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A Study of the Fairness of the Fast Reservation Protocol*

Llorenç Cerdà, Jorge García and Olga Casals
Polytechnic University of Catalonia
Computer Architecture Department
c/ Gran Capitan, Modulo D6, E-08071 Barcelona, Spain
tel : + 34 3 4016798, fax : + 34 3 4017055,
e-mail : llorenc@ac.upc.es

Abstract

Fast Reservation Protocol (FRP) is a Traffic Control scheme intended to multiplex bursty data sources. In this paper we focus on the analysis of the FRP when different sources are multiplexed together in order to study the fairness of the protocol. We present two analytical models to analyse the case in which a set of identical sources is multiplexed with another one of higher rate. Analytical results are compared with simulation results.

1 INTRODUCTION

In order to efficiently multiplex data transfers and LAN-LAN interconnection on the ATM B-ISDN, an in-call bandwidth negotiation called Fast Reservation Protocol (FRP) has been proposed, Boyer (1992). FRP is a kind of Connection Acceptance Control at burst level, that is, when a source wants to transmit a burst it is accepted or blocked depending on the available bandwidth within the link. When a burst is blocked successive reattempts are made until it is accepted. Although the FRP it is not a new proposal, it is still a hot topic because recently it has been included in the ITU-T 371 recommendation to support the ATM Block Transfer Capability.

Performance of an FRP connection is therefore measured in terms of its Burst Blocking Probability (BP) and its Blocking Time (BT, i.e. the time that a blocked burst has to wait until it is eventually accepted). Performance studies of FRP and related protocols have been carried out by several authors, Boyer (1992), Enssle (1994), Suzuki (1992), Bernstein (1994). In those studies however, a set of identical sources is used to model the protocol behaviour. When sources with different parameters (PCR and/or burst duration) are multiplexed together, it is foreseeable that each source type will get a different BP and BT. In this paper we focus on the analysis of the FRP fairness when different sources are multiplexed. We use the term fairness in the sense of discrepancy between BP and

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BT values of different source types. Being all equal, the network would have a fair burst access.

We assume an ON-OFF model for the data sources with exponential ON and OFF time distribution (burst-silence model). In order to assess the burst blocking probability of the sources, two approximations of the protocol are considered. In the first approach we assume that the time between reattempts is zero. With different types of sources this case leads to a Markov chain that does not have a product form solution, so we analyze the simple situation in which a set of identical sources are multiplexed with another source of a higher rate.

In a second approach we consider that the reattempt time and OFF time are identically distributed. This assumption leads to a Markov chain with a simple product form solution even when considering different source types.

In the first approach the time that a burst has to wait when it is blocked until it is accepted is also evaluated. Analytical results are compared with simulation results.

2 OVERVIEW OF THE FRP PROTOCOL

The FRP is described in Boyer (1992). Two variants of the protocol have been proposed. The first, called FRP with Delayed Transmission (FRP/DT), is intended to multiplex the so called Stepwise Variable Bit Rate Sources. These sources are expected to have a stepwise need of bandwidth. However there is a restriction on the sources which must tolerate a delay in the negotiation of an increase of bandwidth. Many data communications are typical examples of such sources.

Basically the FRP/DT works as follows. When a source wants an increase of bandwidth (for example, when it wants to transfer a burst), it sends a Request to the so called FRP Control Unit, situated at the ingress node. This Request is forwarded to the first switching element of the link, which checks whether it can allocate the increase of bandwidth or not. If it has enough bandwidth, the Request is forwarded to the next switching element and so on until it reaches the egress node. Eventually the egress node will send an acknowledgment back to the FRP Unit and the source will be allowed to transfer the burst. The time passed from the FRP Unit sending the request until receiving the acknowledgment is called the Round Trip Time. Note that during this time the switching elements have allocated bandwidth for the source, but the transmission has not started yet. Therefore this time is an overhead introduced by the protocol.

If a switching element is not able to allocate the requested increase of bandwidth, it discards the Request, and by a time-out mechanism the allocated resources are reset to their previous state. In this case the FRP Unit makes successive reattempts until the increase of bandwidth is accepted. The source indicates the FRP Unit when an accepted burst is already transferred in order to release the allocated bandwidth.

