STABILITY AND APPROXIMATIONS OF AN ACOUSTIC-STRUCTURE MODEL

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Abstract: We consider uniform exponential stability and preservation of this property under approximations for a coupled acoustic-structure system with “porous” damped boundary conditions. This system consists of a 2-D acoustic cavity whose portion of its boundary is modeled as a flexible beam. Using a multiplier technique we show that the “porous” boundary condition provides enough damping to yield uniform exponential stability of the model. We also present a polynomial-based Galerkin numerical scheme which results in approximate solutions with a uniform rate of decay.

Key Words: acoustic-structure models, multiplier technique, exponential stability, Galerkin method.

AMS subject classification: 35L05, 93D20, 65N12, 65N30.

1 INTRODUCTION

In this paper, we consider a two dimensional acoustic-structure system which consists of a rectangular acoustic cavity whose two sides are bounded by Euler-Bernoulli beams. The acoustic cavity is $\Omega = (-1, 1) \times (-1, 1)$ and the two beams are located at the sides $x = -1$ and $y = -1$.

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The model systems are given by the following partial differential equations (see [4]): for $t > 0$

$$\partial_t^2 \phi(t, x, y) = \partial_x^2 \phi(t, x, y) + \partial_y^2 \phi(t, x, y), \quad (x, y) \in \Omega, \quad (1.1)$$

$$\partial_t^2 u_1(t, x) = -\partial_x^4 (Eu_1(t, x) + C\partial_t u_1(t, x)) + \partial_t \phi(t, x, -1), \quad x \in (-1, 1), \quad (1.2)$$

$$\partial_t^2 u_2(t, y) = -\partial_y^4 (Eu_2(t, y) + C\partial_t u_2(t, y)) + \partial_t \phi(t, -1, y), \quad y \in (-1, 1), \quad (1.3)$$

where ϕ is the velocity potential function for the acoustic cavity and u_1, u_2 are the transversal displacements of the beams numbered 1 and 2, respectively. The velocity transmission at the interface between the acoustic cavity and the beams is modeled by

$$\partial_y \phi(t, x, -1) = \partial_t u_1(t, x) + \alpha_1 \partial_t \phi(t, x, -1), \quad x \in (-1, 1), \quad (1.4)$$

$$\partial_x \phi(t, -1, y) = \partial_t u_2(t, y) + \alpha_2 \partial_t \phi(t, -1, y), \quad y \in (-1, 1). \quad (1.5)$$

The coefficients $\alpha_i, i = 1, 2$ represent the velocity loss factor. We note that if $\alpha_i = 0$, the boundary conditions above correspond to the continuity of the velocity at the boundaries. Additional boundary conditions are given by

$$\phi(t, x, 1) = \phi(t, 1, y) = 0, \quad x, y \in (-1, 1), \quad (1.6)$$

$$u_1(t, x) = \partial_x u_1(t, x) = 0, \quad x = -1, 1, \quad (1.7)$$

$$u_2(t, y) = \partial_y u_2(t, y) = 0, \quad y = -1, 1. \quad (1.8)$$

In this work we present a summary of the results obtained in [4] on well-posedness and uniform exponential stability of the solutions to the equations (1.1-1.8). We also present a polynomial-based Galerkin scheme that preserves the exponential stability of the solutions under approximations. The model presented here is a modification of the one presented first in [3]. The main difference lies in substituting the condition of continuity of normal velocities of the structure and the acoustic field at the boundary of the acoustic cavity used in [3] by boundary conditions (1.4)–(1.5) that allow for a difference in the two velocities proportional to the acoustic pressure. Recently, several studies have appeared in literature that analyze the stability properties of different models derived from the one in [3], (see [1, 2, 6, 7]). In the same vein the work here also studies the stability of a new model for acoustic-structure interactions, but in addition provides a framework that can be used for proving the existence of a uniform rate of decay for the approximate solutions obtained from a Galerkin method. Since the model equations presented here arise in active noise control problems, for implementation of the linear quadratic regulator control algorithm numerical approximation of the solutions and their exponential stability is of utmost importance. In this regard, this framework facilitates the study of convergence of the approximate Riccati operators and the feedback controls to the infinite-dimensional ones.

