12 Computer-supported mathematical learning in the information age: mathematical modelling learned through hypertexts

Shoichiro Machida

*Department of Mathematics, Faculty of Education
Saitama University
Japan*

Abstract

The educational aims for school mathematics in the information- and knowledge-intensive society require new emphases. Students must be able to grasp phenomena mathematically and systematically and be able to judge the authenticity of this knowledge. The nature of knowledge and information in the 21st Century has changed substantially from that of earlier centuries and mathematics transformed from the study of entities through visual mathematics and mathematical experiments (modelling) facilitated through the use of new visual media - the hypertext electronic textbook.

Keywords

Modelling, hypermedia, information retrieval.

INTRODUCTION

How to offer education in mathematics in 21st century is the common task of mathematics education researchers today. Unlike the 20th century, characterised by the growth of the industrialised society, which is based on production and distribution, the coming century may be characterised as an information- and knowledge-intensive society in an era in which information is converted into knowledge at an increasingly rapid pace, and in which new values are created through the exchange of ideas. These times require a serious examination of such

questions as “What should be the essential qualities of human beings in such an era?” and “To that end, what contributions can mathematical instruction and study make?”

The framework for today’s educational curriculum was proposed at the end of the 19th century in an effort to cope with the switch-over to industrial society. Thereafter, this curriculum continued to develop until it reached the form it has today. Presented in the following is an overall view of the relationship between society and school mathematics in this development process.

On the junior high school level, emphasis is placed on enabling students to mathematically express a world represented by engines and cogwheels, rather than by number theory and Euclidean geometry. With differential and integral calculus serving as the apex, everything is grasped as a function amid changes, making equations and graphs the predominant forms of mathematical expression. In preparation for such an education, students are taught about letters, ratios, and figures first, and then learn geometry, which is represented by diagrams, loci and figure drawings. At the junior high school level, attempts have been made to teach the various types of calculations necessary in modern city life and to foster the ability of students to think logically so that various contracts can be exchanged and promises made among people.

Meanwhile, we are now in the process of switching over to an information- and knowledge-intensive society.

Therefore, the objective of mathematical education must be to teach individuals to cope with such a social transformation. The New Math movement, which was launched in the second half of the 1950s, and lasted through the 1960s and into the early 1970s, stressed the formality of mathematics in an attempt to emphasise a structural understanding of mathematics and make it one of the centrepieces of a good education. Moreover, the educational use of personal computers, begun in the 1970s, flowered after the beginning of the 1980s, making it possible for probability and statistics textbooks to be handled more realistically. Through the use of moving diagrams, various factors could now be grasped in an integrated fashion in terms of their development process. Hypothetical, experimental guidance on functions was provided after repeated trial and error. All this gave rise to numerous proposals and practical examples for educational methods.

At the start of the 1990s, the Graphic User Interface (GUI) environment for computers was streamlined and new teaching topics such as fractals, chaos, fuzzy logic, etc. became the objects of research in the field of mathematical education.

As a result, mathematics was transformed from the study of formulas written in logical and abstract terms to the study of entities through visual mathematics and mathematical experiments.

QUALITY OF STUDENTS' MATHEMATICS REQUIRED BY THE INFORMATION / KNOWLEDGE SOCIETY

The development of the information society appears to require the following types of mathematical education:

To grasp phenomena mathematically and systematically

In today's society, many things - like the LSI chip, to cite one typical example - do not show anything on the outside, and even an internal analysis does not enable understanding. Entities appear as software, and to understand them, one would have to learn to systematically understand how information input and output are connected. In this kind of society, software used in mathematics course needs to be reviewed from an entirely new perspective; especially, how to understand the structure of mathematics must be examined in school.

To produce valuable knowledge based on information, to be able to tell which knowledge is true and which is false

Most people who use the Internet are most impressed by how widely information is dispersed. For information to be really worthwhile, it is far better that no restrictions be placed on it. However, in such a case two questions arise.

How can reflective knowledge be obtained information?

In fostering the ability to make good use of information, the following argument is common to questions about the Internet, electronic calculators and computers: "The use of Internet in education does not necessarily teach students how to make effective use of information. Moreover, those adults who can use the Internet studied how to construct knowledge in an environment in which the Internet did not exist".

A similar argument proposes that allowing students to use electronic calculators without giving them calculation drills on paper makes them incapable of understanding the true concept of mathematics. Adults, however, can understand these concepts precisely because they spent a great deal of time on calculation drills without relying on electronic calculators.

The above arguments are still true to some extent, but are somewhat out of date. A more moderate methodology must be proposed.

It is important to let students experience modelling in various stages. There are plenty of computer software programs today that support such experience.

How can one tell whether the knowledge gained is correct?

The theory of study based on structuralism boils down to how to how students structure knowledge out of information, and how they determine the authenticity of this knowledge. This is precisely what has been required of mathematical studies ever since the days of the ancient Greeks, and what progressive scientists

and philosophers required of mathematics in their arguments with conservative scholars and religious leaders.

Bits of information are labelled in mutual relation to one-another and stored for easy retrieval - chunking.

Information is not stored in one's brain in the way that an object is placed on a shelf. The network of intelligence, whether the student is conscious of it or not, is constantly being changed and restructured by one's interest and concerns. Mathematical studies cannot be pursued without duly considering the process by which the brain obtains knowledge. Independent and self-motivated study, inquisitive joint study, and suitable teaching materials are vitally important for mathematical education in our information society.

