OPTIMAL CONTROL THEORY: FROM FINITE DIMENSIONS TO INFINITE DIMENSIONS *

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1 INTRODUCTION

Optimal control theory has more than 40 years of history. For deterministic finite dimensional case, the works of Pontryagin *et al* (maximum principle [14]), Bellman (dynamic programming [1]) and Kalman (linear quadratic regulator problem [6]) have been regarded as three main milestones in the field.

The purpose of this paper is to give a brief survey on the works done by the people from Fudan Group related to optimal control theory for deterministic infinite dimensional systems.

2 MAXIMUM PRINCIPLE FOR EVOLUTION SYSTEMS

Maximum principle is the necessary conditions of optimal controls for finite dimensional systems. The infinite dimensional version of maximum principle was firstly proved by A. G. Butkovsky. Yu. V. Egorov [3] constructed an example showing that the maximum principle does not necessarily hold for infinite dimensional systems. In past 12 years, the people from Fudan Group makes an effort to derive the Pontryagin type maximum principle for infinite dimensional case under some assumptions.

Let us begin with the following hypothesis.

(H1) Let X be a Banach space, S be a closed and convex subset of $X \times X$, U be a separable metric space. Let e^{At} be a C_0 -semigroup on X with generator $A : \mathcal{D}(A) \subset X \rightarrow X$.

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(H2) Let $f : [0, T] \times X \times U \rightarrow X$ and $f^0 : [0, T] \times X \times U \rightarrow \mathbb{R}$ such that $f(t, y, u)$ and $f^0(t, y, u)$ are strongly measurable in $t \in [0, T]$, continuously Fréchet differentiable in $y \in X$ with $f(t, \cdot, \cdot)$, $f_y(t, \cdot, \cdot)$, $f^0(t, \cdot, \cdot)$ and $f_y^0(t, \cdot, \cdot)$ being continuous. Consider the following evolution equation

$$y(t) = e^{At}y(0) + \int_0^t e^{A(t-s)}f(s, y(s), u(s))ds, \quad t \in [0, T], \quad (2.1)$$

where $u(\cdot) \in \mathcal{U}[0, T] \equiv \{u : [0, T] \rightarrow U \mid u(\cdot) \text{ measurable} \}$.

Any $(y(\cdot), u(\cdot)) \in C([0, T]; X) \times \mathcal{U}[0, T]$ satisfying (1.1) is called a *feasible pair*. The set of all feasible pairs is denoted by $\mathcal{A}$. Next, we let $\mathcal{A}_{ad}$ be the set of all feasible pairs $(y(\cdot), u(\cdot)) \in \mathcal{A}$, such that

$$(y(0), y(T)) \in S, \quad (2.2)$$

For any $(y(\cdot), u(\cdot)) \in \mathcal{A}_{ad}$, we define

$$J(y(\cdot), u(\cdot)) = \int_0^T f^0(s, y(s), u(s))ds. \quad (2.3)$$

Optimal Control Problem (C). Find $(\bar{y}(\cdot), \bar{u}(\cdot)) \in \mathcal{A}_{ad}$, such that

$$J(\bar{y}(\cdot), \bar{u}(\cdot)) = \inf_{\mathcal{A}_{ad}} J(y(\cdot), u(\cdot)). \quad (2.4)$$

Definition. Let (H1)–(H2) hold and $(\bar{y}(\cdot), \bar{u}(\cdot)) \in \mathcal{A}_{ad}$ be an optimal pair. We say that $(\bar{y}(\cdot), \bar{u}(\cdot))$ satisfies the *Maximum Principle*, if there exists a nontrivial pair $(\psi^0, \psi(\cdot)) \in \mathbb{R} \times C([0, T]; X^*)$, i.e., $(\psi^0, \psi(\cdot)) \neq 0$, such that

$$\psi^0 \leq 0, \quad (2.5)$$

$$\begin{aligned} \psi(t) &= e^{A^*(T-t)}\psi(T) + \int_t^T e^{A^*(s-t)}f_y(s, \bar{y}(s), \bar{u}(s))^*\psi(s)ds \\ &\quad + \int_t^T e^{A^*(s-t)}\psi^0 f_y^0(s, \bar{y}(s), \bar{u}(s))ds, \quad \text{a.e. } t \in [0, T], \end{aligned} \quad (2.6)$$