2 WELL-POSEDNESS OF THE MODEL

For these model equations the state space $\mathcal{H}$ is

$$\mathcal{H} = V_a \times H_a \times V_b \times H_b \times V_b \times H_b, \quad (1.9)$$

where the Hilbert spaces $H_a = L^2((-1, 1) \times (-1, 1))$, $H_b = L^2(-1, 1)$ and the Hilbert spaces V_a and V_b are defined by

$$\begin{aligned} V_a &= \{ \phi \in H^1((-1, 1) \times (-1, 1)) : \phi(x, 1) = \phi(1, y) = 0, \forall x, y \in (-1, 1) \}, \\ V_b &= \{ u \in H^2(-1, 1) : u(s) = u'(s) = 0, \text{ for } s = -1, 1 \}, \end{aligned}$$

with inner products defined by

$$\langle \phi, \tilde{\phi} \rangle_{V_a} = \int_{\Omega} \nabla \phi(x, y) \cdot \nabla \tilde{\phi}(x, y) dx dy, \quad \langle u, \tilde{u} \rangle_{V_b} = \int_{-1}^1 E u''(s) \tilde{u}''(s) ds.$$

A solution of the variational form of the model equations is a $\mathcal{H}$ valued function of time t , $\xi(t) = (\phi(t), \theta(t), u_1(t), v_1(t), u_2(t), v_2(t))$, which satisfies the equation

$$\frac{d}{dt} \langle \xi(t), \tilde{\xi} \rangle_{\mathcal{H}} = \sigma(\xi(t), \tilde{\xi}) \quad (1.10)$$

for all $\tilde{\xi} = (\tilde{\phi}, \tilde{\theta}, \tilde{u}_1, \tilde{v}_1, \tilde{u}_2, \tilde{v}_2) \in \mathcal{V}$ and where the Hilbert space $\mathcal{V}$ is defined as $\mathcal{V} = V_a \times V_a \times V_b \times V_b \times V_b \times V_b$ and the sesquilinear form σ on $\mathcal{V} \times \mathcal{V}$ is defined by

$$\begin{aligned} \sigma(\xi, txi) &= \sigma_a(\theta, \tilde{\phi}) - \sigma_a(\phi, \tilde{\theta}) - \rho_1(v_1, \tilde{\theta}) - \rho_2(v_2, \tilde{\theta}) - \gamma_a(\theta, \tilde{\theta}) \\ &\quad + \sigma_b(v_1, \tilde{u}_1) - \sigma_b(u_1, \tilde{v}_1) + \rho_1(\tilde{v}_1, \theta) - \gamma_b(v_1, \tilde{v}_1) \\ &\quad + \sigma_b(v_2, \tilde{u}_2) - \sigma_b(u_2, \tilde{v}_2) + \rho_2(\tilde{v}_2, \theta) - \gamma_b(v_2, \tilde{v}_2). \end{aligned}$$

The above sesquilinear forms are defined by

$$\begin{aligned} \sigma_a(\hat{\phi}, \tilde{\phi}) &= \int_{\Omega} \nabla \hat{\phi}(x, y) \cdot \nabla \tilde{\phi}(x, y) dx dy, \\ \gamma_a(\hat{\phi}, \tilde{\phi}) &= \int_{-1}^1 \alpha_1 \hat{\phi}(x, -1) \tilde{\phi}(x, -1) dx + \int_{-1}^1 \alpha_2 \hat{\phi}(-1, y) \tilde{\phi}(-1, y) dy, \\ \sigma_b(\hat{u}, \tilde{u}) &= \int_{-1}^1 E \hat{u}''(s) \tilde{u}''(s) ds, \quad \gamma_b(\hat{u}, \tilde{u}) = \int_{-1}^1 C \hat{u}''(s) \tilde{u}''(s) ds, \\ \rho_1(\tilde{u}, \tilde{\phi}) &= \int_{-1}^1 \tilde{u}(x) \tilde{\phi}(x, -1) dx, \quad \rho_2(\tilde{u}, \tilde{\phi}) = \int_{-1}^1 \tilde{u}(y) \tilde{\phi}(-1, y) dy. \end{aligned} \quad (1.11)$$

