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Sojourn-time Analysis and CAC in ATM Networks

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Abstract

In this paper, we explore the potential of applying sojourn-time analysis in QoS metrics specification and in CAC algorithm design for ATM networks. We propose using the probability a link overflow occurs during a connection's life as a measure of its QoS. Such *sojourn-time based* QoS metrics have big impacts on many network traffic management components, e.g., CAC and routing. A CAC policy is accordingly designed which guarantees the so specified QoS for the entire life of each connection in the system. For a basic model, simple formulas are obtained for the estimate of the probability that a link overflow occurs within certain time interval. The analytical approach used and presented in the paper is a set of general results for estimating the probability distribution function and the mean of the sojourn-time of a Markov process being in a subset of its state space, which may correspond to a system's *underload* status.

Keywords

ATM, Call admission control, Transient analysis, Sojourn-time, Markov processes, Teletraffic theory, Traffic control

1 INTRODUCTION

The asynchronous transfer mode (ATM) is an emerging standard transfer mode for broadband integrated service digital networks (B-ISDN). An ATM network is expected to be multi-service and multi-quality-of-service since B-ISDN traffic is statistically heterogeneous (e.g. voice, video, data) requiring varied qualities of service (QoS) in terms of cell loss probability and/or cell delay. On one side, ATM has to be capable of offering high efficiency by integrating diverse services onto a common platform, on the other, it has to be able to meet varied and often very stringent QoS requirements.

Call admission control (CAC) is a major component of an ATM network for achieving the goal of both high efficiency and high qualities of service. It makes decisions in real time on whether to admit or block new arriving calls, i.e., sources, based on the sources' traffic characteristics, their QoS requirements and the temporal network conditions. In the following, we discuss three correlated issues that are the major concerns in designing an efficient call admission policy:

The traffic nature and its modeling. It is impossible to guarantee high quality of service if we don't know what the traffic will look like. It is also unlikely that we can design any efficient CAC algorithm if we cannot capture the nature of the traffic in models which are of certain degrees of accuracy and are easy to use. Characterizing and modeling various sources (real time video, LAN to LAN data, etc.) via experiments and analytical models has been a major research area for many years. Despite many recent progresses, still a lot remains unknown.

The QoS metric specifications. Up to now, all the QoS metric specifications are based on the system's steady-state behavior. For instance, the meaning of a specified cell loss ratio of 10^{-9} for a video conference connection is long-term statistical, i.e., for a connection that lasts forever, there would be approximately one cell lost in every 10^9 cells, or, among a large number of connections from the same source, the average cell loss ratio is 10^{-9}. Note that none of these two explanations can tell the user about the cell loss ratio that a specific connection will actually experience in its entire life. Since cell losses tend to occur in clusters, many busy hours will have no cell loss whatsoever while an occasional busy hour will experience a cell loss ratio much higher than 10^{-9}. In fact, steady-state behavior based QoS specifications provide convenient performance criteria for studying how the performance of ATM networks depends on different traffic characteristics. They are not necessarily sufficient, however, to define quality of service: the way cells are lost (the cell loss process) may have a significant impact on perceived quality of service. As was pointed out by the authors of COST 224(1992), the probability a cell loss occurs *during a given period* is a relevant measure of quality of service, therefore, some system transient behavior based QoS metric specifications would be more meaningful from, at least, a user perspective. Lau and Li(1996) also discusses the necessity of sojourn-time based QoS metrics for real-time broadband services and suggests considering a minimum requirement (specified in terms of percentile or average) on the length of non-blocking/no-loss period as one of them.

The efficiency and easy-to-use of the CAC algorithm. This is a methodology issue and is apparently dependent on how the last two issues are addressed. A key goal is to keep things as simple as possible. To achieve this goal, approximation techniques are very often used in traffic and system modeling. Currently proposed CAC algorithms or bandwidth allocation policies fall into two categories. One is the *effective bandwidth* approach, and the other is based on real-time system evaluations. All these algorithms are designed based on a system's steady-state assumption, as maybe partially a result of the standardization of the steady-state based QoS metrics. The effective bandwidth is a natural measure of a connection's bandwidth requirement relative to the desired QoS constraints, e.g., delay and/or cell loss experienced by a connection's cells. The CAC works by virtually assigning each connection its effective bandwidth and rejecting a connection request when the remained capacity is less than the connection's effective bandwidth. In general, the same source with more stringent QoS requirement will demand a larger value of effective bandwidth from the network. But this does not mean that, once admitted, *this* connection will actually receive higher quality of service, because there is no dedicated bandwidth assigned to it to guarantee that. In other words, the ***steady-state based CAC*** does not really guarantee the QoS at the individual connection level in real-time.

This paper explores the potential of using sojourn-time analysis results in QoS metrics specification and in CAC algorithm design. We address the above issues by employing Markov models in traffic modeling, by proposing sojourn-time based QoS metric specifications to supplement long-term steady-state metrics and by using simple and accurate analytical formulas in efficient CAC algorithm design. Our analysis and presentation will be focused on a very useful unbuffered model, but the analytical approach and the idea of sojourn-time based QoS metrics and CAC apply (or can be extended) to more general models.

The rest of the paper is organized as follows. The next section, in the context of a basic ATM multiplexer model, introduces the concept of sojourn-time based QoS metrics and sketches a CAC scheme that takes into account such QoS specifications from all connections. Since sojourn-time analysis is essential to the application of these new concepts, in Section 3 we present some general results on the sojourn-time of a Markov process. This part is of independent interest and is applicable in designing other systems. In Section 4, for our basic model, the general results are applied to obtain simple formulas for estimating the link overflow probabilities. Efficient real-time computation is achieved for the designed CAC. Numerical examples are provided in Section 5 to illustrate the new concepts and to show the accuracy and efficiency of the formulas. The paper is concluded in the Section 6 followed by the Appendix that contains the proof of an asymptotic result given in the formulas.

