum_{k=2}^{N-1} a_k e_k (N-k) P_{a \& b} \times \\ & \left\{ \left[(1 - t_2(k)) \sum_{i=0}^{k-2} \chi(k-2, i, B) \sum_{j=1}^{i+1} G(j|i+1, m) \right. \right. \\ & \quad \left. \sum_{r=0}^{k-i-2} S(i+1-j, r|k-i-2) \times \frac{1}{r+1} \right] \\ & + t_2(k) \left[(1-B) \sum_{i=0}^{k-3} \chi(k-3, i, B) \sum_{j=1}^{i+1} G(j|i+1, m) \right. \\ & \quad \left. \sum_{r=0}^{k-i-2} S(i+1-j, r|k-i-2) \times \frac{1}{r+1} \right. \\ & + B \sum_{i=0}^{k-3} \chi(k-3, i, B) \left(\sum_{j=1}^{i+1} G_{\bar{a}}(j|i+1, m, n) \right. \\ & \quad \left. \sum_{r=0}^{k-i-3} S(i+1-j, r|k-i-3) \times \frac{1}{r+1} \right. \\ & + \sum_{j=1}^{i+1} G_a(j|i+1, m, n) \sum_{s=0}^{k-i-3} \sum_{r=0}^{k-i-3} T(i+1-j, r, s|k-i-3) \\ & \quad \left. \left. \times \frac{1}{r+1} \times \frac{s}{s+1} \right) \right] \right\} \quad (11) \end{aligned}$$

Equation (10) is straightforward. It is just to be noted that the last summation is the probability of reaching j out of i nodes from the end

of the target link (which has m available channels), and reaching the remaining $i - j$ nodes via the $k - i - 1$ nodes that have already been reached directly.

Similar to equation (8), the two terms between the curly brackets in equation (11) correspond to the cases of the node at the end of the adjacent link not being a destination, and being a destination, respectively. And the second term is divided into the probabilities of that node being already reached on the first hop, and being reached on the second hop, but through a different destinations, respectively. However, in all of these terms, we find the probability that the node at the beginning of the target link can reach j other nodes, in addition to the node at the end of this link, and we multiply it by the probability of reaching the rest of the destinations through other nodes. In doing so, we also cater for the probability that the node at the end of the target link can be reached by r other nodes, and that the target link will be used to reach it. In the case in which the node at the end of the adjacent link is a destination which will be reached on the second hop (last term which is multiplied by B for being blocked on the first hop), we distinguish between two cases: the case in which it cannot be reached on the adjacent link because of wavelength discontinuity; and the case in which it can be reached. In this last case we obtain the probability that the node at the end of the adjacent link can be reached by s other nodes, and one of those nodes will be used to reach it.

The case when the adjacent link is blocked can be considered similarly, but we take into account the fact that the node at the end of the adjacent link, if it is a destination, will be reached either on the first hop, or on the second hop but using another link. The expressions for the reduced load are given by

$$\begin{aligned} \lambda_d^{(1)}(m, 0) = & \lambda_d^{(1)}(m, 1) + \\ & + \sum_{k=1}^{N-1} a_k d_k t_1(k) \sum_{i=0}^{k-2} \chi(k-2, i, B(m)) \\ & \cdot \sum_{j=0}^{i+1} F(j|i+1, m) H(i+1-j|k-i-2) \end{aligned} \quad (12)$$

$$\begin{aligned} \lambda_d^{(2)}(m, 0) = & \sum_{k=2}^{N-1} a_k e_k (N-k) P_{a \& b} \\ & \times \left\{ (1 - t_2(k)) \sum_{i=0}^{k-2} \chi(k-2, i, B) \sum_{j=1}^{i+1} G(j|i+1, m) \right. \end{aligned}$$

$$\begin{aligned}
& \sum_{r=0}^{k-i-2} S(i+1-j, r|k-i-2) \times \frac{1}{r+1} \\
& + t_2(k) \left[(1-B) \sum_{i=0}^{k-3} \chi(k-3, i, B) \sum_{j=1}^{i+1} G(j|i+1, m) \right. \\
& \sum_{r=0}^{k-i-2} S(i+1-j, r|k-i-2) \times \frac{1}{r+1} \\
& + B \sum_{i=0}^{k-3} \chi(k-3, i, B) \sum_{j=1}^{i+1} G_a(j|i+1, m, 0) \\
& \left. \sum_{r=0}^{k-i-3} S(i+2-j, r|k-i-3) \times \frac{1}{r+1} \right] \quad (13)
\end{aligned}$$

Equation (12) is derived in a manner similar to that used in deriving equation (10), but taking into account the fact that the adjacent link has no available channels. This requires adding a new term which caters for the case in which the node at the end of the adjacent link is a destination. Equation (13) has been derived similar to equation (11), except that the node at the end of the adjacent node cannot be reached over the adjacent link ($G_a() = 0$).