We define an unbounded linear operator $\mathcal{A} : \mathcal{D}(\mathcal{A}) \subset \mathcal{H} \mapsto \mathcal{H}$ as follows:

$$\mathcal{D}(\mathcal{A}) = \left\{ \begin{array}{l} \xi = (\phi, \theta, u_1, v_1, u_2, v_2) \in \mathcal{V} : \exists \hat{\xi} = (\hat{\phi}, \hat{\theta}, \hat{u}_1, \hat{v}_1, \hat{u}_2, \hat{v}_2) \in \mathcal{H}, \text{ such that} \\ \sigma(\xi, \hat{\xi}) = \langle \hat{\xi}, \tilde{\xi} \rangle_{\mathcal{H}} \text{ for all } \tilde{\xi} = (\tilde{\phi}, \tilde{\theta}, \tilde{u}_1, \tilde{v}_1, \tilde{u}_2, \tilde{v}_2) \in \mathcal{V}. \end{array} \right\},$$

and for all $\xi \in \mathcal{D}(\mathcal{A})$, $\mathcal{A}\xi = \hat{\xi}$ where $\hat{\xi} = (\hat{\phi}, \hat{\theta}, \hat{u}_1, \hat{v}_1, \hat{u}_2, \hat{v}_2)$ satisfies $\sigma(\xi, \tilde{\xi}) = \langle \hat{\xi}, \tilde{\xi} \rangle_{\mathcal{H}}$, for all $\tilde{\xi} = (\tilde{\phi}, \tilde{\theta}, \tilde{u}_1, \tilde{v}_1, \tilde{u}_2, \tilde{v}_2) \in \mathcal{V}$.

The main result of this section is the following theorem.

Theorem 2.1 *The operator $\mathcal{A}$ is the infinitesimal generator of a C_0 -semigroup of contractions denoted by $e^{\mathcal{A}t}$ in $\mathcal{H}$ and for any $\xi_0 = (\phi_0, \theta_0, u_{1,0}, v_{1,0}, u_{2,0}, v_{2,0}) \in \mathcal{D}(\mathcal{A})$, the function $\xi(t) = e^{\mathcal{A}t} \xi_0$ is the unique weak solution of (1.10) with initial value ξ_0 .*

The proof of the above theorem is based on the Lumer-Phillips theorem. The conditions in the Lumer-Phillips theorem are established through the following lemmas, (see [4]).

Lemma 2.1 *The operator $\mathcal{A}$ is dissipative in $\mathcal{H}$.*

Lemma 2.2 *For all real number $\lambda > 0$, the operator $\mathcal{A} - \lambda I : \mathcal{D}(\mathcal{A}) \subset \mathcal{H} \mapsto \mathcal{H}$ is onto.*

3 UNIFORM EXPONENTIAL STABILITY OF THE MODEL

In this section we outline the proof for existence of constants $M \geq 0$ and $\omega > 0$ such that

$$\|e^{\mathcal{A}t}\xi_0\|_{\mathcal{H}} \leq Me^{-\omega t}\|(\xi_0)\|_{\mathcal{H}}, \quad (1.12)$$

for all $\xi_0 = (\phi_0, \theta_0, u_{1,0}, v_{1,0}, u_{2,0}, v_{2,0}) \in \mathcal{H}$. Our approach is based on the use of a multiplier functional $Q(t)$ defined by

$$\begin{aligned} Q(t) &= \frac{t}{2}\|\xi(t)\|_{\mathcal{H}}^2 + \int_{-1}^1 u_1(t, x)v_1(t, x)dx + \int_{-1}^1 u_2(t, y)v_2(t, y)dy \\ &\quad + \int_{\Omega} [2(x-1)\partial_x\phi(t, x, y) + 2(y-1)\partial_y\phi(t, x, y) + \phi(t, x, y)]\theta(t, x, y)dxdy. \end{aligned}$$