2 SOJOURN-TIME BASED QOS AND CAC

In the finite-source version of our model, an ATM link with transmission rate C (cells/second) is shared by heterogeneous, bursty sources which alternate between random periods in the "on" and the "off" states. In the "on" state, a source emits cells at its peak rate, while in the "off" state, it emits no cells. Assume there are J types of sources. For type j K_j denotes the number of sources and d_j its peak rate. Let $N_j(t)$ ($\le K_j$) be the number of active sources of type j being multiplexed at the link at time t and define $N(t) = (N_1(t), N_2(t), \cdots, N_J(t))$. Then the aggregate of the J different source types represents the total load at the link at time t, which is

$$S(t) := \sum_{j=1}^{J} d_j N_j(t). \tag{1}$$

When the load exceeds the link rate, we say the link is overloaded and a link overflow occurs. We assume that a buffer is either not provided or is of a small size to accommodate simultaneous cell arrivals and to cope with cell delay variation. In this case, a link overflow is likely to result in cell losses for some sources. Denote by B^c (i.e., the B complement) the subset of the state space S of $N(t)$ containing all the states representing the status of a link overflow, i.e, for $(n_1, n_2, \cdots, n_J) \in B^c$, $\sum_{j=1}^{J} d_j n_j > C$. Define

$$\tau_B = \inf\{t \geq 0 : S(t) > C\}, \tag{2}$$

then τ is the sojourn-time of $N(t)$ in B, or equivalently the time until the first time a link overflow occurs from an arbitrary reference time point 0. And the probability a link overflow occurs before time T is

$$P_{N(0)}(\tau_B \leq T) = P_{N(0)}(sup_{0<t\leq T} S(t) > C), \tag{3}$$

which is a function of the process $N(t)$'s initial phase $N(0)$. Since when $\tau_B \leq T$ occurs any connection that is alive before time T may suffer from cell loss, it is desirable from a viewpoint of quality of service that the network maintains this probability under certain limit. This limit may vary with time and may be different for different connections when specified as a user QoS requirement. In the above model, for type j sources, this requirement can be specifically given as a pre-defined pair (ϵ_j, T_j) to mean

$$P_{N(0)}(\tau \leq T_j) < \epsilon_j. \tag{4}$$

For example, a two-hour video conference call requiring the chance of any link overflow within its lifetime below 10^{-6}, may specify (10^{-6}, 2hr). This type of QoS metrics is referred to as ***sojourn-time based*** (see also Lau and Li(1996)), as compared to existing ***steady-state based.*** In the above format, the system's initial phase $N(0)$ has to be taken into account. This requires the control schemes to make observations on the transient link load status, which makes things complicated. An alternative is to take an average of $P_{N(0)}(\tau \leq T)$ over the steady-state distribution π of $N(t)$. Consequently, the QoS specification in (4) is modified as

$$P_\pi(\tau \leq T_j) \leq \epsilon_j. \tag{5}$$

This is the approach we are taking in the paper.

The above QoS requirement can be viewed as a constraint to the system on the length of non-blocking/no-loss period. The objective is to guarantee quality of service for a connection's entire life. Therefore, it is particularly meaningful to the sources with known holding-time statistics for which we can estimate the number of lossy periods they most likely encounter. In the model considered above, when a link overflow occurs, connected sources will suffer from cell loss in different degrees depending on their instantaneous activities. The link overflow probability is an upper bound for the connections' loss probabilities. This upper bound is tight for very active sources and not that tight for the idle ones. It is realized that there is no direct translation from the system blocking to the actual cell loss of each connection.

When sojourn-time based QoS is introduced, traffic management elements (e.g., CAC and routing) should be designed accordingly to cope with the new situation. Here we outline a CAC scheme for the above model with an additional assumption that the "on" and the "off" periods are independent and are exponentially distributed. So the the model becomes Markovian. Specifically, $N(t)$ is a multi-dimensional Markov process and is reversible with respect to its steady-state distribution π. The CAC is to make a call admission decision to satisfy all connections' (including the new one's) sojourn-time based QoS requirements. Note that for the existing connections, it makes sense to only guarantee the quality of service for their remained life. Therefore, for these connections, T_j is changed to $T_j - t_j$ assuming t_j is the elapsed time since the individual connection was made. This implies that the CAC needs some timing function or service. Real-time computation of type $P_\pi(\tau_B \leq T_j)$ for all sources is carried out to assess QoS performance. Since the new arrival is taken into account, in the model and the computation, $N_j(t)$ is replaced by $N_j(t) + 1$. If all QoS requirements can still be met, the new request will be admitted, otherwise, rejected. Some computation savings can be obtained when the sources in the same group have the same holding-time statistics, since in which case, computations need only be done for the connection with the longest residual life.

In the next two sections, our focus will be on the computation of $P_\pi(\tau_B \leq T)$, since the efficiency and ease of use of the CAC can only be achieved by the accurate and simple estimate of this probability. As will be seen in the next section, accurate bounds of the probability involving only two parameters, $\lambda_0(B)$ and *Gap*, are available for general Markov models. For both homogeneous and heterogeneous cases of out model, simple formulas for their computation are presented in the followed section. These results show the potential of practical uses of the sojourn-time based QoS and CAC in real networks.