$$\langle \psi(0), x_0 - \bar{y}(0) \rangle - \langle \psi(T), x_1 - \bar{y}(T) \rangle \leq 0, \quad \forall (x_0, x_1) \in S, \quad (2.7)$$

$$\begin{aligned} H(t, \bar{y}(t), \bar{u}(t), \psi^0, \psi(t)) &= \max_{u \in U} H(t, \bar{y}(t), u, \psi^0, \psi(t)), \\ &\text{a.e. } t \in [0, T], \end{aligned} \quad (2.8)$$

where

$$\begin{aligned} H(t, y, u, \psi^0, \psi) &= \psi^0 f^0(t, y, u) + \langle \psi, f(t, y, u) \rangle, \\ \forall (t, y, u) &\in [0, T] \times X \times U, \quad \forall (\psi^0, \psi) \in \mathbb{R} \times X^*. \end{aligned} \quad (2.9)$$

In 1985, Li and Yao [10] consider the problem with the endpoint constraints: $y(0) = x_0$, $y(T) \in Q_1$. They proved that if Q_1 is finite codimensional in X , then, maximum principle holds. In 1987, H.O.Fattorini [4] consider the problem with the endpoint constraints: $y(0) = x_0$, $y(T) = x_1$. He proved that if X is Hilbert space and the associate reachable set is finite codimension in X , then the maximum principle holds.

In 1991, Li and Yong [11] consider the general case. Let $\mathcal{R}$ be the associate reachable set:

$$\begin{aligned} \mathcal{R} = \{ \xi(T) \in X \mid \xi(t) = & \int_0^t e^{A(t-s)} f_y(s, \bar{y}(s), \bar{u}(s)) \xi(s) ds \\ & + \int_0^t e^{A(t-s)} [f(s, \bar{y}(s), u(s)) - f(s, \bar{y}(s), \bar{u}(s))] ds, \\ & t \in [0, T], u(\cdot) \in \mathcal{U}[0, T] \}, \end{aligned} \quad (2.10)$$

and Q be the modified endpoint constraint set:

$$\begin{aligned} Q = \{ y_1 - \eta(T) \mid \eta(t) = & e^{At} y_0 + \int_0^t e^{A(t-s)} f_y(s, \bar{y}(s), \bar{u}(s)) \eta(s) ds, \\ & t \in [0, T], (y_0, y_1) \in S \}. \end{aligned} \quad (2.11)$$

Theorem 2.1 Let (H1)–(H2) hold. Let $(\bar{y}(\cdot), \bar{u}(\cdot))$ be an optimal pair of Problem (C). Assume

(H3) $\mathcal{R} - Q \equiv \{ r - q \mid r \in \mathcal{R}, q \in Q \}$ is finite codimensional in X .

(H4) The dual X^* of X is strictly convex.

Then, the pair $(\bar{y}(\cdot), \bar{u}(\cdot))$ satisfies the Maximum Principle.

For the optimal periodic [9](or anti-periodic) control problem, i.e.,

$$y(0) = y(T) \quad (\text{or } y(0) = -y(T)), \quad (2.12)$$

we have the following

Theorem 2.2 Let e^{At} be a compact semigroup. Let $(\bar{y}, \bar{u})$ be an optimal pair for the optimal periodic (or anti-periodic) control problem. Then, there exists a pair $(\psi^0, \psi(\cdot)) \neq 0$, such that (2.6) and (2.8) hold, and

$$\psi(0) = \psi(T) \quad (\text{or } \psi(0) = -\psi(T)). \quad (2.13)$$

Moreover, if $\mathcal{N}(I - G^*(T, 0)) = \{0\}$ (or $\mathcal{N}(I + G^*(T, 0)) = \{0\}$), then $\psi^0 = -1$.

In 1993, Yong [18] discussed the optimal control problem for infinite dimensional Volterra-Steiltjees evolution equations and derived a maximum principle.