It is easy to see that there exists a constant T such that for all $t \geq T$,

$$\frac{t}{4}E(t) \leq Q(t) \leq tE(t),$$

where $E(t) = \|\xi(t)\|_{\mathcal{H}}^2$. The proof of the inequality (1.12) depends on showing that there exists a constant T' such that for all $t \geq T'$, $Q(t) \leq Q(T')$. Therefore, for all $t \geq \max(T, T')$, we have $\frac{t}{4}E(t) \leq Q(T') \leq T'E(T')$. As a result, we have $E(t) \leq \frac{4}{t}T'\|e^{\mathcal{A}T'}\|_{L(\mathcal{H})}^2 \cdot \|\xi_0\|_{\mathcal{H}}^2$. Using the semigroup property of $e^{\mathcal{A}t}$, we obtain the exponential decay given in inequality (1.12). In order to show that Q is bounded, we shall show $Q'(t)$ is negative for all $t > T'$ for some constant T' . First, take $\xi_0 \in \mathcal{D}(\mathcal{A})$ and let $\xi(t) = e^{\mathcal{A}t}\xi_0$.

We consider the function $Q(t)$. Using the differentiability of the function $(\phi(\cdot), \theta(\cdot), u_1(\cdot), v_1(\cdot), u_2(\cdot), v_2(\cdot))$, we obtain

$$\begin{aligned} \frac{d}{dt}Q(t) &= \frac{1}{2}\|\xi(t)\|_{\mathcal{H}}^2 + t\langle \mathcal{A}\xi(t), \xi(t) \rangle_{\mathcal{H}} \\ &\quad + \int_{-1}^1 \partial_t u_1(t, x)v_1(t, x)dx + \int_{-1}^1 u_1(t, x)\partial_t v_1(t, x)dx \\ &\quad + \int_{-1}^1 \partial_t u_2(t, y)v_2(t, y)dy + \int_{-1}^1 u_2(t, y)\partial_t v_2(t, y)dy \\ &\quad + \int_{\Omega} \partial_t \phi(t, x, y)\theta(t, x, y)dxdy + \int_{\Omega} \phi(t, x, y)\partial_t \theta(t, x, y)dxdy \\ &\quad + \int_{\Omega} [2(x-1)\partial_x\phi(t, x, y) + 2(y-1)\partial_y\phi(t, x, y)]\partial_t \theta(t, x, y)dxdy \\ &\quad + \int_{\Omega} \partial_t [2(x-1)\partial_x\phi(t, x, y) + 2(y-1)\partial_y\phi(t, x, y)]\theta(t, x, y)dxdy. \end{aligned}$$

From the definition of the operator $\mathcal{A}$, integration by parts and by rearranging the terms, we obtain