3 GENERAL RESULTS ON SOJOURN-TIME

In this section, we present general analytical results on the sojourn-time of time-continuous Markov jump processes. Our discussion will be focused on the so-called time-reversible processes since these results are immediately applicable to the previously established ATM CAC model. The readers may find more general results in Iscoe and McDonald(1994) and Qian(1996).

Let $(X_t; t \geq 0)$ be an irreducible finite-state *time-reversible* Markov chain in continuous time. The state space is S and the transition rate matrix (or the infinitesimal generator) is $-L = (q(i,j); i,j \in S)$ where $q(i,i) = -\sum_{j \neq i} q(i,j)$. Let π be the stationary distribution. Because of the reversibility of the chain, the matrix L is symmetrizable and therefore only has real eigenvalues. Since the smallest positive eigenvalue is the distance between the eigenvalue 0 and the rest of the spectrum of L, it is called the *gap* and is denoted by *Gap(L)* or simply *Gap*. Let B be a proper non-empty subset of S and τ_B be the sojourn-time in B. Note that τ_B is also referred to as the *first hitting time* of X_t on B^c (the complement of B). So $0 < E_\pi \tau_B < \infty$. We are interested in the tail of the distribution of τ_B starting from the stationary distribution or $P_\pi(\tau_B > t)$. Let $-L^B$ be the transition rate matrix $-L$ restricted to B and $0 < \lambda_0(B) < \lambda_1(B) \leq \lambda_2(B) \leq \cdots$ be eigenvalues of L^B, then the exact solution of $P_\pi(\tau_B > t)$ can be given as

$$P_\pi(\tau_B > t) = \sum_{i \geq 0} c_i e^{-\lambda_i(B)t}, \tag{6}$$

where c_i are positive coefficients that can be determined by the corresponding eigenvectors of $\lambda_i(B)$ and the stationary distribution. Apparently, it is not practical to use this expression for large systems. In most cases, we seek approximation approaches such as asymptotics or bounds. The following general results on the mean of the sojourn-time τ_B and its probability distribution function $P_\pi(\tau_B \leq t)$ are found in Aldous and Brown(1993), Iscoe and McDonald(1994) and Qian(1996).

Lemma 1 *For the sojourn-time τ_B defined above, the following inequalities provide upper and lower bounds over its probability distribution and its mean:*

$$\left(1 - \frac{\lambda_0(B)}{Gap}\right) e^{-\lambda_0(B)t} \leq P_\pi(\tau_B > t) \leq e^{-\lambda_0(B)t}, \tag{7}$$

$$\frac{1}{\lambda_0(B)} - \frac{1}{Gap} \leq E_\pi \tau_B \leq \frac{1}{\lambda_0(B)}. \tag{8}$$

In Qian(1996), these results with additional negligible higher order terms are also obtained for general *nonreversible* Markov jump processes. Here $\lambda_0(B)$ and *Gap* are the only two parameters that need some computation. These results are therefore easy to use since various methods are available to obtain bounds or estimates for these parameters. The *Gap* is such an important parameter since a strictly positive *Gap* makes the chain converge to stationarity exponentially fast. The following useful result regarding $Gap(L)$ of a multi-dimensional Markov process is provided by Liggett(1989). Let $-L$ be the generator of a vector Markov process whose components are independent Markov processes with generators $-L_k$ and stationary distribution π_k.

Lemma 2 *Assume that the stationary distribution π of the vector process is the product of the π_k's, then*

$$Gap(L) = \inf_k Gap(L_k). \tag{9}$$

Since $\lambda_0(B)$ is the smallest eigenvalue of L^B which is symmetrizable, standard algorithms are available for its efficient calculation. The computational complexities of the algorithms are well understood. For small models with not too many sources, it is possible to use some of the algorithms in real-time.

For large models, the following simple formula can be applied to estimate $\lambda_0(B)$ (see Qian(1996)).

Lemma 3 *Let ϕ_0 be the nonnegative eigenvector of L^B corresponding to the principal eigenvalue $\lambda_0(B)$ such that $\sum_{i\in B}\phi_0(i)\pi(i) = 1$. Then*

$$\lambda_0(B) = \sum_{i\in B}\Big(\sum_{j\in B^c} q(i,j)\Big)\phi_0(i)\pi(i). \tag{10}$$

This result is especially useful if the process has no long-range correlation, e.g. the birth and death process. In this case, the "killing" rates $\sum_{j\in B^c} q(i,j)$ are zero except at the boundary of B where they are just the birth rates. So this proposition says that $\lambda_0(B)$ very much depends on the values of the stationary distribution and the eigenvector at the boundary of B. Therefore, capturing the boundary behavior of those values will help in estimating $\lambda_0(B)$.

Many teletraffic models that have applications in communications and computer systems, such as ATM and wireless communications, belong to the general class of continuous-time Markov jump processes. The loss or blocking problem arises in a system with shared resource may often be characterized by the *first hitting time* of a rarely-visited set (B^c) of states of the modeling process. The results presented in this section are of general interest to the system analysts.

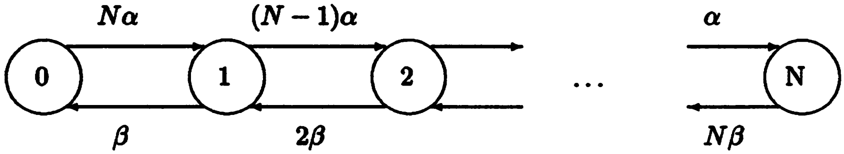

Figure 1 State transition diagram

4 COMPUTATIONS IN CAC

In this section, we address the computation problem involved in the CAC scheme designed in the second section. For both the homogeneous and the heterogeneous cases, simple formulas are obtained for efficiently calculating the estimate of the probability a link overflow occurs in a time interval corresponding to a connection's residual life.