Finally, the shared load between the target and the adjacent links is

$$\begin{aligned}
\lambda_s^{(1)}(m, n) = & \sum_{k=2}^{N-1} a_k d_k t_1(k) \sum_{i=0}^{k-2} \chi(k-2, i, B(m)) \\
& \sum_{j=0}^{i+1} F(i, j|m) H(i-j|k-i-1) \quad , \quad (14)
\end{aligned}$$

$$\begin{aligned}
\lambda_s^{(2)}(m, n) = & \sum_{k=3}^{N-1} a_k e_k (N-k) t_2(k) P_{a \& b} B(0) \\
& \times \left[\sum_{i=0}^{k-3} \chi(k-3, i, B) \sum_{j=1}^{i+1} G_a(j|i+1, m, n) \right. \\
& \left. \sum_{s=0}^{k-i-3} \sum_{r=0}^{k-i-3} T(i+1-j, r, s|k-i-3) \times \frac{1}{r+1} \times \frac{1}{s+1} \right] \quad (15)
\end{aligned}$$

for $m, n > 0$. The derivation of the above equations follows earlier derivations, except that in equation (15) the node at the end of the

adjacent link is reached over that link, even if that node can be reached by r other nodes.

In the Appendix II we show the derivation of $b^{(l)}(n|m)$, $F(j|i, m)$, $G(j|i, m)$, $G_a(j|i, m, n)$, $G_a(j|i, m, n)$, $S(i, n|l)$, $T(i, n, p|l)$ and $H(i|l)$.

From the above,

$$P(\text{success}) = \sum_{k=1}^{N-1} r_k \left\{ B \sum_{i=0}^{k-2} \chi(k-1, i, B(0)) H(i+1|k-i-1) + \sum_{m=1}^W q(m) \sum_{i=0}^{k-1} \chi(k-1, i, B(m)) \sum_{j=0}^i F(j|i, m) H(i-j|k-i-1) \right\} \quad (16)$$

5. NUMERICAL RESULTS

The expressions for the reduced load derived in the previous section assume the knowledge of the channel availability distribution. Since these probabilities depend in turn on the knowledge of the reduced loads, through the matrix $\mathcal{P}$, an iterative solution procedure must be used, in which λ is computed and fed back to the model. The iterative procedure ends when all λ 's converge, and the convergence criterion, ϵ , was chosen as 10^{-3} in all of the examples shown below. Convergence usually occurred very fast, and with a number of iterations that rarely exceeded 10.

We first validate the model by comparing the analytical results against results from simulation. In Table 1 we consider a network with six nodes, and three channels per fiber. Different load vectors are used, namely $V_1 = (1, 0, 0, 0, 0)$, $V_2 = (.2, .2, .2, .2, .2)$, $V_3 = (0, 0, 0, 0, 1)$ and $V_4 = (.5, .5, 0, 0, 0)$. The corresponding blocking probabilities are shown in the table. Most of the analytical results shown in the table are within 10% of simulation results. Table 2 show the corresponding examples for the case of no wavelength conversion.

From the above tables we conclude two things. First, the use of indirect routing rather than direct routing enhances the performance of the system. Indirect routing with no hop limit results in a performance which is marginally better than that under indirect routing with a two hop limit. Therefore, it is not really justifiable to use a significantly more complex control function in order to achieve that marginal enhancement. Second, the use of wavelength conversion reduces the probability of call blocking as compared to the absence of wavelength conversion.

In Figure 1 we study the effect of the number of channels per fiber on the call blocking probability for all routing strategies. We assume

Table 1 Blocking probabilities for a network with wavelength conversion; $N = 6$ and $W = 3$

Network parameters			P(Blocking):model		P(Blocking):simulation		
Load per node	Load vector	Load per ch.	direct routing	indirect with 2 hops	direct routing	indirect with 2 hops	indirect with no limit
1	V_1	.06667	.00109	.00109	.00103	.00103	.00103
1	V_2	.2	.0371	.0069	.0343	.0079	.0078
1	V_3	.3333	.0623	.0619	.0625	.0625	.0624
1	V_4	.1	.0049	.0017	.0049	.0017	.0017
2	V_1	.1333	.007156	.007156	.007268	.007268	.007268
2	V_2	.4	.1378	.0423	.1320	.0514	.048
2	V_3	.6667	.21056	.2105	.21169	.2106	.2106
2	V_4	.2	.0279	.01129	.0279	.0114	.0114