3 MAXIMUM PRINCIPLE FOR ELLIPTIC SYSTEMS

The optimal control problem of a semilinear second order elliptic partial differential equation as follows: minimize the cost functional

$$J(y, u) = \int_{\Omega} f^0(x, y(x), u(x)) dx. \quad (3.1)$$

subject to

$$\begin{aligned} Ay(x) &= f(x, y(x), u(x)), & \text{in } \Omega, \\ y|_{\partial\Omega} &= 0. \end{aligned} \quad (3.2)$$

Assumptions:

(S1) $\Omega \subset \mathbb{R}^n$ is a bounded region with $C^{1,\gamma}$ boundary $\partial\Omega$, for some $\gamma > 0$ and U is a separable metric space.

(S2) Operator A is defined by

$$Ay(x) = - \sum_{i,j=1}^n (a_{ij}(x) y_{x_j}(x))_{x_i}, \quad (3.3)$$

with $a_{ij} \in C(\overline{\Omega})$, $a_{ij} = a_{ji}$, $1 \leq i, j \leq n$ and for some $\lambda > 0$,

$$\sum_{i,j=1}^n a_{ij}(x) \xi_i \xi_j \geq \lambda \sum_{i=1}^n |\xi_i|^2, \quad \forall x \in \Omega, (\xi_1, \xi_2, \dots, \xi_n) \in \mathbb{R}^n. \quad (3.4)$$

(S3) $f : \Omega \times \mathbb{R} \times U \rightarrow \mathbb{R}$ satisfies

$$f_y(x, y, u) \leq 0, \forall (x, y, u) \in \Omega \times \mathbb{R} \times U.$$

(S4) $\mathcal{Y}$ is a Banach space with strict convex dual $\mathcal{Y}^*$, $F : W_0^{1,p}(\Omega) \rightarrow \mathcal{Y}$ is continuously Fréchet differentiable and $Q \subset \mathcal{Y}$ is closed and convex.

Let $\mathcal{A}_{ad}$ be the set of all pair (y, u) , such that $u(\cdot) \in \mathcal{U} = \{u : \Omega \rightarrow U \mid u \text{ is measurable}\}$ and the state constraint $F(y) \in Q$.

Problem (SD). Find an admissible pair $(\bar{y}, \bar{u}) \in \mathcal{A}_{ad}$, such that $J(\bar{y}, \bar{u}) = \inf_{\mathcal{A}_{ad}} J(y, u)$.

Let $z = z(\cdot; u) \in W_0^{1,p}(\Omega)$ be the unique solution of *variational system*:

$$\begin{aligned} Az(x) &= f_y(x, \bar{y}(x), \bar{u}(x))z(x) + f(x, \bar{y}(x), u(x)) \\ &\quad - f(x, \bar{y}(x), \bar{u}(x)), & \text{in } \Omega, \\ z|_{\partial\Omega} &= 0. \end{aligned} \quad (3.5)$$

$\mathcal{R}$ be the *reachable set* and the *Hamiltonian* H as follows:

$$\mathcal{R} = \{z(\cdot; u) \mid u \in \mathcal{U}\}; \quad (3.6)$$

$$\begin{aligned}
H(x, y, u, \psi^0, \psi) &= \psi^0 f^0(x, y, u) + \psi f(x, y, u), \\
\forall (x, y, u) &\in \Omega \times \mathbb{R} \times U, (\psi^0, \psi) \in \mathbb{R} \times \mathbb{R}.
\end{aligned} \tag{3.7}$$

In 1992, Yong [17] proved

Theorem (Maximum Principle) *Let (S1)–(S5) hold. Let $(\bar{y}, \bar{u}) \in \mathcal{A}_{ad}$ be an optimal pair of Problem (SD). Let $F'(\bar{y})\mathcal{R} - Q$ be finite codimensional in $\mathcal{Y}$. Then, there exists a triplet $(\psi^0, \psi, \varphi) \in [-1, 0] \times W_0^{1,p'}(\Omega) \times \mathcal{Y}^*$, such that $(\psi^0, \varphi) \neq 0$,*

$$\langle \varphi, \eta - F(\bar{y}) \rangle_{\mathcal{Y}^*, \mathcal{Y}} \leq 0, \quad \forall \eta \in Q. \tag{3.8}$$

$$\begin{aligned}
A\psi(x) &= f_y(x, \bar{y}(x), \bar{u}(x))\psi(x) \\
&+ \psi^0 f_y^0(x, \bar{y}(x), \bar{u}(x)) - F'(\bar{y}(\cdot))^* \varphi, \quad \text{in } \Omega, \\
\psi|_{\partial\Omega} &= 0.
\end{aligned} \tag{3.9}$$

$$\begin{aligned}
H(x, \bar{y}(x), \bar{u}(x), \psi^0, \psi(x)) &= \max_{u \in U} H(x, \bar{y}(x), u, \psi^0, \psi(x)), \\
&\text{a.e. } x \in \Omega.
\end{aligned} \tag{3.10}$$