4.1 Homogeneous case

The simple homogeneous case of our model is when all the sources are identical in terms of traffic characteristics as well as QoS requirements. Assume the peak rate of the sources is normalized to one and the link rate is devided by the peak rate. Denote by α and β the transition rates from off to on and from on to off respectively. Let N be the total number of sources and $N(t)$ be the number of sources in the system at time t. Then $N(t)$ is a birth and death process on the state space $S = \{0, 1, 2, \cdots, N\}$. The transition diagram of this process is shown in Figure 1. It is known that $N(t)$ in steady state has a binomial distribution:

$$
\begin{aligned}
\pi(k) &= \binom{N}{k} p^k (1-p)^{N-k}, \\
p &= \frac{\alpha}{\alpha+\beta}.
\end{aligned}
$$

Define $B := \{0, 1, 2, \cdots, [C]\}$ and $\tau_B = \inf\{t \geq 0 : N(t) \geq [C] + 1\}$, we apply (7) to the link overflow probability, i.e.,

$$1 - e^{-\lambda_0(B)t} \leq P_\pi(\tau_B \leq t) \leq 1 - \left(1 - \frac{\lambda_0(B)}{Gap}\right) e^{-\lambda_0(B)t}. \tag{11}$$

Easy computations of the values or the estimates of *Gap* and $\lambda_0(B)$ are essential in applying the bounds in real-time CAC. Simple formulas are given in the following two propositions.

Proposition 1 *In the homogeneous case of out model, for the generator $-L$ of $N(t)$, the gap between the smallest eigenvalue 0 and the rest of the spectrum is given by*

$$Gap(L) = \alpha + \beta. \tag{12}$$

Proof Let λ be an eigenvalue of the generator $-L$ of $N(t)$ and ϕ be the corresponding right eigenvector. Then, for $0 \leq k \leq N$,

$$(\lambda + (N-k)\alpha + k\beta)\phi(k) = (N-k)\alpha\phi(k+1) + k\beta\phi(k-1). \tag{13}$$

Define the weighted generating function of ϕ as following

$$\Phi(z) := \sum_{k=0}^{N} \phi(k) z^k \pi(k).$$

Multiply $z^k\pi(k)$ on both sides of equation (13) and sum over k from 0 to N we get

$$(\lambda + Na)\Phi(z) + (\beta - \alpha)z\Phi'(z) = (\beta - \alpha z^2)\Phi'(z) + N\alpha z\Phi(z).$$

This is equivalent to

$$\frac{\Phi'(z)}{\Phi(z)} = \frac{N\alpha(z-1) - \lambda}{\alpha z^2 + (\beta - \alpha)z - \beta}$$

A solution to this equation is

$$\Phi(z) = (\alpha z + \beta)^{N + \frac{\lambda}{\alpha+\beta}}(1-z)^{\frac{\lambda}{\alpha+\beta}}.$$

Since ϕ is polynomial function, $\frac{\lambda}{\alpha+\beta}$ must be a non-negative integer. This implies that the eigenvalues of $-L$ must be multiples of $\alpha + \beta$, in particular, $Gap(L) = \alpha + \beta$. ■

Proposition 2 *Let $\pi(B) = \sum_{i \in B} \pi(i)$, then the smallest positive eigenvalue $\lambda_0(B)$ of L^B has an upper bound as in the following.*

$$\lambda_0(B) < (N - [C])\alpha \binom{N}{[C]} p^k (1-p)^{N-[C]} / \pi(B). \tag{14}$$

In addition, as $N \to \infty$ and $C \to \infty$,

$$\lambda_0(B) \sim (N - [C])\alpha \binom{N}{[C]} p^k (1-p)^{N-[C]}. \tag{15}$$

Proof Applying Equation (10) in Lemma 3 to the birth and death process $N(t)$, we immediately have

$$\begin{aligned} \lambda_0(B) &= q([C], [C]+1))\pi([C])\phi_0([C]) \\ &= (N - [C])\alpha \binom{N}{[C]} p^k (1-p)^{N-[C]} \phi_0([C]). \end{aligned}$$

In the following, we only prove the upper bound of $\lambda_0(B)$ by showing $\phi_0([C]) < 1/\pi(B)$, and leave the proof of the asymptotic property to the Appendix at the end of the paper. Essentially what we explain there is the approximation of $\phi_0([C]) \sim 1$ in the above equation.

Since $-L^B$ is a bounded matrix, there exists a positive number M such that $-L^B + MI$ is a positive matrix, where I is the identity matrix. By Perron-Frobenius theorem, this matrix has a maximum positive eigenvalue of multiplicity one associated with the positive right eigenvector ρ. Apparently, ρ is also an eigenvector for L^B and is associated with the least positive eigenvalue, i.e., $\lambda_0(B)$. Let ϕ_0 be ρ and normalized so that

$$\sum_{i \in B} \phi_0(i)\pi(i) = 1, \tag{16}$$

then $\phi_0(i) \geq 0$ for all $i \in B$. Let ϕ_0 be extended such that $\phi_0([C]+1) = \phi_0(-1) = 0$, then similar to Equation (13), for $0 \leq k \leq [C]$,

$$-\lambda_0(B)\phi_0(k) = (N-k)\alpha(\phi_0(k+1) - \phi_0(k)) - k\beta(\phi_0(k) - \phi_0(k-1)).$$

