Adaptive Tool Path Planning for 5-Axis Sculptured Surface Machining by Machining Strip Evaluation

Yuan-Shin Lee, Ph.D., P.E. (e-mail: yslee@eos.ncsu.edu; <http://www.ie.ncsu.edu/yslee/>)
IE Dept., North Carolina State Univ., Raleigh, NC 27695-7906, U.S.A.

Abstract: This paper presents a new approach to non-isoparametric 5-axis NC tool path generation for complex surface machining by using the techniques of feasible machining strip evaluation. A searching algorithm is developed to find the parameter increments of adjacent cutter locations along orthogonal path intervals for non-isoparametric path generation. The techniques presented in this paper can be used to automate the non-isoparametric 5-axis cutter path generation and reduce the total machining time for CAD/CAM systems.

1. INTRODUCTION

Tool paths for sculptured surface machining are traditionally generated by guiding cutters to trace along constant parametric curves on the surfaces [Deng 96, Lee 95a]. An example of the conventional constant parametric cutter path generation for sculptured surface machining is shown in Figure 1. The distance between the parallel trajectories is the CC path interval, which depends on the local surface shape, cutter size and the allowable scallop height remaining on the surface after machining [Chen 98, Jerard 91, Lin 96, Lee 96b]. To achieve machined surface quality, the scallops left on the part surface need to be controlled within allowable tolerance. In the constant parametric (isoparametric) path generation methods, tool path distribution is determined by calculating, at each path, the smallest tool path interval and using it as a constant interval for the next tool path [Altan 93, Ma 98]. One serious problem of the traditional (isoparametric) method is the inefficient machining due to the non-predictable scallop remaining on the part surface [Lin 96, Lee 97b, Suresh 98]. Since the next tool path is generated by the smallest path interval between adjacent tool paths, redundant machining overlap occurs between the two adjacent tool paths, which causes machining inefficiency. In this paper, we adopt a new approach for the determination of efficient tool paths by using the machining strip evaluation method and a non-isoparametric path generation algorithm, which results in shorter total tool path than the conventional constant parametric tool path methods achieve.

Figure 1. Constant parameter (Isoparametric) tool path distribution

The original version of this chapter was revised: The copyright line was incorrect. This has been corrected. The Erratum to this chapter is available at DOI: [10.1007/978-0-387-35392-0_40](https://doi.org/10.1007/978-0-387-35392-0_40)

2. FEASIBLE MACHINING STRIP EVALUATION

Effective cutting shape analysis was developed in our earlier work presented in [Lee 96a, 97a, 98b]. During machining, a surface tolerance h is given to maintain the machined surface accuracy. During machining, the machined region that lies within the required surface tolerance is called the *machining strip* (w) [Lee 97a], as shown in Figure 2. In Figure 2, the local surface curvature (normal to the cutting direction $\vec{X}_L$) is represented as κ . The radius of surface curvature can then be calculated as $\frac{1}{\kappa}$. A local coordinate system is defined on the cutter contact point C^* . The effective cutting edge $E(\theta)$ intersects the offset surface at points C_a and C_b . The region between points C_a and C_b is the feasible machining strip, and the machined surface errors within the machining strip are within the given tolerance h , as shown in Figure 2. Details of finding the feasible machining strip w can be found in our earlier work in [Lee 97a]. As shown in Figure 2, the machining strip width w at the current cutting position C^* with tool orientation (λ_L, ω_L) can be found as:

$$w = w_a + w_b \quad (1)$$

It is noticed that, given the same tool orientation (λ_L, ω_L) , the feasible machining strip width w of the same effective cutting shape $E(\theta)$ in different surface regions has the relationship of $(w_{concave} > w_{planar} > w_{convex})$ [Lee 97a].

Cutting direction (X_L) out from the paper

Figure 2. Machining strip width w at cutter contact point C^* [Lee 97a]

3. TOOL PATH DISTRIBUTION BY MACHINING STRIP EVALUATION

To generate cutter paths to achieve the required surface quality, the machining strips of the two adjacent cutter paths have to overlap each other to ensure no machined surface error exceeds the tolerance limit h . As shown in Figure 3, given a surface tolerance h , the machining strip width w_{i+1} of the new adjacent tool path S_{i+1} should be connected to the machining strip w_i of the current tool path S_i to ensure the required surface finish is satisfied. To ensure the scallop height is no larger than the required surface finish, a distance D between the current cutter contact point $C_{i,j}$ and the adjacent cutter contact point $C_{i+1,j}$ should be no larger than the connected machining strips as follows [Lee 98a]:

$$w_{b,i} + w_{a,i+1} \geq D \quad (2)$$

where, $w_{b,i}$ is the right portion of the machining strip w_i at cutter contact point $C_{i,j}$, and $w_{a,i+1}$ is the left portion of the adjacent machining strip w_{i+1} at $C_{i+1,j}$, as shown in Figure 3.