In addition, if $F'(\bar{y})^*$ is injective (i.e., $\mathcal{N}(F'(\bar{y})^*) = \{0\}$), then, $(\psi^0, \psi) \neq 0$.

In 1996, Gao [5] discussed the optimal control problem when the operator A dependent to the control variable u and derived a maximum principle.

4 RELATIONS BETWEEN MP AND DP

The another important approach to the optimal control problems was originated by R. Bellman for finite dimensional optimal control problems and is called the *dynamic programming method*. Recent works done by M. G. Crandall and P. L. Lions on the *viscosity solutions* for Hamilton-Jacobi-Bellman equations is a breakthrough in this direction.

Both maximum principle (MP) and dynamic programming (DP) are all necessary conditions for optimal controls. Thus, it is very natural to ask: Are there any relations between them? The objective of this section is to answer this question.

Let X be a Banach space. Suppose $\varphi : X \rightarrow \mathbb{R}$ and $x_0 \in X$. We define

$$D^+ \varphi(x_0) = \{p \in X^* \mid \overline{\lim}_{x \rightarrow x_0} \frac{\varphi(x) - \varphi(x_0) - \langle p, x - x_0 \rangle}{|x - x_0|} \leq 0\}, \tag{4.1}$$

and

$$D^- \varphi(x_0) = \{p \in X^* \mid \lim_{x \rightarrow x_0} \frac{\varphi(x) - \varphi(x_0) - \langle p, x - x_0 \rangle}{|x - x_0|} \geq 0\}. \tag{4.2}$$

We call $D^+ \varphi(x_0)$ and $D^- \varphi(x_0)$ the *superdifferential* and *subdifferential* of φ at x_0 , respectively. It should be pointed out that $D^+ \varphi(x_0)$ and/or $D^- \varphi(x_0)$ could be empty.

For functions $\varphi : [0, T] \times X \rightarrow \mathbb{R}$, fix t_0 , $\varphi(t_0, x)$ is a function of x . We may define its superdifferential and subdifferential in x , denoted by $D_x^+ \varphi(t_0, x_0)$ and $D_x^- \varphi(t_0, x_0)$, respectively.

We consider the following family of optimal control problems: For any given $(t, x) \in [0, T] \times X$, let us consider the following state equation:

$$y_{t,x}(s) = e^{A(s-t)}x + \int_t^s e^{A(s-r)}f(r, y_{t,x}(r), u(r))dr, \quad s \in [t, T]. \quad (4.3)$$

with $u(\cdot) \in \mathcal{U}[t, T]$ and the cost functional

$$J_{t,x}(u(\cdot)) = \int_t^T f^0(r, y_{t,x}(r), u(r))dr + h(y_{t,x}(T)). \quad (4.4)$$

And define the *value function* $V : [0, T] \times X \rightarrow \mathbb{R}$ by the following:

$$V(t, x) = \inf_{u(\cdot) \in \mathcal{U}[t, T]} J_{t,x}(u(\cdot)), \quad V(T, x) = h(x). \quad (4.5)$$

The following Theorem (see Li-Yong [12]) gives the result concerning the relation between MP and DP. That is an infinite version of the result of Zhou [20](which was for finite dimensional cases).