Table 2 Blocking probabilities for a network without wavelength conversion; $N = 6$ and $W = 3$

Network parameters			P(Blocking): model		P(Blocking):sim.		
Load per node	Load vector	Load per ch.	direct routing	indirect with 2 hops	direct routing	indirect with 2 hops	indirect with no limit
1	V_1	.06667	.00109	.00109	.00107	.00107	.00107
1	V_2	.2	.0371	.00777	.0343	.01016	.00875
1	V_3	.3333	.0623	.06263	.0625	.06248	.06244
1	V_4	.1	.0049	.001775	.0049	.0019	.0019
2	V_1	.1333	.007156	.007156	.007268	.007067	.007067
2	V_2	.4	.1378	.055355	.1320	.06653	.05817
2	V_3	.6667	.21056	.21052	.21169	.21052	.21051
2	V_4	.2	.0279	.01196	.0279	.01328	.01328

that the network has 10 nodes, with each node offering a total load of 2 Erlangs. Since in most networks, unicast applications are expected to dominate, we assume that the probability of a unicast call is 0.5, while the probability of a call with k destinations for $k = 2, 3, \dots, 9$ is equal to 0.0625. As expected, increasing the number of channels per fiber significantly reduces the call blocking probability. Adding one more channel per fiber reduces the call blocking probability by almost one order of magnitude for all cases. For example, under wavelength conversion, and with indirect routing with a limit of two hops, increasing the number of fibers per channel from 6 to 7 reduces the call blocking probability from 4.9×10^{-5} to 5×10^{-6} . We notice also that direct routing results in the worst performance, while the other indirect routing strategies result in almost the same performance. This confirms our earlier conclusion that the complexity of indirect routing with a number of hops beyond two is not justifiable.

Figure 1 Blocking probabilities versus the number of wavelength channels per fiber, when the node offered load is 2 Erlangs

Figure 2 Blocking probabilities versus the number of wavelength channels per fiber, when the node offered load is 2 Erlangs

In the last example, we study the effect of the number of destinations per call. We consider a network with 8 nodes, and 4 channels per fiber. Seven experiments are performed in which the number of destinations is fixed at k , where $k = 1, 2, \dots, 7$. Similar to the above, we assume a fixed offered load per node that is equal to 2 Erlangs. The results are shown in Figure 2. First, it should be noticed that in both the unicast and the broadcast cases, all routing strategies yield exactly the same

performance. This is true for the broadcast case since all links leaving any of the sources will be either all blocked, or all unblocked. It is also noticed that with direct routing, increasing the number of destinations increases the blocking probability. However, the broadcast case, $k = 7$, results in a slightly better performance because of the above mentioned correlation. With the indirect routing strategies, increasing the number of destinations from the unicast case improves the performance first, before it starts to degrade. This can be explained since with two destinations, a source has three alternative routes of reaching all the destinations compared to just one route for the case of one destination. This reduces the probability of call blocking. After that, the increased loading on the links degrades the performance.

6. CONCLUSIONS

In this paper we derived expressions for the call blocking probabilities in optical networks under multicast service in a completely connected network topology. We considered the case in which full wavelength conversion is allowed, and the case in which no wavelength conversion is used. For both cases, we considered the direct routing method, and the indirect routing method in which the maximum number of hops is restricted to two hops. The case in which there is no restriction on the number of hops is very involved, and the blocking probabilities were obtained using simulation only. The solution method was based on the reduced load approach, and was iterative in nature.

Based on the numerical examples, it was found that although indirect routing improved the blocking probabilities, the enhancement achieved by using more than two hops is marginal compared to the enhancement achievable with two hops compared to direct routing. Therefore, it is advantageous to use indirect routing with multicast service, but restricting the maximum number of hops to two usually yields a sufficiently good performance. It was also shown that increasing the number of destinations does not necessarily result in a degraded performance. On the contrary, since increasing the number of destinations, when employing indirect routing, increases the number alternative routes from the source to all the destinations, a better performance can be achieved.

Appendix: I

This appendix gives the definitions and notations used in this paper.

N number of nodes in the system.