S_i : Current tool path
 S_{i+1} : Adjacent new tool path
 $C_{i,j}$: Cutter contact point on tool path S_i
 $C_{i+1,j}$: Adjacent cutter contact point on tool path S_{i+1}
 For overlapping of machining strip: $w_{b,i} + w_{a,i+1} > D$

Figure 3. Overlapping of machining strips on adjacent tool paths

S_i, S_{i+1} : Adjacent tool paths
 $C_0(u_0, v_0)$: Current cutter contact point on tool path S_i
 $C^*(u^*, v^*)$: Adjacent cutter contact point on tool path S_{i+1}
 R_u, R_v : Tangential vectors at C_0 along u - and v -isoparametric directions
 δ_k : Searching step size
 X_{L0}, X_{L*} : Cutting directions at C_0 and C^*
 D : Path interval orthogonal to cutting direction X_L

Figure 4. Searching for the new adjacent cutter contact point $C^*(u^*, v^*)$ on path S_{i+1}

To calculate the precise tool path interval D , the direction of the path interval has to be orthogonal to the tool path itself [Lee 98a]. The orthogonal path interval cannot be adopted directly in parametric domains (u, v) because the direction of the path interval is usually not oriented in the same direction as the constant parametric curve, as shown in Figure 4. Therefore, to find the optimal tool path distribution, we must solve the tool path interval D and the parametric interval $(\Delta u$ or $\Delta v)$ in the orthogonal direction. As shown in Figure 4, a searching process is needed so that the feasible orthogonal path interval D is equivalent to the

feasible machining strip width as being constrained by Equation (2). The searching starts from the current cutter contact point C_0 and moves along the direction orthogonal to the cutting direction $\vec{X}_L$ to find the adjacent cutter contact point C^* . To maintain the accuracy, we calculate a searching step size δ_k based on the local surface curvature κ and the predefined searching tolerance τ , where τ is a sufficiently small tolerance. Given the current cutter contact point $C_0(u_0, v_0)$, the local surface curvature κ along the orthogonal direction $\vec{Z}_L$ -axis can be found. From Figure 5, one has the following relationship:

$$\tau = \frac{1}{\kappa} (1 - \cos \psi) \quad (3)$$

The feasible step size δ_k can be calculated as follows (also in Figure 5):

$$\delta_k = 2 \frac{1}{\kappa} \sin \psi \quad (4)$$

where $\psi = \cos^{-1}(1 - \tau \kappa)$ from Equation (3). A conversion from the physical domain of step forward size ($\delta_k \vec{Z}_L$) to the parameter increments ($\Delta u_k, \Delta v_k$) in the parametric domain is needed in order to place the tool on the calculated adjacent path.

Figure 5. Determination of searching step size δ_k along path interval

4. SOLVING PARAMETER INCREMENTS OF THE SEARCHING STEPS

The parameter increments ($\Delta u_k, \Delta v_k$) of a searching step δ_k along the orthogonal path interval direction can be found by using geometric analysis of the local area. As shown in Figure 6, the searching vector $\vec{Z}_L$ is defined by the current cutting direction $\vec{X}_L$ and the surface normal $\vec{Y}_L$, ($\vec{Z}_L = \vec{X}_L \times \vec{Y}_L$). On the parametric part surface $R(u, v)$, the adjacent cutter contact point C^* can be found by searching a series of *marching points* with a step size δ_k along the searching direction $\vec{Z}_L$ from the current cutter contact point $C_0(u_0, v_0)$, as shown in Figure 6(a). Figure 6(b) shows that, when moving along the searching direction $\vec{Z}_L$ with a step size δ_k , it is equivalent to move ($\Delta u \vec{R}_u$) along $\vec{R}_u$ direction and ($\Delta v \vec{R}_v$) along $\vec{R}_v$ direction, where $\vec{R}_u$ and $\vec{R}_v$ are the tangential vectors along u - and v -parametric direction respectively. The parameter increments Δu_k and Δv_k in the parameter domain can be determined by solving the following two linear equations (see Figure 6):

$$\Delta u_k (\vec{R}_u \cdot \vec{X}_L) + \Delta v_k (\vec{R}_v \cdot \vec{X}_L) = 0 \quad (5)$$