$$\begin{aligned}
 \frac{d}{dt}Q(t) &= \frac{1}{2} \int_{\Omega} \nabla \phi(t, x, y) \cdot \nabla \phi(t, x, y) dx dy + \int_{\Omega} \phi(t, x, y) \Delta \phi(t, x, y) dx dy \\
 &+ \int_{\Omega} [2(x-1) \partial_x \phi(t, x, y) + 2(y-1) \partial_y \phi(t, x, y)] \Delta \phi(t, x, y) dx dy \\
 &+ \int_{\Omega} [2(x-1) \partial_x \theta(t, x, y) + 2(y-1) \partial_y \theta(t, x, y)] \theta(t, x, y) dx dy \\
 &+ \frac{3}{2} \int_{\Omega} \theta^2(t, x, y) dx dy - t \int_{-1}^1 \alpha \theta^2(t, x, -1) dx - t \int_{-1}^1 \alpha \theta^2(t, -1, y) dy \\
 &+ \frac{3}{2} \int_{-1}^1 v_1^2(t, x) dx + \frac{3}{2} \int_{-1}^1 v_2^2(t, y) dy \\
 &- \frac{1}{2} \int_{-1}^1 E(\partial_x^2 u_1(t, x))^2 dx - \frac{1}{2} \int_{-1}^1 E(\partial_y^2 u_2(t, y))^2 dy \\
 &+ (-tC) \int_{-1}^1 (\partial_x^2 v_1(t, x))^2 dx + (-tC) \int_{-1}^1 (\partial_y^2 v_2(t, y))^2 dy \\
 &- C \int_{-1}^1 \partial_x^2 v_1(t, x) \partial_x^2 u_1(t, x) dx - C \int_{-1}^1 \partial_y^2 v_2(t, y) \partial_y^2 u_2(t, y) dy \\
 &+ \int_{-1}^1 \theta(t, x, -1) u_1(t, x) dx + \int_{-1}^1 \theta(t, -1, y) u_2(t, y) dy = \sum_{k=1}^{17} I_k,
 \end{aligned} \tag{1.13}$$

where I_k is the k -th integral in the above equality. It is easy to see that there exists $T_1 > 0$ such that for all $t > T_1$,

$$\begin{aligned}
 \sum_{k=8}^{15} I_k &\leq -\frac{1}{4} \int_{-1}^1 E(\partial_x^2 u_1(t, x))^2 dx - \frac{1}{4} \int_{-1}^1 E(\partial_y^2 u_2(t, y))^2 dy \\
 &- \frac{t}{2} \int_{-1}^1 (\partial_x^2 v_1(t, x))^2 dx - \frac{t}{2} \int_{-1}^1 (\partial_y^2 v_2(t, y))^2 dy.
 \end{aligned}$$

Using the divergence theorem, we can also show that there exists a constant T_2 such that for all $t > T_2$

$$I_4 + I_5 \leq \frac{t\alpha}{2} \int_{-1}^1 \theta^2(t, x, -1) dx + \frac{t\alpha}{2} \int_{-1}^1 \theta^2(t, -1, y) dy.$$

On the other hand, for the terms I_{16} and I_{17} , it is easy to see that

$$\begin{aligned}
 &\int_{-1}^1 \theta(t, x, -1) u_1(t, x) dx + \int_{-1}^1 \theta(t, -1, y) u_2(t, y) dy \\
 &\leq \epsilon \int_{-1}^1 u_1^2(t, x) dx + \epsilon \int_{-1}^1 u_2^2(t, y) dy \\
 &\quad + \frac{1}{\epsilon} \int_{-1}^1 \theta^2(t, x, -1) dx + \frac{1}{\epsilon} \int_{-1}^1 \theta^2(t, -1, y) dy.
 \end{aligned}$$

For the first 3 terms, we use a sequence of functions $\phi_n \in H^2(\Omega) \cap V_a$ that approximate ϕ and also have enough smoothness to justify application of the divergence theorem, (see [4]). Therefore, one can show there exists a constant T_3 such that for all $t > T_3$,

$$\begin{aligned} I_1 + I_2 + I_3 \leq & \frac{t\alpha}{2} \int_{-1}^1 \theta^2(t, x, -1) dx + \frac{t\alpha}{2} \int_{-1}^1 \theta^2(t, -1, y) dy \\ & + \frac{t}{2} \int_{-1}^1 (\partial_x^2 v_1(t, x))^2 dx + \frac{t}{2} \int_{-1}^1 (\partial_y^2 v_2(t, y))^2 dy. \end{aligned} \quad (1.14)$$

By combining all these results, we get the desired uniform exponential stability.

