FILLET OPERATIONS WITH RECURSIVE SUBDIVISION SURFACES

Zheng XU

Dept.of Information and Computer Sciences, SAITAMA University

xuz@ke.ics.saitama-u.ac.jp

Kunio KONDO

Dept.of Information and Computer Sciences, SAITAMA University

kondo@ke.ics.saitama-u.ac.jp

Abstract Recursively subdividing polyhedral networks, often called polyhedral subdivision, has become one of the basic tools in Computer Aided Geometric Design(CAGD) for modeling complex surfaces. In this paper, we propose a set of rules for making fillet operations on polyhedral networks. The fillet operations are implemented by subdividing a polyhedral network one step with given fillet values of edges of the polyhedral network. After fillet operations, we tried two basic recursive subdivision methods Doo-Sabin and Catmull-Clark to generate surfaces. Generally, the number of edges that meet at one vertex is no restriction and the fillet values of every edge can be different. The fillet operations proposed here will strengthen the functions of Computer Graphics systems which use subdivision surfaces to model surfaces.

Keywords: Recursive Subdivision Surfaces, Doo-Sabin surface, Catmull-Clark surface, B-Spline Curves, Fillet Operations

1. INTRODUCTION

1.1. BACKGROUND

As polyhedral subdivision process provides a simple and efficient way to generate surfaces over polyhedral networks, it has been interested by a lot of researchers since the first two methods were proposed by E.E.Catmull, J.H.Clark[2] and D.Doo, M.Sabin[3] in 1978. A lot of approaches were made or are being made to follow or extend polyhedral subdivision method. Nasri extended Doo-Sabin method to generate a surface which interpolates some or all vertices of a polyhedron[4],More-

over, he proposed an algorithm to generate subdivision surfaces which interpolate B-spline curves[5][6]. J.Peters[9] extended the subdivision technology to generate surfaces by separating singular regions after a few steps of Doo-Sabin subdivision. M.Halstead et al.[10] proposed an interpolation method using Catmull-Clark surfaces. N.Dyn, D.Levin and J.Gregory[8] proposed a butterfly subdivision scheme. U. Reif[7] gave a serious analysis on the smoothness properties of subdivision surfaces such as Doo-Sabin and Catmull-Clark surfaces. Recently the success of CG animation system using subdivision surfaces developed by Pixar Animation Studios[11] showed the recursive subdivision surfaces will be an essential role in modeling surfaces.

1.2. DOO-SABIN AND CATMULL-CLARK SUBDIVISION SURFACES

As in our approach, the surfaces are generated by Doo-Sabin and Catmull-Clark subdivision methods, we overview these two methods in this section.

1.2.1 Chaikin's algorithm. The fundamental idea of both Doo-Sabin surfaces and Catmull-Clark surfaces goes back to Chaikin's [1] algorithm which generates a quadratic B-spline curve from a polygon by successively cutting its corners. Each subdivision generates two new points on each polygon leg at $(1/4, 3/4)$. For a polygon with n vertices $(V_i)_{1 \leq i \leq n}$, on polygon leg (V_i, V_{i+1}) , two new points are given by the following:

$$V_i'' = \frac{3}{4}V_i + \frac{1}{4}V_{i+1}, \quad V_{i+1}' = \frac{1}{4}V_i + \frac{3}{4}V_{i+1}.$$

Then the new polygon is linked by the new generated points V_i', V_i'' corresponding to $(V_i)_{1 \leq i \leq n}$. Repeat the subdivision process, we can get a quadratic B-spline curve.

1.2.2 Doo-Sabin and Catmull-Clark subdivision method.

In 1978, D.Doo, M.Sabin[3] applied Chaikin's idea to generating surfaces. In Doo-Sabin's method, surfaces are generated from polyhedral networks by successively cutting polyhedron's corners and edges. The algorithm can be described as follows and illustrated in Fig.1, some terms proposed by Nasri[4] are used hereunder:

- 1 For every vertex V_i of the polyhedron P_i , a new vertex V_i' , termed *image*, is generated on each face adjacent to V_i .