EXAMPLES

Example 1: authenticity of knowledge

Authenticity of knowledge may be endorsed by logical verification, verification by numerical values and calculations, verification by graphs and diagrams, and computer simulation. For people living in today's society, a sense of numerical values in term of geometric series may be considered an extremely important part of verification. The following story which is a mimicry of an old Japanese tale gives us a clue regarding the problems that we face today.

A shogun told a little boy to name anything he wished and the shogun would give it to him as a reward. The boy said: "I wish for the number of grains of rice placed on a Japanese chessboard, which has a total of 81 squares, one grain of rice placed on the first square, two grains on the next square, four grains on the third square, eight grains on the fourth square, and so on, multiplying in that way until all squares are filled." The shogun said, "Is that all you want?" and had his retainer make the necessary calculations. Learning that the amount of rice requested would be too great even for a shogun to give away, he apologised.

We should actually make the necessary calculations to see how much astonishment this witty story would evoke. Personal computers of the past had difficulty even with problems of this level. Let's calculate this on the TI 92.

$$2^0 + 2^1 + 2^2 + \dots + 2^{80} = 2,417,851,639,229,258,349,412,351 \approx 2418 \times 10^{21}$$

This figure really doesn't mean much to us. Therefore, to estimate how much this really is we need something for comparison. Assuming that a grain of rice is 1cm in length, and is arranged horizontally, it would total about 161,621,098,879 times the average distance between the earth and sun, which is about 14,960,000,000,000cm.

The major axis of the ellipse constituted by the galactic system is about 98,000 light years or 92.70×10^{21} cm, and even that can hardly match the length of the

rice grains. It would be 2^6 times the length of the major axis of the galactic system.

If we were to consider this topic in terms of the propagation of a rumour on the Internet, it would spread at a far greater speed, and the influence would be far more serious. Suppose, for example, a warning about viruses dubbed 'Penpal Greeting' is sent to a mailing list to which I belong. This is followed immediately afterward by a message from another person saying, "Penpal Greeting is not a virus, but a hoax." Assuming that propagation on the Internet spreads on the basis of a factor of 5, the propagation will have covered $(5^{11}-1)/4 = 12,207,031$ persons after 10 generations. It would spread throughout Tokyo in no time. Recent bankruptcies stemming from the problem of bad loans may be partly ascribed to an education that does not emphasise this sense of compound interest calculation.

In order to develop this type of software as teaching material, a computer capable of undertaking multiple calculations, and a database software capable of retrieving various practical data would be necessary.

Example 2: Creation of effective knowledge out of information

According to Gibson's Affordance Theory, things appeal to human beings in many different ways. If weeds in early spring put out tiny lovely flowers, people would get a good feeling from them and would hesitate to pull them out. If we wanted to select only necessary information out of continuous flow of information, organise it and thereby obtain only necessary knowledge, we would have to have a good sense of what is important in each bit of information received.

Problem of the trailer's length

"You are driving five meters behind a large trailer. Its visible shape is as shown in Figure 1, however, since you don't know how long the trailer is, you are afraid to pass it. How can you measure the length of the trailer? A hint is contained in the visual information given in Figure 1.

Figure 1

In order to obtain effective knowledge out of the given information, it would be necessary to eliminate unnecessary information and prepare a mathematical model (Machida and Kurihara, 1994). This problem can be solved through use of the proportional model. In order to perceive such a structure, it would have to be examined from various angles. Software to support such an investigation includes Macromedia Director, Visual Basic, 3D animation software, and a data base of photographs.

THE HYPERTEXT ELECTRONIC TEXTBOOK

Textbooks are a form of printed media. In Japan, it is believed that a good textbook is one that is thin and easy for teacher to handle. However, this type of textbook, which is designed primarily for teachers, cannot serve as a good source of information for students with diversified interests who wish to study on their own. Accordingly, an electronic textbook based on hypertext can be cited as a textbook to be used in school in an information society where students with diverse abilities, interest, and a level of enthusiasm can pursue autonomous and creative studies (Tiffin and Rajashingham, 1995; Koschmann, 1996). Electronic textbooks should have the following characteristics (Machida, 1996):

- Offered via CDROM or Internet communications.
- The screen should look like hypertext, but should provide a more elaborate explanation when the student comes across unfamiliar terminology or concepts.
- The concept of a certain unit is positioned in a network of concepts taught at all levels of education, ranging from primary school to high school.
- As in the case of an ordinary printed textbook, the first picture has a simple configuration.
- Satisfies key principles and embodies structural techniques such as those delineated in the Information Mapping Method (Horn, 1995).

REFERENCES

- Horn, R. E. (1995) *Mapping Hypertext*, Japanese edition, Nikkei BP Publishing.
- Koschmann, T. (1996) *CSCL: Theory and practice of an emerging paradigm*, Laurence Erlbaum Associates.
- Machida, S. and Kurihara, H. (1994) Seiji Moriya Mathematical Modelling, Technology in the Service of the Mathematics Curriculum. *Proceedings of Working Group 17 at ICME 7*, Freie Universitat Berlin.
- Machida, S. (1996) The Role of Technology in Cooperative Learning and Connecting Plural Mathematics Subjects. *The Role of technology in the mathematics classroom, Working Group 16 at ICME8*.

Tiffin, J. and Rajasingham, L. (1995). *In Search of The Virtual Class, Education in an Information Society*, Routledge.

Shoichiro Machida is a Professor of the Graduate School at Saitama University, Japan. He graduated from Keio University, specialising in mathematics education, computing and mathematics methodology. He has researched the educational features and role of computers in Japan and has presented papers at ICME on mathematical modelling (1992) and the role of technology in co-operative learning and connecting mathematics subjects (1996).