4 GALERKIN APPROXIMATIONS

The Galerkin approximation of the variational form of the model equations requires the use of polynomial functions for the approximation of V_a . In particular, let H_a^N be a subspace of V_a given by

$$H_a^N = \left\{ \begin{array}{l} \phi^N : \phi^N(x, y) \in \text{span}\{x^k y^j\}_{0 \leq k \leq N_x, 0 \leq j \leq N_y}, \text{ and } \\ \phi^N(x, 1) = \phi^N(1, y) = 0, \text{ for all } x, y \in (-1, 1) \end{array} \right\}, \quad (1.15)$$

where N_x and N_y are positive integers. There are no special restriction on the selection of the subspaces H_b^N of V_b for the discussion of the exponential stability of solutions of the finite dimensional approximations of the model equations. A solution $\xi^N(t) = (\phi^N(t), \theta^N(t), u_1^N(t), v_1^N(t), u_2^N(t), v_2^N(t))$ to the Galerkin approximation on the subspace $\mathcal{H}^N$ defined by

$$\mathcal{H}^N = H_a^N \times H_a^N \times H_b^N \times H_b^N \times H_b^N \times H_b^N \quad (1.16)$$

is a $\mathcal{H}^N$ valued function of time t which satisfies the equation:

$$\frac{d}{dt} \langle \xi^N(t), \tilde{\xi}^N \rangle_{\mathcal{H}} = \sigma(\xi^N(t), \tilde{\xi}^N) \quad (1.17)$$

for all $\tilde{\xi}^N = (\tilde{\phi}^N, \tilde{\theta}^N, \tilde{u}_1^N, \tilde{v}_1^N, \tilde{u}_2^N, \tilde{v}_2^N) \in \mathcal{H}^N$. The equation above is a linear evolution equation in $\mathcal{H}^N$. As a consequence, there exists a semi-group $S^N(t)$ of linear operators on $\mathcal{H}^N$ such that for every given initial state vector $\xi_0^N = (\phi_0^N, \theta_0^N, u_{1,0}^N, v_{1,0}^N, u_{2,0}^N, v_{2,0}^N) \in \mathcal{H}^N$ the function defined by $\xi^N(t) = S^N(t)\xi_0^N$ is a solution of the dynamical equation in $\mathcal{H}^N$. The main result on the uniform exponential decay rate of the approximate solution semigroup $S^N(t)$ is given by the following theorem.

Theorem 4.1 *For the polynomial based Galerkin scheme there exist constants $L \geq 1$ and $\beta > 0$ such that*

$$\|S^N(t)\|_{L(\mathcal{H}^N)} \leq L e^{-\beta t}$$

for all $t \geq 0$ and for all N .

For the proof of this theorem, see [5].

For computational purposes, we represent the model equations in $\mathcal{H}^N$ in their coordinate vector form with respect to selected bases for H_a^N and H_b^N which are subspaces of polynomials. For this purpose, we let $\{w_i^n\}_{i=1}^n$ be a basis for H_b^N and let $\{\psi_k^m\}_{k=1}^m$ denote the 2-D polynomial basis functions which span H_a^N . By expanding the components of the approximate state vector ξ^N in terms of these basis functions, we obtain

$$\begin{aligned}\phi^N(t, x, y) &= \sum_{k=1}^m \phi_k^N(t) \psi_k^m(x, y), & \theta^N(t, x, y) &= \sum_{k=1}^m \theta_k^N(t) \psi_k^m(x, y), \\ u_j^N(t, x) &= \sum_{i=1}^n u_{j,i}^N(t) w_i^n(x), & v_j^N(t, x) &= \sum_{i=1}^n v_{j,i}^N(t) w_i^n(x), \quad j = 1, 2\end{aligned}$$

From equation (1.17), we can write the following first order approximate system

$$\mathcal{M}^N \dot{\xi}^N(t) = \mathcal{A}^N \bar{\xi}^N(t), \quad (1.18)$$

where $\bar{\xi}^N(t) = (\bar{\phi}^N(t), \bar{\theta}^N(t), \bar{u}_1^N(t), \bar{v}_1^N(t), \bar{u}_2^N(t), \bar{v}_2^N(t)) \in R^N$, with $N = 2(m + 2n)$ is the coordinate vector representation of the approximate state.