- 2 For each face F_i of P_i , a new face, termed *F-face*, is made by connecting the *images*, the vertices V'_i 's generated in step 1.
- 3 For each edge E_i common to two faces F_i and F'_i , a new 4-sided face, termed *E-face*, is made by connecting the *images* of the end vertices of E_i on the faces F_i and F'_i .
- 4 For each vertex V_i , where n faces meet, a new face, termed *V-face*, is made by connecting the *images* of V_i on the faces meeting at V_i .

The *image* vertex V'_i generated in step 1 is functions only of the vertices of P_i . That is :

$$V'_i = \sum_{j=1}^n a_{ij} V_j$$

Where V_j are the vertices of the old faces and V'_i is the new vertex of V_j , and a_{ij} are weights.

$$a_{ij} = \frac{n+5}{4n} \quad \text{for } i = j,$$

$$a_{ij} = \frac{3 + 2 \cos(2\pi(\frac{i-j}{n}))}{4n} \quad \text{for } i \neq j.$$

Fig.1(a) illustrates the *F-face*, *E-face* and *V-face* of Doo-Sabin's subdivision method. The face of a polyhedron is closed in solid nodes and generated *F-face*, *E-face* and *V-face* are closed in hollow nodes. Fig.1(b)-(e) illustrate a cube and make Doo-Sabin's subdivision process over it. Doo-Sabin surfaces are composed of biquadratic splines. Using similar idea, Catmull and Clark implemented a cubic spline surface simultaneously in 1978. Fig.2 illustrates a Catmull-Clark surface generated from a cube.

1.3. MOTIVATIONS

Surfaces can be generated easily by subdividing polyhedral networks recursively, but complex or precise polyhedral networks have to be defined. For example, for getting a cylinder-like shape (Fig.3(d)) with Doo-Sabin subdivision method, a polyhedron shown in Fig.3(b) has to be defined. we can not get cylinder-like shape by subdividing a simple cube (Fig.3(a)). Over simple or rough polyhedral network, it is difficult to get the surfaces which designers want to model. According to the subdivision method(Doo-Sabin, Catmull-Clark etc.), special polyhedral networks have to be defined. Obviously, it is troublesome and inefficient. So for a system that uses subdivision method to generate surfaces, some

Figure 1 Doo-Sabin's method.(a) Three types of faces.(b)A cube. (c)The cube and its first subdivision.(d)The cube and its second subdivision. (e)The cube and its fourth subdivision.

Figure 2 Catmull-Clark's method. (a) The cube and its first subdivision. (b) The cube and its fourth subdivision.

operations have to be developed for supporting the subdivision method. In this paper, we propose a set of rules for implementing fillet operations, an essential process in design systems. In our method, according to fillet values of every edge of polyhedral networks, the initial polyhedral networks are subdivided one step using the rules proposed here, and then Doo-Sabin subdivision or Catmull-Clark process is carried out over the modified polyhedral networks. The number of edges that meet at one vertex is no restriction and the fillet values of every edge can be different.

2. THE RULES OF SUBDIVISION IN FILLET OPERATIONS

2.1. DEFINITIONS AND REGULAR PROCESS

Before giving our rules, some terms should be defined. First is the *fillet value*. It describes the sharpness of an edge and is defined as the distance between an edge and a new parallel line, termed *fillet line*, on the faces that the edge is common to. Each edge has two *fillet values* and two *fillet lines*. The two *fillet values* can be different. The *fillet value* of an edge also can be 0. Now we assume the fillet value is not equal to 0. Similar to Doo-Sabin subdivision method, the fillet operations will subdivide the initial polyhedral networks and generate new faces *F-face*, *E-face* and *V-face* according to initial polyhedra's faces, edges and vertices. For describing our method smoothly, first, we make fillet operations on a cube with same fillet value for all edges. The fillet process can be described as follow steps (Referring to Fig.4):

Figure 3 A Cylinder-like shape generated by Doo-Sabin subdivision method. (a) A Cube. (b) The modified cube. (c) The cube and its third subdivision. (d) The fourth subdivision.

Figure 4 The regular subdivision of fillet operations. Solid circle nodes are the vertices of the cube, hollow circle nodes are the fillet edge points. Solid square nodes are fillet face points.