$$\Delta u_k (\vec{R}_u \cdot \vec{Z}_L) + \Delta v_k (\vec{R}_v \cdot \vec{Z}_L) = \delta_k \quad (6)$$

where, $\vec{X}_L$ is the current cutting direction, $\vec{Z}_L$ is the searching direction (orthogonal to $\vec{X}_L$), $\mathbf{R}_u$ and $\mathbf{R}_v$ are the tangential vectors along u and v , respectively. By solving Equations (5) and (6), we can find the parameter increments Δu_k and Δv_k as follows:

$$\Delta u_k = \frac{\begin{vmatrix} 0 & (\mathbf{R}_v \vec{X}_L) \\ \delta_k & (\mathbf{R}_v \vec{Z}_L) \end{vmatrix}}{\begin{vmatrix} (\mathbf{R}_u \vec{X}_L) & (\mathbf{R}_v \vec{X}_L) \\ (\mathbf{R}_u \vec{Z}_L) & (\mathbf{R}_v \vec{Z}_L) \end{vmatrix}} = \frac{-\delta_k (\mathbf{R}_v \vec{X}_L)}{(\mathbf{R}_u \vec{X}_L)(\mathbf{R}_v \vec{Z}_L) - (\mathbf{R}_u \vec{Z}_L)(\mathbf{R}_v \vec{X}_L)} \quad (7)$$

$$\Delta v_k = \frac{\begin{vmatrix} (\mathbf{R}_u \vec{X}_L) & 0 \\ (\mathbf{R}_u \vec{Z}_L) & \delta_k \end{vmatrix}}{\begin{vmatrix} (\mathbf{R}_u \vec{X}_L) & (\mathbf{R}_v \vec{X}_L) \\ (\mathbf{R}_u \vec{Z}_L) & (\mathbf{R}_v \vec{Z}_L) \end{vmatrix}} = \frac{\delta_k (\mathbf{R}_u \vec{X}_L)}{(\mathbf{R}_u \vec{X}_L)(\mathbf{R}_v \vec{Z}_L) - (\mathbf{R}_u \vec{Z}_L)(\mathbf{R}_v \vec{X}_L)} \quad (8)$$

Let (u_k, v_k) denote the current marching point C_k , then the new marching point $C_{k+1}(u_{k+1}, v_{k+1})$ for the next searching can be found as follows:

$$u_{k+1} = u_k + \Delta u_k \quad \text{and} \quad v_{k+1} = v_k + \Delta v_k \quad (9)$$

C_0 : Current cutter contact point
 XL : Cutting direction
 YL : Surface normal
 ZL : Path interval direction
 $E(\theta)$: Effective cutting shape on $YL-ZL$ plane

(a)

C_0 : Current cutter contact point at $R(u_0, v_0)$
 $R(u, v_0)$: Iso-parametric curve along $v = v_0$
 $R(u_0, v)$: Iso-parametric curve along $u = u_0$
 δ_k : Step size along path interval direction ZL
 $\mathbf{R}_u, \mathbf{R}_v$: Tangential vectors along u - and v -parametric curves

(b)

Figure 6. Solving parameter increments u and v of path interval along ZL axis

5. FINDING THE ADJACENT CUTTER LOCATIONS

Given a cutter contact point $C_0(u_0, v_0)$ on tool path S_i , we would like to find the adjacent cutter contact point C^* on the new path S_{i+1} . The searching starts by finding an initial guess point C_g on the adjacent tool path S_{i+1} . As shown in Figure 7, the machining strip width w_{C_0} at the current cutter contact point C_0 is calculated by offsetting a given tolerance h from part surface. Assume D is the path interval between the current cutter contact point C_0 and the guess point C_g , one can find the following relationship from Figure 7 as:

$$\frac{D}{\left| \frac{1}{\kappa} \right|} = \frac{w_{C0}}{\left| \frac{1}{\kappa} - h \right|} \quad (10)$$

where, $\frac{1}{\kappa}$ is the surface curvature normal to cutting direction $\vec{X}_L$. The distance D between C_0 and C_g can be represented as:

$$D = \frac{w_{C0}}{1 - \kappa h} \quad (11)$$

C₀: Current cutter contact point (on path S_i)
C_g: Initial guess point for new cutter contact point on adjacent path S_{i+1}

Figure 7 Determination of the initial guess point C_g using w_{C0} for adjacent tool path S_{i+1}