Theorem *Let (H1) and (H2) hold. In addition, we assume that $h(x)$ are Fréchet differentiable in x with $h_x(x)$ being continuous in (x, u) . Let $V(t, x)$ be the value function of optimal control problem. Suppose $(\bar{y}(\cdot), \bar{u}(\cdot))$ is an optimal pair for the problem starting from $(0, x)$. Let $\psi(\cdot)$ be the solution of the following equation*

$$\begin{aligned} \psi(t) = & -e^{A^*(T-t)}h_x(\bar{y}(T)) + \int_t^T e^{A^*(r-t)}f_x(r, \bar{y}(r), \bar{u}(r))^* \psi(r)dr \\ & - \int_t^T e^{A^*(r-t)}f_x^0(r, \bar{y}(r), \bar{u}(r))dr, \quad t \in [0, T]. \end{aligned} \quad (4.6)$$

Then,

$$\begin{aligned} & \langle \psi(t), f(t, \bar{y}(t), \bar{u}(t)) \rangle - f^0(t, \bar{y}(t), \bar{u}(t)) \\ & = \max_{v \in U} [\langle \psi(t), f(t, \bar{y}(t), v) \rangle - f^0(t, \bar{y}(t), v)] \\ & \equiv -H(t, \bar{y}(t), -\psi(t)), \quad \text{a.e. } t \in [0, T]. \end{aligned} \quad (4.7)$$

$$D_x^- V(t, \bar{y}(t)) \subset \{-\psi(t)\} \subset D_x^+ V(t, \bar{y}(t)), \quad \forall t \in [0, T], \quad (4.8)$$

$$\begin{aligned} D^- V(t, \bar{y}(t)) \subset & \left\{ \left(\langle \psi(t), A\bar{y}(t) \rangle - H(t, \bar{y}(t), -\psi(t)), -\psi(t) \right) \right\} \\ & \subset D^+ V(t, \bar{y}(t)), \quad \forall t \in \mathcal{T}(\bar{y}(\cdot), \bar{u}(\cdot)), \end{aligned} \quad (4.9)$$

where for any admissible pair $(y(\cdot), u(\cdot))$,

$$\begin{aligned} \mathcal{T}(y(\cdot), u(\cdot)) \equiv & \left\{ t \in [0, T] \mid y(t) \in \mathcal{D}(A), \right. \\ & \left. \lim_{\delta \downarrow 0} \frac{1}{\delta} \int_t^{t+\delta} |f(r, y(r), u(r)) - f(t, y(t), u(t))|dr = 0 \right\}. \end{aligned} \quad (4.10)$$

5 LQ PROBLEM WITH UNBOUNDED CONTROL AND INDEFINITE COST FUNCTIONAL

The linear quadratic optimal control (LQ, for short) problems in finite dimensional system was first studied by Bellman-Glilksberg-Gross. In 1960, Kalman [6] founded the relation between the LQ problems and Riccati equation.

The LQ problems in infinite dimensional systems discussed by Lions [13], Lukes-Russell, Curtain-Pritchard, Balakrishnan, and Lasiecka-Triggiani [7, 8] etc. Flandoli and Da Prato-Ichikawa as well as Bensoussan, Da Prato, Delfour and Mitter had done a direct study of the Riccati equation of infinite dimensional case. However, all the above works are deal with the nonnegative cost functional.

Li-Yong [12] developed the results of You [19] and Chen [2] into the LQ problem with indefinite cost functional and unbounded control in Hilbert space including Neumann boundary control or pointwise control (for space dimension $n = 1$) for a parabolic equation.

Recently, Wu-Li [15] generalized and improve the results of You [19], Chen [2], Li-Yong [12] and Lasiecka-Triggiani [7, 8] etc into the finite horizon LQ problems with indefinite cost functional and unbounded control in Hilbert space, which covers Dirichlet, Neumann boundary control and pointwise control (for $n \leq 3$) for a parabolic equation. Wu-Li [16] also established the necessary-sufficient frequency theorem or called positive real lemma, and derive the synthesis for optimal control of the corresponding infinite horizon LQ problem, which does not require the assumption for the exact controllability of the system.