Figure 5 The regular fillet operation on a cube. (a) A Cube.(b)The modified cube after fillet operations. (c)The fourth Doo-Sabin subdivision over modified cube.

- 1 For each vertex V_i on a face F_s , an intersection point of two *fillet lines*, corresponding to the two contiguous edges that have common end point V_i , is generated. The intersection point called *fillet face point* termed W_{si} (Solid square nodes in Fig.4).
- 2 For each edge E_i linking vertex V_k, V_j and shared by face F_s, F_t , four intersection points of the E_i and fillet lines on the two faces F_s, F_t are generated, respectively. They are called *fillet edge points*, termed $P_{ij}^s, P_{ij}^t, P_{ik}^s$ and P_{ik}^t (Hollow circle nodes in Fig.4). Corresponding to V_j , there are two points P_{ij}^s and P_{ij}^t are generated on edge E_i , but only one point between P_{ij}^s and P_{ij}^t will be used to generate new faces, the useful point is termed P_{ij} . Similarly corresponding to V_k , the useful point is P_{ik} . Here, the three points P_{ij}^s, P_{ij}^t and P_{ij} are same point. Three points P_{ik}^s, P_{ik}^t and P_{ik} are same point too. For example, in Fig.4, P_{00}^1, P_{00}^0 and P_{00} are same point.
- 3 For every face F_i , a new face, termed as $(F - face)_i$ is generated by linking all *fillet face points* W_{ij} (Solid square nodes in Fig.4) on the face F_i .
- 4 For each edge E_i linking vertices V_j, V_k , common to two faces F_s, F_t , two new 4-sided faces, termed $(E - face)_{sj}$ and $(E - face)_{ti}$ are generated by linking $(W_{sj}, W_{sk}, P_{ik}, P_{ij})$ and $(W_{tj}, W_{tk}, P_{ik}, P_{ij})$, respectively.
- 5 For each vertex V_i , a 4-sided face termed $(V - face)_{ji}$, is generated on every face F_j that V_i is common to. It is linked by V_i , the fillet face point W_{ji} and two fillet edge points. For example, in Fig.4 the $(V - face)_{00}$ is linked by $(V_0, P_{00}, W_{00}, P_{20})$.

Figure 6 Rule 1. Fillet values are not equal each other. Solid circle nodes are vertices of the polyhedron, hollow circle nodes are fillet edge points.

After subdividing the cube according to above steps, subdivision process will be carried out recursively. Fig.5 illustrates the procedure of making fillet operations on a cube with Doo-Sabin surface.

2.2. RULE 1

The above process is a special case. In general, the fillet values of the edges of a polyhedron are not equal each other. Here we give the rule 1. Referring to Fig.6, for every edge E_i linking vertices (V_k, V_j) , common to faces F_s and F_t , the fillet values, distances between P_{ij}^s and V_j , P_{ij}^t and V_j , are D_s and D_t , respectively. If D_s is larger than D_t , P_{ij}^s will be treated as useful fillet edge point P_{ij} . Here the V_j must be a convex point on both F_s and F_t .

2.3. RULE 2

Referring to Fig. 7, for a face F_s and a vertex V_i of F_s , two edges of F_s having common end point V_i are E_j and E_k . Corresponding to E_j and E_k , the two fillet lines on F_s are L_j and L_k . If the useful fillet edge points P_{ji} and P_{ki} are not P_{ji}^s and P_{ki}^s , respectively, through point P_{ji} make a line parallel to E_k on F_s and get the intersection point A (hollow square node) on line L_j . Similarly, corresponding to P_{ki} , we get another intersection point B (hollow square node) on line L_k . Then the (*V-face*) on F_s corresponding to V_i will be two 4-sided faces (V_i, P_{ji}, A, W_{si}) and (V_i, W_{si}, B, P_{ki}) .

Figure 7 Rule 2. V-face is constructed by two 4-sided faces. Solid square nodes are *fillet face points*. Hollow circle nodes are the *fillet edge points*. Small solid circle nodes means there are some faces between F_t and F_q .

Figure 8 Rule 3. The fillet subdivision process of a concave point.