To find the initial guess point C_g , with a searching step size δ_k calculated by Equation (4), one needs to know the number of searching steps from C_0 to C_g . The number of searching steps, n_s , can be found by the following procedure as shown in Figure 8. For each searching step of the marching point C_k with step size δ_k , one can find the following relationship from Figure 8:

$$\psi_k = 2 \sin^{-1} \left(\frac{\delta_k}{2 \left| \frac{1}{\kappa} \right|} \right) \quad (12)$$

For the distance D calculated by Equation (11), we have the following relationship (Figure 8):

$$\psi_{Total} = 2 \sin^{-1} \left(\frac{D}{2 \left| \frac{1}{\kappa} \right|} \right) \quad (13)$$

From Figure 8, the following relationship exists:

$$\psi_{Total} = n_s \psi_k \quad (14)$$

Substituting Equations (12) and (13) into Equation (14), we can solve the number of searching steps, n_s , as follows:

$$n_s = \frac{\sin^{-1} \left(\frac{\left| \kappa \right| D}{2} \right)}{\sin^{-1} \left(\frac{\left| \kappa \right| \delta_k}{2} \right)} \quad (15)$$

After marching along $\vec{Z}_L$ direction with n_s searching steps, we can find the guess point $C_g(u_g, v_g)$ as follows (in Figure 8):

$$C_g = R(u_g, v_g) = R\left(u_0 + \sum_{k=1}^{n_s} \Delta u_k, v_0 + \sum_{k=1}^{n_s} \Delta v_k\right) \quad (16)$$

Using Equation (16), the machining strip width w_{C_g} at the cutter contact point C_g can then be calculated. If the machining strip width between C_0 and C_g is not equivalent to the path interval, i.e., $(w_{b,C_g} + w_{a,C_0} < D - \varepsilon)$ or $(w_{b,C_g} + w_{a,C_0} > D + \varepsilon)$ and ε is a predefined searching tolerance, then the guess point C_g is adjusted to find the new guess point C_g' . The searching process is repeated until a satisfiable new guess point C_g is found. Finally, the estimated guess point $C_g(u_g, v_g)$ is refined to obtain a "true" adjacent cutter contact point $C^*(u^*, v^*)$. For each cutter contact point C_{ij} of the current tool path S_i , the feasible path interval w_{ij} and the adjacent new cutter contact point C_{i+1j} on the next tool path S_{i+1} can be calculated using the proposed non-isoparametric offset method, as shown in Figure 9.

$$\psi_k = 2 \sin^{-1} \left(\frac{\delta_k}{2 \left| \frac{1}{K} \right|} \right)$$

$$\text{Number of searching steps } n_s = \frac{\sin^{-1} \left(\left| \frac{1}{K} \right| \frac{D}{2} \right)}{\sin^{-1} \left(\left| \frac{1}{K} \right| \frac{\delta_k}{2} \right)}$$

$$\psi_{Total} = 2 \sin^{-1} \left(\frac{D}{2 \left| \frac{1}{K} \right|} \right)$$

Cutting direction

Figure 8. Determination of number n_s of searching steps

As shown in Figure 10, if a non-isoparametric offset point C' that has at least one parameter value exceeds the parameter limits (either u or v parameter), then C' is skipped without being

stored as an element of the new path S_{i+1} . If the error f between any two cutter contact points, $(C_{i+1,j}, C_{i+1,j+1})$ in sequence, is larger than the given tolerance, a new cutter contact point C_j' is inserted between $C_{i+1,j}$ and $C_{i+1,j+1}$. C_j' can be defined as follows:

$$C_j' = R\left(\frac{(u_{i+1,j} + u_{i+1,j+1})}{2}, \frac{(v_{i+1,j} + v_{i+1,j+1})}{2}\right) \quad (17)$$

where $R(u, v)$ is the surface description.

6. COMPUTER IMPLEMENTATION AND RESULTS

The methodology presented in this paper has been implemented using C programming language on Motorola PowerPC DT604 workstations. Some testing examples were reported in [Lee 98a]. For the testing example, the total tool path length generated by the traditional constant parametric method is 106.73 units. For the same example, the total length of the tool path generated by the proposed non-isoparametric method is 82.89 units. Comparing the result generated by the proposed non-isoparametric method and the one by the traditional method, one can easily find that the proposed method reduces the total tool path length of the example sculptured surface by 22.3%. The reduction of total tool length by using the proposed non-isoparametric method should increase when the sculptured surface becomes more complex, due to the difference between the largest and smallest feasible machining strips along the constant parametric path. Details of the computer implementation and the testing results can be found in [Lee 98a].