Multiplying $\pi(k)$ to both sides of the above equation, we obtain

$$\begin{aligned} -\lambda_0(B)\pi(k)\phi_0(k) &= (N-k)\alpha\pi(k)(\phi_0(k+1) - \phi_0(k)) - \\ &\quad\ k\beta\pi(k)(\phi_0(k) - \phi_0(k-1)) \\ &= (N-k)\alpha\pi(k)(\phi_0(k+1) - \phi_0(k)) - \\ &\quad\ (N-k-1)\alpha\pi(k-1)(\phi_0(k) - \phi_0(k-1)), \end{aligned} \tag{17}$$

here the following local balance equation of the reversible process is used

$$k\beta\pi(k) = (N-k-1)\alpha\pi(k-1).$$

In Equation (17), sum up both sides from $k = 0$, then

$$-\lambda_0(B)\sum_{i=0}^{k}\pi(i)\phi_0(i) = (N-k)\alpha\pi(k)(\phi_0(k+1)-\phi_0(k)).$$

Since $\lambda_0(B) > 0$ and ϕ_0 is positive, the left-hand side, and therefore the right-hand side, of the equation is less than 0. This implies that, for all $k \in B$, $\phi_0(k+1) < \phi_0(k)$, i.e., $\phi_0(k)$ is a strictly decreasing function of k. Especially we have, for $k < [C]$, $\phi_0([C]) < \phi_0(k)$. Applying this in Equation (16), we obtain

$$\begin{aligned}\phi_0([C]) &= \sum_{k\in B}\phi_0([C])\pi(k)/\pi(B)\\ &< \sum_{k\in B}\phi_0(k)\pi(k)/\pi(B)\\ &= 1/\pi(B).\end{aligned}$$

This completes the proof of the first part of the proposition. ■

It is important to note that, using the upper bound of $\lambda_0(B)$ in the upper bound in the link overflow probability in (11), we get a conservative estimate. This can be seen from the fact that $f(x) = 1-(1-x)e^{-xt}$ is an increasing function of x when $t > 0$ and $0 < x < 1$ (which is true for a well engineered link since $E_\pi\tau > 1/\lambda_0(B) - 1/Gap$).

4.2 Heterogeneous case

In the heterogeneous case, our model contains J different types of sources. $N_j(t)$ is the number of active sources of type j currently being multiplexed at the link at time t. The aggregate of the J different source types represents the total load at the link at time t, which is $S(t) := \sum_{j=1}^{J} d_jN_j(t)$. $N(t) = (N_1(t), N_2(t), \cdots, N_J(t))$ is a multi-dimensional reversible Markov process which takes values in the state space $S = S_1 \times S_2 \times \cdots \times S_J$ with $S_i = \{0, 1, \cdots, N_i\}$, and has stationary distribution π given by, for $\vec{n} = (n_1, n_2, \cdots, n_J) \in S$,

$$\pi(\vec{n}) = \prod_{i=1}^{J}\pi_i(n_i) \tag{18}$$

where

$$\pi_i(n_i) = \binom{N_i}{n_i}p_i^{n_i}(1-p_i)^{N_i-n_i},$$

$$p_i = \frac{\alpha_i}{\alpha_i + \beta_i}.$$

For $\vec{n} = (n_1, n_2, \cdots, n_J) \in S$, we define $f(\vec{n}) = \sum_{i=1}^{J} d_i n_i$ and

$$\pi^*(r) = \sum_{\vec{n} \in S: f(\vec{n}) = r} \pi(\vec{n}). \tag{19}$$

Let $B := \{\vec{n} \in S : f(\vec{n}) < C\}$, then the sojourn-time τ_B from an arbitrary reference time point 0 to the first time link overflow or cell loss occurs is $\tau_B = \inf\{t \geq 0 : f(N(t)) \in B\}$. We now present formulas for the computation of the two parameters *Gap* and $\lambda_0(B)$.

Since the steady state distribution (18) has a product form, combining Equation (9) in Lemma 2 with the result (12) in Proposition 1 we obtain the following.

Proposition 3 *In the heterogeneous case of our model, for the generator $-L$ of $N(t)$, the gap between the smallest eigenvalue 0 and the rest of the spectrum is given by*

$$Gap = \min\{\alpha_i + \beta_i, i = 1, 2, \cdots, J\}. \tag{20}$$

Let $\phi_0(\vec{n})$ be the eigenvector corresponding to the principal eigenvalue $\lambda_0(B)$ and

$$\sum_{\vec{n} \in B} \phi_0(\vec{n}) \pi(\vec{n}) = 1.$$

Then, according to Equation (10) in Lemma 3,

$$\lambda_0(B) = \sum_{\vec{n} \in B} \sum_{\vec{m} \in B^c} q(\vec{n}, \vec{m}) \phi_0(\vec{n}) \pi(\vec{n}).$$

Since $N(t)$ is a multi-dimensional birth and death process, $q(\vec{n}, \vec{m}) = 0$ unless $\vec{n} = \vec{m} \pm \delta_i$, where δ_i are the basis vectors. Therefore,

$$\lambda_0(B) = \sum_{\vec{n} \in B} \sum_{i: f(\vec{n}) \leq C, f(\vec{n} + \delta_i) > C} (N_i - n_i) \alpha_i \phi_0(\vec{n}) \pi(\vec{n}).$$

For the same reason as in the homogeneous case, we have:

Proposition 4 *Let $\pi(B) = \sum_{\vec{n} \in B} \pi(\vec{n})$, then the smallest positive eigenvalue*

$\lambda_0(B)$ of L^B has an upper bound as in the following.

$$\lambda_0(B) \leq \sum_{\vec{n}\in B} \sum_{i:C-d_i<f(\vec{n})\leq C} (N_i - n_i)\alpha_i \pi(\vec{n})/\pi(B).$$

In addition, if for all i, $N_i \to \infty$ and $C \to \infty$, then

$$\lambda_0(B) \sim \sum_{\vec{n}\in B} \sum_{i:C-d_i<f(\vec{n})\leq C} (N_i - n_i)\alpha_i \pi(\vec{n}).$$

Before closing off this section, we make a few remarks on the accuracy of the bounds and approximations in order to qualitatively justify their use in real-time CAC.