W number of wavelength channels per link.

a offered load per node.

r_k probability that a call is destined to k destinations.

a_k offered load per node with k destinations = ar_k .

d_k probability that the node at the end of the link is chosen as a destination, given that the call is destined to k nodes out of $N - 1$ nodes = $\frac{k}{N-1}$.

e_k probability that the two nodes connected by a given link. are chosen as destinations, given that the call is destined to k nodes out of $N - 1$ nodes = $\frac{k(k-1)}{(N-1)(N-2)}$.

$t_i(k)$ probability that a given node is chosen as a destination for a call with k destinations, given i known destinations = $\frac{k-i}{N-i-1}$.

$\lambda^{(h)}(m, n)$ load carried on the link², given that the target and the adjacent links have m and n available channels, respectively, and the load is experiencing its h th hop.

$\pi_{m,n,l}$ the steady state probability that the target and the adjacent links have m and n available channels, respectively, and that l channels are being used for common calls.

$q(m)$ probability that the link has m idle wavelengths.

$B(m)$ probability that a link is blocked, given that the adjacent target link has m available channels = $\frac{\sum_{k=0}^{W-m} \pi_{0,m,k}}{q(m)}$.

B the unconditional link blocking probability = $q(0) = \sum_{m=0}^W B(m)q(m)$.

$P_{a\&b}$ probability that a link is not blocked, while its adjacent link is blocked = $\sum_{m=1}^W B(m) \cdot q(m)$.

$P_{b\&b}$ probability that the target and the adjacent links are both blocked = $B(0)q(0)$.

$\chi(i, j, p)$ probability of choosing j out of i events according to the binomial distribution with parameter $p = \binom{i}{j} p^j (1-p)^{i-j}$.

$H(i|l)$ probability of reaching i out of i destinations, given that l nodes have been reached.

²The load carried on a link is the reduced load offered to the link.

$U(k)$ probability of using the target link to reach the node at its end, given that k other links are available for that purpose

$$= \sum_{i=0}^k \binom{k}{i} (1-B)^i B^{k-i} \times \frac{1}{i+1} = \frac{1-B^{k+1}}{(k+1)(1-B)}.$$

$c(n|m, f, W)$ probability of n common available channels between two links, given m available channels on the first, f available channels on the second, and a total of W channels per link =

$$\binom{m}{n} \binom{W-m}{f-n} / \binom{W}{f}.$$

$b^{(l)}(n|m)$ probability that a link with m available channels has n available channels in common with l other links originating from the end of this particular link.

$F(j|i, m)$ P(reaching j destinations out of i destinations from the end of the first link, given m wavelengths on the link).

$G(j|i, m)$ P(reaching j out of i , including the node at the end of a particular second link, given m wavelengths on that particular link, and the first link is not blocked).

$G_{\bar{a}}(j|i, m, n)$ P(reaching j out of i nodes from the node at the end of the first link, and **not reaching** the node at the end of the adjacent link, given that m and n channels are available on the target and adjacent links, respectively).

$G_a(j|i, m, n)$ P(reaching j out of i nodes from the node at the end of the first link, and **reaching** the node at the end of the adjacent link, given that m and n channels are available on the target and adjacent links, respectively). P(reaching i out of i over the second hop, via l nodes

$T(i, n, p|l)$ P(reaching i out of i nodes over the second hop, via l nodes that are reached directly, and n of the l nodes can also reach the destination at the end of the target link, while p of the l nodes will reach the destination at the end of the adjacent node).

$S(i, n|l)$ P(reaching i out of i nodes over the second hop, via l nodes that are reached directly, and n out of the l nodes can also reach the destination at the end of the target link) = $\sum_{p=0}^l T(i, n, p|l)$.

Appendix: II

In this appendix we derive the auxiliary probabilities $b^{(l)}(n|m)$, $F(j|i, m)$, $G(j|i, m)$, $G_{\bar{a}}(j|i, m, n)$, $G_a(j|i, m, n)$, $S(i, n|l)$, $T(i, n, p|l)$ and $H(i|l)$

which are required in the derivation of the blocking probability under multicast service when no wavelength conversion is used, and when a maximum of two hops is imposed.