We represent the linear system (1.18) more explicitly as the following equation:

$$\begin{aligned}& \begin{bmatrix} K_a^N & 0 & 0 & 0 & 0 & 0 \\ 0 & M_a^N & 0 & 0 & 0 & 0 \\ 0 & 0 & K_b^N & 0 & 0 & 0 \\ 0 & 0 & 0 & M_b^N & 0 & 0 \\ 0 & 0 & 0 & 0 & K_b^N & 0 \\ 0 & 0 & 0 & 0 & 0 & M_b^N \end{bmatrix} \frac{d}{dt} \begin{bmatrix} \bar{\phi}^N(t) \\ \bar{\theta}^N(t) \\ \bar{u}_1^N(t) \\ \bar{v}_1^N(t) \\ \bar{u}_2^N(t) \\ \bar{v}_2^N(t) \end{bmatrix} \\ &= \begin{bmatrix} 0 & K_a^N & 0 & 0 & 0 & 0 \\ -K_a^N & -A_a^N & 0 & -A_{ab}^N & 0 & -A_{ab}^N \\ 0 & 0 & 0 & K_b^N & 0 & 0 \\ 0 & A_{ab}^{N^T} & -K_b^N & -A_b^N & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & K_b^N \\ 0 & A_{ab}^{N^T} & 0 & 0 & -K_b^N & -A_{ab}^N \end{bmatrix} \begin{bmatrix} \bar{\phi}^N(t) \\ \bar{\theta}^N(t) \\ \bar{u}_1^N(t) \\ \bar{v}_1^N(t) \\ \bar{u}_2^N(t) \\ \bar{v}_2^N(t) \end{bmatrix}.\end{aligned} \quad (1.19)$$

The component matrices can be computed from the sesquilinear forms given by (1.11):

$$\begin{aligned} [K_a^N]_{k,\ell} &= \sigma_a(\psi_k^m, \psi_\ell^m) & [K_b^N]_{i,j} &= \sigma_b(w_i^n, w_j^n) \\ [M_a^N]_{k,\ell} &= \int_{\Omega} \psi_k^m \psi_\ell^m dx dy & [M_b^N]_{i,j} &= \int_{-1}^1 w_i^n(s) w_j^n(s) ds \\ [A_a^N]_{k,\ell} &= \gamma_a(\psi_k^m, \psi_\ell^m) & [A_{ab}^N]_{i,\ell} &= \rho_1(w_i^n, \psi_\ell^m) & [A_b^N]_{i,j} &= \gamma_b(w_i^n, w_j^n) \end{aligned} \quad (1.20)$$

5 NUMERICAL RESULTS

In this section, we demonstrate that the approximate solutions obtained from the Galerkin method described in the previous section have a uniform rate of

decay. In carrying out this goal, our approach is to calculate the eigenvalues of the matrix

$$A^N = (\mathcal{M}^N)^{-1} \mathcal{A}^N,$$

for increasing values of N , and observe the trend in the location of the eigenvalues, and the margin of stability which is defined as the magnitude of the maximum of the real part of the eigenvalues. These computations are performed using MATLAB on a SPARC 10 workstation.

For the parameters, we chose $\alpha_1 = \alpha_2 = 0.1$, and E and C to be 1. We calculated the eigenvalues and the margin of stability using the *eig* routine in MATLAB. The results summarized in the table below, clearly indicate that a uniform margin of stability is preserved as the dimension of approximation is increased.

$N_a = N_b$	$\max\{Re\lambda, \lambda \in \sigma(A^N)\}$
10	-0.07786
14	-0.05948
18	-0.05372
22	-0.05174
26	-0.05092

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