Figure 9 Finding the adjacent non-isoparametric tool path S_{i+1} based on the calculated feasible path intervals

7. CONCLUSIONS

This paper presents a new approach to generating non-isoparametric tool paths for 5-axis sculptured surface machining. Two major contributions are explained in this paper. First, this paper deploys the new concept of machining strip analysis in non-isoparametric 5-axis tool path planning. Second, this paper presents a method that successfully searches for the parameter increments (u, v) along an orthogonal path interval that enables a non-isoparametric path planning strategy to come true. The proposed method can be used to improve the 5-axis machining efficiency for CAD/CAM systems.

Figure 10. Skipping the infeasible non-isoparametric offset points for path generation

ACKNOWLEDGMENT

This work was partially supported by the National Science Foundation (NSF) *Faculty Early CAREER Award* (DMI-9702374) to Dr. Y.S. Lee. Their support is greatly appreciated.

REFERENCES

- Altan, T., et. al., "Advanced techniques for Die and Mold Manufacturing," *Annals of CIRP*, Vol.42, No.2, 1993, pp. 707-716.
- Chen, Y.H., Lee, Y.S., and Fang, S.C., "Optimal Cutter Selection and Machining Plane Determination for Process Planning and NC Machining of Complex Surfaces," *Journal of Manufacturing Systems*, Vol. 17, No. 5, 1998, pp. 371-388.
- Deng, Z., Leu, M.C., Wang, L., and Blackmore, D., "Determination of Flat-end Cutter Orientation in 5-Axis Machining," ASME International Mechanical Engineering Congress and Exposition, Atlanta, GA, November 1996, MED-Vol.4, pp. 73-80.
- Jerard, R.B., Angleton, J.M., and Drysdale, R.L., "Sculptured Surface Tool Path Generation with Global Interference Checking," ASME Design Productivity International Conference, Honolulu, Hawaii, February 1991, pp. 737-742.
- Lee, Y.S., and Chang, T.C., "Application of Computational Geometry in Optimizing 2.5 D and 3D NC Surface Machining," *Computers in Industry*, Vol. 26, No. 1, 1995a, pp. 41-59.
- Lee, Y.S., and Chang, T.C., "Two-Phase Approach to Global Tool Interference Avoidance in 5-axis Machining," *Computer Aided Design*, Vol. 27, No. 10, 1995b, pp. 715-729.
- Lee, Y.S., and Chang, T.C., "Machined Surface Error Analysis For 5-axis Machining," *International Journal of Production Research*, Vol. 34, No. 4, 1996a, pp. 977-998.
- Lee, Y.S., and Chang, T.C., "Automatic Cutter Selection for 5-axis Sculptured Surface Machining," *the International Journal of Production Research*, Vol. 34, No. 4, 1996b, pp. 977-998.
- Lee, Y.S. and Ji, H., "Surface Interrogation and Machining Strip Evaluation for Multi-axis CNC Die/Mold Machining," *International Journal of Production Research*, Vol. 35, No. 1, 1997a, pp. 225-252.
- Lee, Y.S., "Admissible Tool Orientation Control of Gouging Avoidance for 5-Axis Complex Surface Machining," *Computer-Aided Design*, Vol. 29, No. 7, 1997b, pp. 507-521.
- Lee, Y.S., "Non-isoparametric Tool Path Planning by Machining Strip Evaluation for 5-Axis Sculptured Surface Machining," *Computer-Aided Design*, Vol. 30, No. 7, 1998a, pp. 559-570.
- Lee, Y.S., "Mathematical Modeling of Using Different Endmills and Tool Placement Problems For 4- and 5-Axis NC Complex Surface Machining," *International Journal of Production Research*, Vol. 36, No. 3, 1998b, pp. 785-814.
- Li, S., and Jerard, R.B., "5-axis Machining of Sculptured Surfaces with a Flat-end Cutter," *Computer Aided Design*, Vol. 26, No. 3, 1994, pp. 165-178.

14. Lin, R.S., and Koren, Y., "Efficient Tool-path Planning for Machining Free-form Surfaces," *Transactions of the ASME*, Vol. 118, No. 1, 1996, pp. 20-28.
15. Ma, Y., and Lee, Y.S., "Detection of Loops and Singularities of Surface Intersections," (in press) *Computer-Aided Design*, Accepted on September 22, 1998.
16. Suresh, K., and Yang, D.C.H., "Constant Scallop Height Machining of Free-form Surfaces," *ASME Journal of Engineering for Industry*, Vol. 116, No. 2, 1994, pp. 253-259.
17. Vickers, G.W., and Quan, K.W., "Ball-mills versus end-mills for curved surface machining", *Journal of Engineering for Industry*, Vol.111, February, 1989, pp. 22-26.