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Continuity and Self-intersections of Variable Radius Rolling Ball Blend Surfaces

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EXTENDED ABSTRACT

Variable radius rolling ball (VRRB) blend surfaces can be considered as envelopes of one parameter families of varying radius balls. As compared to circular blends Joseph Pegna called this type of surface a "spherical (tubular) blend" (Pegna, 1990). Here spherical VRRB surfaces are analyzed on the basis of the theory of envelopes (see e.g (Zalgaller, 1975)). Envelope surfaces are special cases of discriminant sets which have several useful properties, above all under certain natural conditions they are G^1 continuous provided the defining equations are G^1 (and piecewise C^2) continuous as well. In addition to spherical blends, offset surfaces can be defined as envelopes. Voronoi surfaces can be considered as discriminant sets.

The VRRB surface will be the envelope of the family of spheres centered on a spatial spine curve $\mathbf{c}(t)$ and touching a given surface $\mathbf{s}(u,v)$. Using a ***sufficient*** condition for the existence of the envelope surface it is shown that local self-intersection of the VRRB surface in $\mathbf{p}$ can occur only if either the $\mathbf{c}(t)$ spine curve is not smooth or

1. $\dot{\mathbf{c}}$ is parallel to $\mathbf{c}-\mathbf{s}$, where $\mathbf{s}$ is the contact point of the blend on the surface, or

2. $\langle \mathbf{s}-\mathbf{p}, \ddot{\mathbf{c}} \rangle + \dfrac{(E-rL)\langle \dot{\mathbf{c}}, \mathbf{s}^v \rangle^2 - 2(F-rM)\langle \dot{\mathbf{c}}, \mathbf{s}^v \rangle \langle \dot{\mathbf{c}}, \mathbf{s}^u \rangle + (G-rN)\langle \dot{\mathbf{c}}, \mathbf{s}^u \rangle^2}{(E-rL)\cdot(G-rN)-(F-rM)^2} <= 0,$

   where $r = |\mathbf{c}-\mathbf{s}|$ and E,F,G and L,M,N are the coefficients of the first and second fundamental forms of $\mathbf{s}(u,v)$ resp.

These conditions can be simply checked for any fixed t for all corresponding $\mathbf{p}$ by testing whether the halfspace defined by (2.) intersects the characteristic circle of the blend.

From (1.) and (2.) it follows that if one defines a VRRB surface between two surfaces then under certain simple local criteria the VRRB surface will be well defined and smooth in a neighborhood of the contact point unless the two surfaces are locally parallel. If these local criteria hold for *both* surfaces then the regularity of the blend surface in arbitrary points depends basically on the magnitude of the *geodesic curvature of the spine curve on the Voronoi surface.* Moreover the principal curvatures of these VRRB surfaces can be computed in arbitrary points and it turns out that the curvatures of the blend surface in a contact point *do not depend on the other base surface, nor on the curvature of the spine curve,* merely on the curvature of the base surface and the projection of $\dot{\mathbf{c}}$ to the tangent plane.

It is shown that the Voronoi surface between two surfaces being piecewise C^2 but as a whole only G^1 will be G^1 itself. Consequently the VRRB surface defined by a smooth piecewise C^2 spine curve running on the Voronoi surface will be G^1 unless self-intersection ((1.) or (2.)) occurs.

These considerations underline the concept that VRRB surfaces should be designed in such a way that one starts from a space curve and projects it to the Voronoi surface to get the smooth spine curve of the blend surface. The condition (2.) can be easily formulated in terms of the original space curve instead of the projected one. This idea serves as the basis of an experimental VRRB surface generator, see (Hermann, Lukács and Várady, 1994).

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