The other variant, called FRP with immediate transmission (FRT/IT), is intended for sources more sensitive to a time delay. In this case the source transfers the burst immediately after the reservation request. If the reservation fails in any of the nodes, the whole burst is discarded.

3 MODEL DESCRIPTION AND ANALYSIS

In our analysis we consider an isolated node. We assume an ON-OFF model for the data sources with exponential ON and OFF time distribution (burst-silence model). The parameters of the sources are the bitrate within a burst period Λ; the mean burst duration t^{on} and the mean silence duration t^{off}.

In the model N_l identical sources (we will refer to them as *ltype* sources) with parameters Λ_l, t_l^{on} and t_l^{off}, are multiplexed with another different source (we will refer to it as *htype* source) with parameters Λ_h, t_h^{on} and t_h^{off}.

Being all time intervals exponentially distributed, the activation rate α of a source is given by $\alpha = 1/t^{off}$. Let the service time be the time that a node allocates bandwidth for a non blocked source. Clearly, for the FRP/IT the mean service time is the mean burst duration t^{on}. For the FRP/DT a non blocked source has to wait a deterministic time equal to the round trip time t_{rt} before transferring a burst, so the mean service time is given by $t^{on} + t_{rt}$. However, in this paper we do not study the influence of the round trip time time, so we will assume it to be zero. Therefore the service rate μ of a source is given by $\mu = 1/t^{on}$. Assuming $t_{rt} = 0$ our model makes no distinction between the FRP/DT and FRP/IT. Refer to Enssle (1994) for a contrast of both variants of the protocol.

3.1 Approximation by zero time between reattempts

In this approximation we suppose that when a burst is blocked, the time between the successive requests that are made until the burst is accepted is zero. This is equivalent to considering a blocked burst being kept in a queue until there is enough bandwidth left by the other sources in the link.

Let K_1 be the maximum number of ltype sources that can be simultaneously transferring a burst without exceeding the link capacity, when the htype source is also transferring a burst. Let K_2 be the same, but when the htype source is silent or blocked. Let us further suppose that the htype source transmits at a higher rate than the ltype source such that $K_2 > K1 + 1$. In this case when an ltype and htype sources are blocked, the ltype source will be accepted first (i.e. the htype source does not see a FIFO queue). Clearly, if the link capacity is C

$$K_1 = \left\lfloor \frac{C - \Lambda_h}{\Lambda_l} \right\rfloor \tag{1}$$

$$K_2 = \left\lfloor \frac{C}{\Lambda_l} \right\rfloor \tag{2}$$

With these assumptions an isolated node can be described by the Markov chain of figure 1 with state space $\{(i,j) \ : \ i = 0,1,2 \ ; \ 0 \leq j \leq N_l\}$, where j is the number of ltype active sources (transferring or blocked) while the htype source is silent ($i = 0$), transferring a burst ($i = 1$) or blocked ($i = 2$). This Markov chain does not have a product form solution for the stationary probabilities π_{ij}, so they have to be calculated numerically solving the global balance equations.

The ltype and htype source blocking probability (P_l and P_h) can be obtained from the