Remark 1. Both the lower bound and the upper bound given in (11) on the link overflow probability are tight. In fact, they are different in a factor of $1 - \lambda_0(B)/Gap$ which is very close to 1. This is due to the fact that $\lambda_0(B)$ has to be extremely small to have a large value of $E_\pi(\tau)$ (see Lemma 1), a true fact in a well engineered system. Meanwhile, the values of *Gap* are known before hand. The minimum value of *Gap* can not be too small, at least not at the same order as $\lambda_0(B)$. In addition, the upper bound of the link overflow probability provides conservative estimate when used with the upper bound on $\lambda_0(B)$.

Remark 2. Whenever appropriate, the model should be modified to reduce the number of underload states (see a numerical example later). This procedure may result in a small matrix L^B, espesially when the link carries a small number of connections with medium to large values of peak rates (relative to link rate). Then the exact value of $\lambda_0(B)$ can be computed using well known standard algorithms. When a large number of small to medium connections are present, the asymptotics of $\lambda_0(B)$ would be accurate based on the asymptotic property. Numerical examples shown in the following section provide additional evidence to this fact.

5 NUMERICAL EXAMPLES

To illustrate the sojourn-time based QoS concepts and to show the accuracy of the formulas presented in the previous sections, we consider the following model of a single connection type (i.e., the homogeneous case): link rate=45Mb/s, source peak rate=1.5Mb/s and source activity $\rho = 0.1$. Let the burst length be the time unit and divide the link rate by the source peak rate. We leave the number of connections in the system as a variable to construct examples at different link utilizations. Using the notation introduced in early sections, we have $C = 30$, $\beta = 1$ and $\alpha = \rho/(1-\rho) = 0.11$. Therefore, $Gap = 1.1111$. The matrix L^B is of dimension 31×31, which is not too big

Table 1 Comparison of basic statistics.

No. conn.	$\lambda_0(B)$	$\lambda_0^*(B)$	$1-\pi(B)$	$E_\pi\tau$
60	1.48E-14	1.67E-14	5.99E-16	6.74E13
70	3.09E-12	3.64E-12	1.35E-13	3.23E11
80	2.06E-10	2.54E-12	9.85E-12	4.85E09
90	6.24E-09	8.06E-09	3.26E-10	1.60E08
100	1.05E-07	1.43E-07	6.05E-09	9.49E06
110	1.13E-06	1.62E-06	7.18E-08	8.83E05
120	8.51E-06	1.29E-05	5.99E-07	1.18E05
130	4.77E-05	7.71E-05	3.75E-06	2.10E04
140	2.09E-04	3.62E-04	1.85E-05	4.78E03
150	7.43E-04	1.39E-03	7.49E-05	1.34E03
160	2.21E-03	4.47E-03	2.56E-04	4.52E02

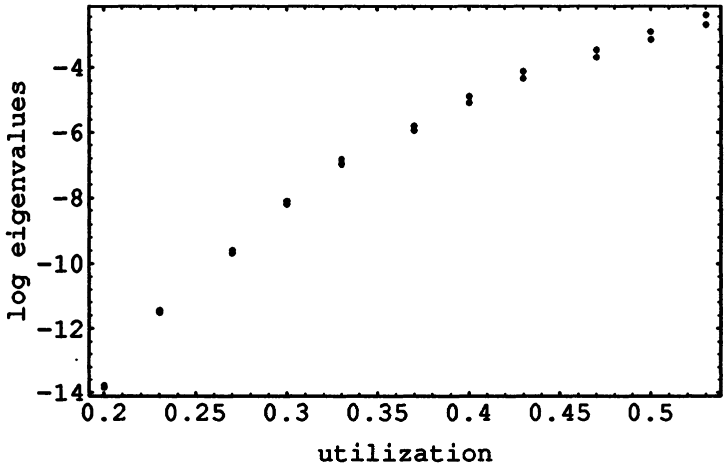

Figure 2 Comparison of exact eigenvaules and the upper bounds.

for the standard algorithms to find exact eigenvalues. Table 1 lists some basic statistics for the number of sources from 60 to 160. In this table, $\lambda_0(B)$ and $\lambda_0^*(B)$ are respectively the exact eigenvaule and its upper bound computed from Proposition 2, $1-\pi(B)$ is the link overflow probability in steady-state and $E_\pi\tau$ is the mean time until the overflow occurs. From the Table 1 we see that the upper bound on $\lambda_0(B)$ is tight. Note that it does not show the convergence to $\lambda_0(B)$ as the number of connections increases. This is because the link rate is fixed. The tightness can also be seen in Figure 2 where the logarithm of the eigenvalues is plotted against the link utilization. In Figure 3 and Figure 4, in the time interval of from 0 to 1200 bursts, we plotted the bounds (in logarithm)of the probability distribution functions for the number

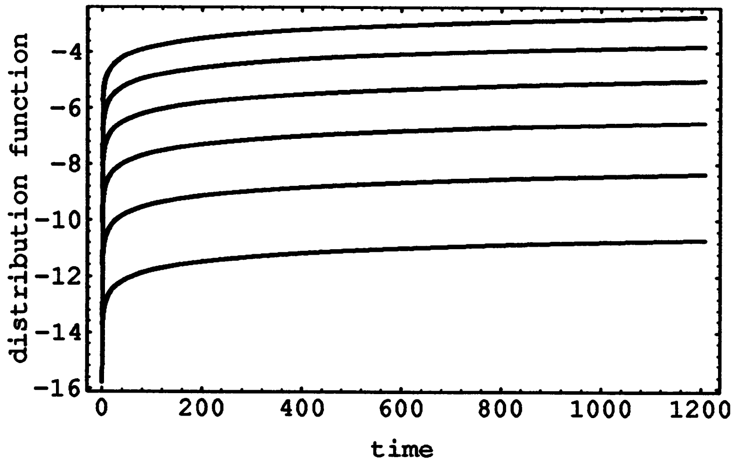

Figure 3 Log distribution functions for N=60 to N=100.