Figure 9 Rule 4. Fillet subdivision process will not be made on *un-boundary edges*.

2.4. RULE 3

If a vertex V_i on a face F_s is a concave point, two edges of F_s linking V_i are E_j and E_k . Corresponding to E_j and E_k , the two fillet lines on F_s are L_j and L_k . Because the V_i is a concave point on the face F_s , the fillet edge points P_{ji}^s and P_{ki}^s do not exist, the fillet edge points P_{ji} and P_{ki} are decided by the fillet lines on other two faces sharing E_j and E_k with face F_s , respectively. Similar to the rule 2, through point P_{ji} make a line parallel to E_k and get the intersection point A on L_j . Similarly we can get intersection point B. Then the (*V-face*) on the face F_s corresponding to V_i is two 4-sided face (V_i, W_{si}, A, P_{ji}) and (V_i, P_{ki}, B, W_{si}) . (Referring to Fig.8)

2.5. RULE 4

Referring to Fig.9, in some cases, for controlling recursive subdivision surfaces, some faces or concave faces are divided, even on a same plane, this means there are some edges shared by two faces that are on a same plane. In this case, the fillet operations will not be carried out and these edges are called *un-boundary edges*. *Boundary edges* are the edges shared by two faces which are not on a same plane. The fillet operations will be done only on the *boundary edges*. After fillet subdivision, the *un-boundary edges* are linked by new generated *fillet face points* or the new points on the *fillet lines*, correspondingly. For example, in Fig.9(a), F_s is a face including a concave point V_i , for controlling the final generated shape, an edge linking V_i and V_j is added. The fillet operations will not be done on this *un-boundary edge*. It will be modified by linking W_{sj} and W_{si} . The Fig.9(b) illustrates another case.

In rule 1 to rule 4, we assume the fillet values of the edges are not equal to 0. If the fillet values of some edges are equal to 0, this means the fillet operations are not carried out on these edges. The edge will be made round by subdivision process. We give rule 5 to deal with this case.

279

Figure 11 Rule 5. The two fillet values of an edge are equal to 0.

V -faces of two end points V_j and V_k will not be generated. Fig.11(a) shows the case that fillet values on both side of an edge are equal each other. Fig.11(b) shows the general case.

3. EXAMPLES

In this section, we make fillet operations on some polyhedra with the above rules. For demonstrating the effect of fillet operations, we use Doo-Sabin and Catmull-Clark methods to generate surfaces. Fig.12 illustrates fillet operations on a polyhedron with some vertices having valence four. The fillet values of each edge are different. Fig.12(a) is the original polyhedron. Fig.12(b),(c) are Doo-Sabin surface and Catmull-Clark surface generated from Fig.12(a) after 5 iterations, respectively. Fig.12(d) is the polyhedron after fillet subdivision process. Fig.12(e),(f) are Doo-Sabin surfaces generated from Fig.12(d). Fig.12(g),(h) are Catmull-Clark surfaces generated from Fig.12(d). Fig.13 illustrates fillet operations on a polyhedron with a vertex having valence six. Fig.14 illustrates filler operations for modeling a mouse. Fig.15 shows two other examples. Fig.15(c),(d) show two schemes of making fillet operations on same initial polyhedron with different fillet values. Fig.15(g) shows a club. Making comparison between the models generated from original polyhedra and the models generated from the polyhedra modified by fillet operations, we can see the effect of fillet operations clearly.

Fig. 12(a)Original polyhedron

Fig. 12(b)Doo/Sabin surface of Fig. 12(a)

Fig. 12(c)Catmull/Clark surface of Fig. 12(a)

Fig. 12(d)After fillet operations on Fig. 12(a)

Fig. 12(e)Doo/Sabin surface of Fig. 12(d)

Fig. 12(f)Doo/Sabin surface of Fig. 12(d)

Fig. 12(g)Catmull/Clark surface of Fig. 12(d)

Fig. 12(h)Catmull/Clark surface of Fig. 12(d)

Figure 12. Fillet operation on a polyhedron with some vertices having valence 4.