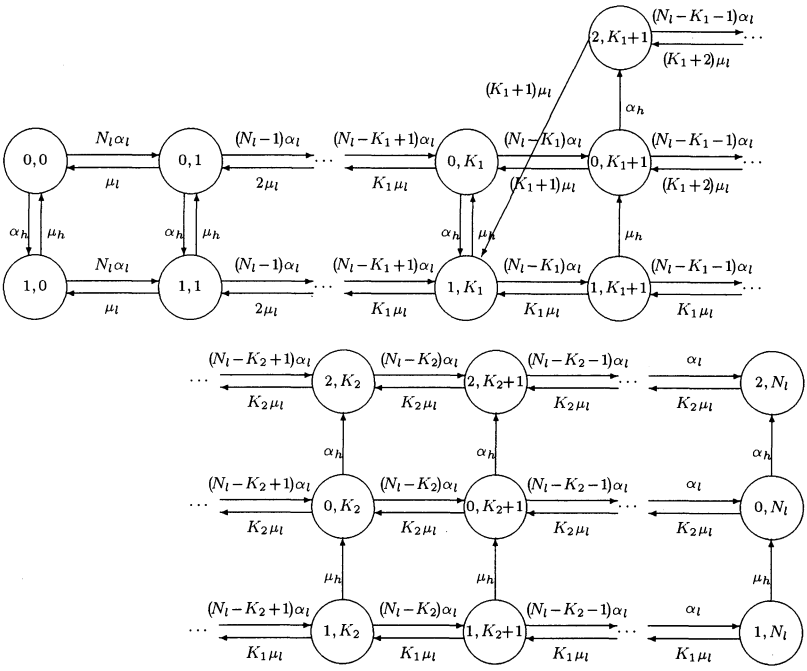

Figure 1 State-transition diagram assuming zero time between reattempts

stationary probabilities π_{ij}. The blocking probability is given by the probability that an arriving burst is blocked, divided by the probability of a burst arrival. Thus

$$P_h = \frac{\sum_{j=K_1+1}^{N_l} \pi_{0j}}{\sum_{j=0}^{N_l} \pi_{0j}} \tag{3}$$

$$P_l = \frac{\sum_{j=K_2}^{N_l-1} (N_l - j)\pi_{0j} + \sum_{j=K_1}^{N_l-1} (N_l - j)\pi_{1j} + \sum_{j=K_2}^{N_l-1} (N_l - j)\pi_{2j}}{\sum_{j=0}^{N_l-1} (N_l - j)\pi_{0j} + \sum_{j=0}^{N_l-1} (N_l - j)\pi_{1j} + \sum_{j=K_1+1}^{N_l-1} (N_l - j)\pi_{2j}} \tag{4}$$

3.2 Approximation by identically reattempt and OFF time distribution

In this approximation we assume that when a burst is blocked, the time between the successive requests that are made until the burst is accepted is exponentially distributed with a mean equal to the OFF time distribution, i.e. we assume an identically reattempt and OFF time distribution. This is equivalent to considering that a blocked burst is lost.

Let K_1 and K_2 be the same as in the previous section. Because a blocked burst can be considered as lost, with this approach an isolated node can be described by the Markov chain with state space $\{(i,j) \,:\, i=0,1\,;\, 0 \le j \le K_2\}$ of figure 2, where j is the number of ltype sources transferring a burst while the htype source is silent ($i=0$) or transferring a burst ($i=1$). The stationary probabilities π_{ij} of the Markov chain has a straightforward product form solution given by

$$\pi_{ij} = \frac{1}{G}\binom{N_l}{j}\rho_h^i\,\rho_l^j \tag{5}$$

where G is the normalization constant, $\rho_l = \mu_l/\alpha_l$ and $\rho_h = \mu_h/\alpha_h$. We note that considering more than one htype source or even considering more than two types of sources, a product form solution would still apply.

In this model we make no distinction between a burst or a reattempt arrival. So we calculate the blocking probability as the probability that a burst or a reattempt arrival is blocked, divided by the probability of a burst or a reattempt arrival. Such blocking probability for the ltype and htype sources (P_l and P_h) is given by

$$P_h = \frac{\sum\limits_{j=K_1+1}^{K_2} \pi_{0j}}{\sum\limits_{j=0}^{K_2} \pi_{0j}} \tag{6}$$

$$P_l = \frac{(N_l-K_2)\pi_{0K_2} + (N_l-K_1)\pi_{1K_1}}{\sum\limits_{j=0}^{K_2}(N_l-j)\pi_{0j} + \sum\limits_{j=0}^{K_1}(N_l-j)\pi_{1j}} \tag{7}$$

Note that in the previous section we do not count the reattempts to calculate the blocking probability (considering a zero time between reattempts implies considering ∞ reattempts after a blocked burst). If the reattempt time is not zero, the following relation applies for the P_l^{init} and P_l^{total} blocking probabilities of an ltype source, calculated counting and not counting the reattempts respectively. Let $\bar{r}_l$ be the mean number of reattempts that a blocked burst of an ltype source do until it is accepted. It can be derived that

$$P_l^{init} = \frac{P_l^{total}}{\bar{r}_l\,(1-P_l^{total})} \tag{8}$$

Obviously, an analogous relation holds for the htype source. If the blocking probability is small and the reattempt time is high enough such $\bar{r} \approx 1$ (i.e. a blocked burst is almost always accepted at the first reattempt), $P^{init} \approx P^{total}$. These conditions are foreseeable in the approximation by identically reattempt and OFF time distribution. So, this approximation can be used to asses P_h^{init} and P_l^{init} from the probabilities calculated with equations 6 and 7.