Let X and U be real Hilbert spaces, $-A$ generate an analytic semigroup e^{-At} on X and A^γ be defined for all $\gamma \in \mathbb{R}$. Denote F^* be the adjoint operators of F . Consider the evolution system

$$y(t) = e^{-At}x + \int_0^t A^\alpha e^{-A(t-s)}Bu(s)ds, \quad (5.1)$$

where $\alpha \in [0, 1)$ is constant, $x \in X$ and $B \in \mathcal{L}(U, X)$. In the case of parabolic equation on a bounded domain $\Omega \subset \mathbb{R}^n$, the relevant values of α are as follows: $\alpha = \frac{3}{4} + \varepsilon$, $\forall \varepsilon > 0$, for Dirichlet boundary control with $U = L^2(\partial\Omega)$, $X = L^2(\Omega)$; $\alpha = \frac{1}{4} + \varepsilon$, $\forall \varepsilon > 0$ for Neumann boundary control with $U = L^2(\partial\Omega)$, $X = L^2(\Omega)$; and $\frac{n}{4} < \alpha < 1$, for pointwise control with $U = H^{-2\alpha}(\Omega)$, $X = L^2(\Omega)$.

Let the cost functional be

$$J(x; u(\cdot)) = \int_0^\infty \{ \langle Qy(t), y(t) \rangle + 2 \operatorname{Re} \langle Sy(t), u(t) \rangle + \langle Ru(t), u(t) \rangle \} dt,$$

where $S \in \mathcal{L}(X, U)$. A control $u(\cdot)$ is called admissible at an initial state condition $x \in X$ if $u(\cdot) \in L^2(0, \infty; U)$ and $y(\cdot) \in L^2(0, \infty; X)$.

Problem LQ For given $x \in X$, find an admissible $u(\cdot) \in L^2(0, \infty; U)$ so that the cost functional is minimized.

Wu-Li [16] assume that e^{-At} is exponentially stable. Let

$$[Lu(\cdot)](t) \triangleq \int_0^t A^\alpha e^{-A(t-s)} Bu(s) ds, \quad \forall u(\cdot) \in L^2(0, \infty; U). \quad (5.2)$$

$$\begin{aligned} \tilde{\Phi}(\omega) &\triangleq R + B^*(A^*)^\alpha(-i\omega + A^*)^{-1}QA^\alpha(i\omega + A)^{-1}B \\ &+ SA^\alpha(i\omega + A)^{-1}B + B^*(A^*)^\alpha(-i\omega + A^*)^{-1}S^*, \quad \forall \omega \in \mathbb{R}, \end{aligned} \quad (5.3)$$

where $\tilde{\Phi}(\cdot)$ is called the frequency characteristic associated with Problem LQ . [16] proved

Theorem *The following statements are equivalent:*

(I) *Problem LQ is uniquely solvable at any $x \in X$ with the optimal pair $(\bar{u}(\cdot), \bar{y}(\cdot))$;*

(II) *The algebraic Riccati equation*

$$\begin{aligned} -\langle Ax, Px \rangle - \langle Px, Ax \rangle + \langle Qx, x \rangle \\ - \langle [S + B^*(A^*)^\alpha P]^* R^{-1} [S + B^*(A^*)^\alpha P]x, x \rangle = 0, \quad \forall x \in D(A), \end{aligned} \quad (5.4)$$

admits a solution P satisfying $(A^)^\alpha P \in \mathcal{L}(X)$ and*

$$\tilde{A} \triangleq -A^\alpha \{A^{1-\alpha} + BR^{-1}[S + B^*(A^*)^\alpha P]\}$$

with

$$D(\tilde{A}) = \{x \in D(A^{1-\alpha}) \mid [A^{1-\alpha} + BR^{-1}[S + B^*(A^*)^\alpha P]]x \in D(A^\alpha)\}$$

generates an exponentially stable and analytic semigroup. In this case, the state feedback optimal control of Problem LQ is given by

$$\bar{u}(t) = -R^{-1}[S + B^*(A^*)^\alpha P]\bar{y}(t), \quad \text{for } t \geq 0;$$

(III) *There exists some constant $\sigma > 0$ such that*

$$\langle (R + L^*QL + SL + L^*S^*)u(\cdot), u(\cdot) \rangle \geq \sigma \langle u(\cdot), u(\cdot) \rangle, \quad \forall u(\cdot) \in L^2(0, \infty; U);$$

(IV) *There exists some constant $\sigma > 0$ such that*

$$\langle \tilde{\Phi}(\omega)u, u \rangle \geq \sigma \langle u, u \rangle, \quad \forall u \in U, \omega \in \mathbb{R}.$$

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