Fig. 13(a)Original polyhedron

Fig. 13(b) Doo/Sabin surface after 5 iterations on Fig. 13(a)

Fig. 13(c)Catmull/Clark surface after 5 iterations on Fig. 13(a)

Fig. 13(d)After fillet operations on Fig. 13(a)

Fig. 13(e)Doo/Sabin surface after 1 iteration on Fig. 13(d)

Fig. 13(f)Doo/Sabin surface after 5 iterations on Fig. 13(d)

Fig. 13(g)Catmull/Clark surface after 1 iteration on Fig. 13(d)

Fig. 13(h)Catmull/Clark surface after 5 iterations on Fig. 13(d)

Figure 13. Fillet operation on a polyhedron with a vertex having valence 6.

4. CONCLUSIONS

In this paper, some rules for implementing the fillet operations with recursive subdivision surfaces are proposed. The polyhedral networks modified by fillet operations can be used to generate both Doo-Sabin surfaces and Catmull-Clark surfaces. The operations are simple and effective. Generally, there is no restriction on the number of edges that meet at one vertex, and fillet values on each edge can be different. It will strengthen the functions of systems that use subdivision method to model surfaces. There are some restrictions here. For example, if the polyhedral networks modified by fillet operations are used by Doo-Sabin method. We can not reach the round areas defined. For solving this problem, we will try to apply Non-Uniform Recursive Subdivision Surfaces[14] to our system in next step.

References

- [1] George Merrill Chaikin: *An Algorithm for High-Speed Curve Generation*, Computer Graphics and Image Processing, No.3, pp.346-349,(1974).
- [2] Catmull, E. and Clark, J.: *Recursively generated B-spline surfaces on arbitrary topological meshes*, Computer Aided Design, Vol.10, No.6, pp.350-355,(Nov.1978).
- [3] Doo, D. and Sabin, M.: *Behaviour of recursive division surfaces near extraordinary points*, Computer Aided Design, Vol.10, No.6, pp.356-360,(Nov.1978).
- [4] Nasri, Ahmad H.: *Polyhedral Subdivision Methods for Free-Form Surfaces*, ACM Transaction on Graphics, Vol.6, No.1,pp.29-73, (Jan.1987).
- [5] Nasri, A.: *Curve interpolation in recursively generated B-spline surfaces over arbitrary topology*, Computer aided Geometric Design, No.14, pp.13-30, (1997).
- [6] Nasri, A.: *interpolation of Open Curves by Recursive Subdivision Surfaces*, Mathematics of Surfaces VII, Information Geometers, pp.173-188, (1997).
- [7] Reif, U.: *A unified approach to subdivision algorithms near extraordinary vertices*, Computer aided Geometric Design, No.12,pp.153-174,(1995).
- [8] Dyn, N., Levin, D., Gregory, J. A.: *A Butterfly Subdivision Scheme for Surface Interpolation with Tension Control* ACM Transactions on Graphics, Vol.9, No.2, pp.160-169, (Apr.1990)

- [9] Peters, J.: *Smoothing Polyhedra Made Easy*, ACM Transactions on Graphics, Vol.14.No.2.pp.162-170,(Apr.1995).
- [10] Halstead, M., Kass, M. and DeRose, T.: *Efficient, Fair Interpolation Using Catmull-Clark Surfaces*, Computer Graphics Proceedings, Annual Conference Series,(1993).
- [11] T.DeRose, M.Kass and T. Truong: *Subdivision Surfaces in Character Animation*, Computer Graphics Proceedings, Annual Conference Series, SIGGRAPH'1998, pp.85-94.
- [12] Zorin,D., Schroder,P. and Sweldens,W.: *Interpolating Subdivision for Meshes with Arbitrary Topology*, Computer Graphics Proceedings, Annual Conference Series, pp.189-192,(1996).
- [13] Farin, G.: *Curves and surfaces for Computer Aided Geometric Design:a practical guide*,4th Edition, Academic Press.(1996).
- [14] Thomas W.Sederberg, Jianmin Zheng,David Sewell, Malcolm Sabin :*Non-Uniform Recursive Subdivision Surfaces*, Computer Graphics Proceedings, Annual Conference Series,SIGGRAPH'1998,pp.387-394.