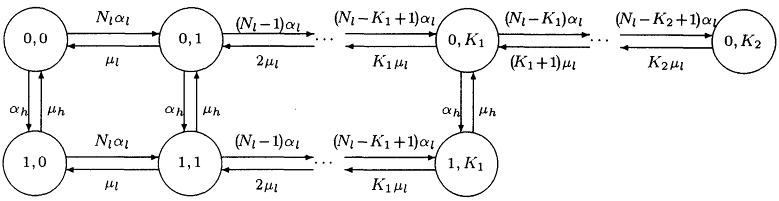

Figure 2 State-transition diagram assuming identically reattempt and OFF time distribution

3.3 Blocking time in the approximation by zero time between reattempts

In this section we calculate the time that an arriving burst that is blocked has to wait until it is eventually accepted (we refer to it as blocking time). We calculate this time assuming the approximation by zero time between reattempts, so the referred states are those of figure 1. We do not use the approximation by identically reattempt and OFF time distribution to assess the blocking time, because in general it would be inaccurate.

Let T_h and T_l be the blocking time of an htype source and ltype source respectively. Let $B_{ij} = (i, j)$ be entering state resulting from the blocking transition. Clearly

$$P(T_h \le x) = \sum_{j=K_1+1}^{N_l} P(T_h \le x|B_{2j})\, P(B_{2j})\,, \tag{9}$$

$$P(B_{2j}) = \frac{\pi_{0j}}{\sum\limits_{k=K_1+1}^{N_l} \pi_{0k}} \tag{10}$$

and

$$P(T_l \le x) = \sum_{\forall B_{ij}} P(T_l \le x|B_{ij})\, P(B_{ij})\,, \tag{11}$$

$$P(B_{ij}) = \frac{(N_l - j + 1)\pi_{ij-1}}{\sum\limits_{k=K_2}^{N_l-1} (N_l - k)\pi_{0k} + \sum\limits_{k=K_1}^{N_l-1} (N_l - k)\pi_{1k} + \sum\limits_{k=K_2}^{N_l-1} (N_l - k)\pi_{2k}} \tag{12}$$

$P(T_h \le x|B_{ij})$ is the distribution of the time that a blocked burst of an htype source has to wait until it is accepted, when the entering state in the blocking transition is B_{ij}. $P(T_l \le x|B_{2j})$ is the same for an ltype source. Formulas for this probabilities are derived in appendixes 1 and 2.

4 RESULTS

In this section we present a numerical study of the FRP fairness using the models described above. We evaluate the fairness of the protocol in terms of the burst blocking probability and the mean blocking time. Blocking time is specially important using the FRP/IT scheme in which the sources are supposed to be time sensitive. We also compare analytical and simulation results.

Figures 3 and 4 (model parameters are summarized in Table 2) plot the blocking probability and the mean blocking time of the two source types considered, when the htype source varies the mean burst duration (i.e. the mean ON time t_h^{on}) †. Varying t_h^{on} from 0 (the source is always silent) to ∞ (the source is always active), blocking probability of the ltype sources will increase between the one obtained when sharing a link of capacity varying from C to $C - \Lambda_h$. Figure 3 shows that the blocking probability of the ltype sources increases within these limits, while the blocking probability of the htype source remains constant. The blocking probability is assessed using the approximation by zero time between reattempts (section 3.1) ‡, and the approximation by identical reattempt and OFF time distribution (section 3.2).

Figures 5 and 6 plot the blocking probability and the mean blocking time of the two source types, when the htype source varies the bitrate within a burst period. Each time that the htype source bitrate reaches a multiple of the ltype source bitrate, there is a decrement on the maximum number of sources that can be simultaneously transferring a burst. This causes an increasing step on the blocking probability and the blocking time.