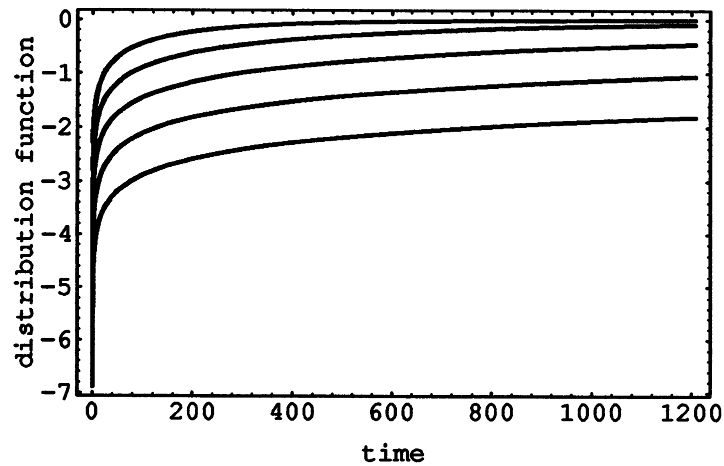

Figure 4 Log distribution functions for N=110 to N=160.

of connections $N = 60$, $N = 70$ and up to $N = 160$, in a sequence of from bottom up. If the connection's QoS is specified as $(10^{-4}, 600)$, i.e., the probability of a link overflow occurs in 600 bursts is less than or equal to 10^{-4}, then only less than 100 connections could be accepted by the system. However, it is interesting to note that the steady-state blocking probability is only 6.05E-09 even when all 100 connections are in the system. This means that the requirement of guaranteeing QoS continuously for certain time is much more stringent than that of maintaining a long-term average QoS.

6 CONCLUSION

This paper explores the potential of applying sojourn-time analysis in QoS metrics specification and in CAC algorithm design for ATM networks. It discusses the necessity of using sojourn-time based QoS metrics and corresponding traffic control schemes to supplement existing steady-state based ones. Specifically, we propose using the probability a link overflow occurs during a connection's lifetime as a measure of its QoS. A CAC policy is sketched which is aimed at guaranteeing specified minimum link overflow for the entire life of each connection in the system. Applying these novel ideas, we are essentially able to implement a CAC scheme for a basic model, for which simple formulas are obtained for estimating, in real-time, the probability a link overflow occurs within certain time interval. These results are accurate as shown by the theory and by the numerical examples.

Also presented in the paper are some general results for estimating the probability distribution function and the mean of the sojourn-time of a Markov process being in a subset of its state space. They can be applied to other areas such as congestion control or priority systems, and are therefore of independent interest.

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7 APPENDIX

The use of the approximations $\phi_0([C]) \sim 1$ and $\pi(B) \sim 1$ in the Proposition 2 is explained in this appendix.

In the case of an infinite-source version of the homogeneous model, bursts arrive in a Poisson stream of a constant rate. The burst durations are independent (and independent of the arrival process) and are exponentially distributed. Let a and $1/b$ respectively denote the arrival rate and the mean duration period, and also $\rho = a/b$. In steady state, the superposed demand process $N_\infty(t)$ now has a Poisson(ρ) distribution, i.e., for $k \leq 0$,

$$\pi(k) = \frac{\rho^k}{k!}e^{-\rho}.$$

It is well known that this model can be obtained as the Poisson limit of the finite-source model by making $N \to \infty$ and $p \to 0$ such that $Np = \rho$. Therefore, statistics of the infinite-source model are asymptotics of the same statistics of the finite-source model. For example, taking the limit of $p \to 0$ (same as $\alpha \to 0$) in $Gap = \alpha + \beta$ in the finite-source model, we obtain the $Gap = \beta = b$ for the infinite-source model. Let ℓ be the link rate and $B = \{0, 1, 2, \cdots, \ell - 1\}$ be the set of underload states of the process $N(t)_\infty$. It is easy to see that $\pi(B) \sim 1$, as $\ell \to \infty$. We now prove $\phi^\ell(\ell - 1) \sim 1$, as $\ell \to \infty$, where ϕ^ℓ is the eigenvector of the principal eigenvalue $\lambda_0(\ell) := \lambda_0(B)$ and is normalized so that $\phi^\ell(0) = 1$. *Proof* Let $(x, y)_\pi$ be the inner product of vectors x and y with respect to the distribution π restricted on B, and $\|x\|_\pi^2 = (x, x)_\pi$ then

$$\begin{aligned}
\lambda_0(\ell) &= (\phi^\ell, L^B\phi^\ell)_\pi / \|\phi^\ell\|_\pi^2 \\
&= \sum_{k=0}^{\ell-1} \frac{1}{2\|\phi^\ell\|_\pi^2} \left[(\phi^\ell(k+1) - \phi^\ell(k))^2 a + (\phi^\ell(k-1) - \phi^\ell(k))^2 kb\right] \pi(k) \\
&= \sum_{k=0}^{\ell-1} (\phi^\ell(k+1) - \phi^\ell(k))^2 a\pi(k) / \|\phi^\ell\|_\pi^2, \text{ (by reversibility)} \qquad (21)
\end{aligned}$$