We first define $\eta_{i,j}$ and $\delta_{i,j}$ in exactly the same way e_i and d_i are defined, except that we condition on having a total of j possible destinations. Therefore, we have

$$\eta_{i,j} = \frac{i}{j}$$

$$\delta_{i,j} = \frac{i(i-1)}{j(j-1)}$$

The probability $b^{(l)}(n|m)$ can be obtained recursively as follows:

$$b^{(1)}(n_1|m) = \sum_{f_1=n_1}^{W-m+n_1} c(n_1|m, f_1, W)q(f_1)$$

$$b^{(l)}(n_l|m) = \sum_{n_{l-1}=n_l}^m \sum_{f_l=n_{l-1}}^{W-n_{l-1}+n_l} c(n_l|n_{l-1}, f_l, W)b^{(l-1)}(n_{l-1}|m)q(f_l)$$

for $l > 1$

$F(j|i, m)$ can now be computed using the following equation:

$$F(j|i, m) = \binom{i}{j} \sum_{n=1}^m b^{(j)}(n|m)F(0|i-j, n) \quad (17)$$

with the initial condition

$$F(0|i, m) = \left[\sum_{f=0}^{W-m} c(0|m, f, W)q(f) \right]^i = [b^{(1)}(0|m)]^i$$

To compute $G(j|i, m)$ it is required that out of the i destinations which must be reached on the second hop, j will be reached, including the one at the end of the target link which has m available wavelength channels. $G(j|i, m)$ can therefore be expressed as

$$G(j|i, m) = \sum_{f=1}^W \sum_{n=1}^{\min(m, f)} c(n|m, f, W) \frac{q(f)}{1-B} F(j-1|i-1, n) \quad (18)$$

In equation (18), $c(n|m, f, W)$ is the probability that, given that the considered second link has m available channels, and the direct link has

f channels, they both have n common available channels. $F(j-1|i-1, n)$ indicates the probability of reaching $j-1$ destinations out of the remaining $i-1$ destinations given those available n channels.

Once $G(j|i, m)$ has been calculated, one can proceed to calculate

$$G_a(j|i, m, n) = c(0|m, n, W)G(j|i, m), \quad \text{and}$$

$$G_a(j|i, m, n) = \sum_{r=1}^{\min(m, n)} c(r|m, n, W)G(j|i, r)$$

Finally,

$$S(i, n|l) = \sum_{j=0}^i \left(\sum_{f=1}^W F(j|i+1, f) \frac{q(f)}{1-B} \right) \cdot \frac{i+1-j}{i+1} S(i-j, n|l-1)$$

$$+ \sum_{j=0}^i \left(\sum_{f=1}^W F(j+1|i+1, f) \frac{q(f)}{1-B} \right) \cdot \frac{j+1}{i+1} S(i-j, n-1|l-1)$$
(19)

with $S(0, 0|0) = 1$, $S(i, n|l) = 0$ for $n > l$, $S(i, n|0) = 0$ for $i > 0$, $n \geq 0$, $S(0, 0|l) = 1$ for $l \geq 0$, $S(0, n|l) = 0$ for $l, n > 0$ and

$$S(i, 0|l) = \sum_{j=0}^i \left(\sum_{f=1}^W F(j|i+1, f) \frac{q(f)}{1-B} \right) \cdot \frac{i+1-j}{i+1} S(i-j, 0|l-1) . \quad (20)$$

We can also obtain

$$T(i, n, p|l) =$$

$$\sum_{j=0}^i \left(\sum_{f=1}^W F(j|i+2, f) \frac{q(f)}{1-B} \right) (1 - \eta_{j, i+2} - 2\delta_{j, i+2}) T(i-j, n, p|l-1)$$

$$+ \sum_{j=0}^i \left(\sum_{f=1}^W F(j+1|i+2, f) \frac{q(f)}{1-B} \right) g_{j+1, i+2} \sum_{k=0}^1 T(i-j, n-k, p|l-1)$$

$$+ \sum_{j=0}^i \left(\sum_{f=1}^W F(j+2|i+2, f) \frac{q(f)}{1-B} \right) \eta_{j+2, i+2} T(i-j, n-1, p-1|l-1)$$
(21)

with the following initial conditions:

$$\begin{aligned} T(i, n, p|0) &= 0 && \text{for } i \text{ or } n \text{ or } p > 0 \\ T(i, n, p|l) &= 0 && \text{for } n \text{ or } p > l \\ T(0, 0, 0|l) &= 1 && \text{for } l \geq 0 \\ T(0, n, p|l) &= 0 && \text{for } n \text{ or } p > 0 \end{aligned}$$

Similarly, but with less involvement we can compute the following:

$$H(i|l) = \sum_{j=0}^i \left(\sum_{f=1}^W F(j|i, f) \frac{q(f)}{1-B} \right) H(i-j|l-1) \quad (22)$$

with $H(0|l) = 1$ for $l \geq 0$, and $H(i|0) = 0$ for $i > 0$.

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