where ϕ^ℓ is extended so that $\phi^\ell(\ell) = \phi^\ell(-1) = 0$. Since $\phi^\ell(k)$ is a decreasing function of k (same as in the proof of Proposition 2) and $\phi^\ell(0) = 1$, we know that ϕ^ℓ is uniformly bounded, i.e. $\phi^\ell(k) \leq 1$ for all k and ℓ. In addition,

$$\|\phi^\ell\|_\pi^2 = \sum_{k=0}^{\ell-1} (\phi^\ell(k))^2 < 1.$$

Therefore

$$0 < (\phi^\ell, \mathbf{1})^2_\pi \le \|\phi^\ell\|^2_\pi \|\mathbf{1}\|^2_\pi \le 1,$$

where $\mathbf{1}$ is the vector with all its elements equal to 1. By a result in Qian(1996), we have

$$1 \ge \frac{(\phi^\ell, \mathbf{1}_B)^2_\pi}{\|\phi^\ell\|^2_\pi} \ge 1 - \frac{\lambda_0(\ell)}{Gap(L)} \to 1, \text{ as } \ell \to \infty$$

since $Gap(L) = b$, and $\lambda_0(\ell) \sim a\pi(\ell - 1) \to 0$ as can be seen in the expression below. Applying (10) we get

$$\lambda_0(\ell) = \frac{(\phi^\ell, K_B)_\pi}{(\phi^\ell, \mathbf{1}_B)_\pi} \sim \frac{a\phi^\ell(\ell-1)\pi(\ell-1)}{\|\phi^\ell\|_\pi}, \text{ as } \ell \to \infty.$$

So

$$\frac{\lambda_0(\ell)\|\phi^\ell\|^2_\pi}{a\pi(\ell-1)} \sim \phi^\ell(\ell-1)\|\phi^\ell\|_\pi, \text{ as } \ell \to \infty.$$

By (21) and $\phi^\ell(\ell-1)\|\phi^\ell\|_\pi \le 1$, we conclude that there is a constant M such that, for ℓ large enough,

$$\sum_{k=0}^{\ell-1}(\phi^\ell(k+1) - \phi^\ell(k))^2 \frac{\pi(k)}{\pi(\ell-1)} = \frac{\lambda_0(\ell)\|\phi^\ell\|^2_\pi}{a\pi(\ell-1)} \le M < \infty,$$

and, as $\pi(k)$ is decreasing in k after some large value,

$$\sum_{k=0}^{\ell-2}(\phi^\ell(k+1) - \phi^\ell(k))^2 \frac{\pi(\ell-2)}{\pi(\ell-1)} \le \sum_{k=0}^{\ell-2}(\phi^\ell(k+1) - \phi^\ell(k))^2 \frac{\pi(k)}{\pi(\ell-1)} \le M$$

Since $\pi(\ell-2)/\pi(\ell-1) = (\ell-1)/\lambda \to \infty$ as $\ell \to \infty$, we have

$$\sum_{k=0}^{\ell-2}(\phi^\ell(k+1) - \phi^\ell(k))^2 \to 0, \qquad \text{as } \ell \to \infty.$$

So, uniformly for $k = 0, 1, \cdots, \ell - 2$,

$$|\phi^\ell(k+1) - \phi^\ell(k)| \to 0, \qquad \text{as } \ell \to \infty.$$

Also

$$\sum_{k=0}^{\ell-3}(\phi^\ell(k+1)-\phi^\ell(k))^2\frac{\pi(\ell-3)}{\pi(\ell-1)} \le \sum_{k=0}^{\ell-3}(\phi^\ell(k+1)-\phi^\ell(k))^2\frac{\pi(k)}{\pi(\ell-1)} \le M$$

and

$$\begin{aligned}
(\phi^\ell(\ell-2)-1)^2 &= (\phi^\ell(\ell-2)-\phi^\ell(0))^2 \\
&= \left[\sum_{k=0}^{\ell-3}(\phi^\ell(k+1)-\phi^\ell(k))\right]^2 \\
&\le (\ell-2)\sum_{k=0}^{\ell-3}(\phi^\ell(k+1)-\phi^\ell(k))^2 \text{ (Cauchy-Schwarz)} \\
&\le (\ell-2)M\frac{\pi(\ell-1)}{\pi(\ell-3)} \\
&= M\frac{\lambda^2}{\ell-1} \to 0, \qquad \text{as} \ell \to \infty.
\end{aligned}$$

Finally,

$$|\phi^\ell(\ell-1)-1| \le |\phi^\ell(\ell-1)-\phi^\ell(\ell-2)| + |\phi^\ell(\ell-2)-1| \to 0, \text{as } \ell \to \infty.$$

We conclude that

$$\lim_{\ell\to\infty}\max_{0\le k<\ell}|\phi^\ell(k)-1| = 0. \blacksquare$$

8 BIOGRAPHY

Kun Qian received his PhD in mathematics from the University of Ottawa, Ontario, Canada, in 1993. Since September 1993, he has been with Nortel Technology, Ottawa, Ontario, Canada, where he now is a senior engineer in the System Engineering Core. He has been working on ATM switching, network planning tools and simulation tools. In addition to these activities, his current research interests are in the area of performance analysis and modeling high-speed networks, in particular, congestion control, resource allocation, routing, and their impacts on network dimensioning and planning.