Table 1 compares analytical and simulation results (given with 95% confidence intervals). To calculate the blocking probabilities in the simulation, the reattempts have been not counted in order to compare with the approximation by zero time between reattempts (these probabilities are referred to as "init." in the table), and have been counted to compare with the approximation by identically reattempt and OFF time dist. (referred to as "tot." in the table, cfr. section 3.2). Increasing the reattempt time decreases the blocking probability. So, the first approximation can be considered as an upper bound for the "init." probabilities, and, for a reattempt time lower than the mean OFF time, the second approximation can be considered as a lower bound for the "tot." probabilities.

A deterministic and an exponentially distributed reattempt time has been considered in the simulation. It can be seen that the exponentially distributed approximation for the reattempt time gives accurate results for the blocking probabilities, but the blocking time. Simulation results show that the mean blocking time increases rapidly with increasing the reattempt time.

5 CONCLUSIONS

We have analyzed the behaviour of the FRP when different source types are multiplexed together. We have considered the case in which a set of identical sources is multiplexed with another one of higher bitrate. To assess the blocking probability we have considered

† We note that the burstiness, defined as $b = (t^{on} + t^{off})/t^{on}$ is a decreasing function with increasing t^{on}.

‡ To calculate the stationary probabilities using this approximation, we have solved the global balance equations using a Gaussian elimination method

Table 1 Comparison of analytical and simulation results

	Analytical		Simulation					
	Zero time between reatt.	Id. reatt. and OFF time dist.	Reattempt time					
			5 ms		20 ms		50 ms	
			Exp. dist.	Det.	Exp. dist.	Det.	Exp. dist.	Det.
P_h init.	$7.63\ 10^{-3}$		$7.00\ 10^{-3}$ $\pm 2.83\ 10^{-4}$	$6.86\ 10^{-3}$ $\pm 4.30\ 10^{-4}$	$7.31\ 10^{-3}$ $\pm 5.33\ 10^{-4}$	$7.28\ 10^{-3}$ $\pm 6.46\ 10^{-4}$	$7.00\ 10^{-3}$ $\pm 2.49\ 10^{-4}$	$7.16\ 10^{-3}$ $\pm 8.72\ 10^{-4}$
P_l init.	$8.22\ 10^{-4}$		$5.76\ 10^{-4}$ $\pm 1.19\ 10^{-5}$	$5.92\ 10^{-4}$ $\pm 1.78\ 10^{-5}$	$5.13\ 10^{-4}$ $\pm 4.89\ 10^{-5}$	$4.98\ 10^{-4}$ $\pm 5.28\ 10^{-5}$	$4.28\ 10^{-4}$ $\pm 1.10\ 10^{-5}$	$4.31\ 10^{-4}$ $\pm 4.37\ 10^{-5}$
P_h tot.		$7.57\ 10^{-3}$	$29.7\ 10^{-3}$ $\pm 1.65\ 10^{-3}$	$25.6\ 10^{-3}$ $\pm 1.75\ 10^{-3}$	$14.4\ 10^{-3}$ $\pm 1.33\ 10^{-3}$	$12.1\ 10^{-3}$ $\pm 1.34\ 10^{-3}$	$10.1\ 10^{-3}$ $\pm 0.35\ 10^{-3}$	$8.52\ 10^{-3}$ $\pm 1.12\ 10^{-3}$
P_l tot.		$4.15\ 10^{-4}$	$13.9\ 10^{-4}$ $\pm 2.32\ 10^{-5}$	$12.4\ 10^{-4}$ $\pm 4.29\ 10^{-5}$	$6.80\ 10^{-4}$ $\pm 6.36\ 10^{-5}$	$5.90\ 10^{-4}$ $\pm 6.57\ 10^{-5}$	$4.90\ 10^{-4}$ $\pm 1.09\ 10^{-5}$	$4.47\ 10^{-4}$ $\pm 4.63\ 10^{-5}$
T_h (ms)	15.24		22.0 ± 0.32	19.13 ± 0.48	40.8 ± 0.95	33.6 ± 0.88	73.2 ± 1.23	59.9 ± 0.92
T_l (ms)	6.822		12.3 ± 0.07	10.51 ± 0.16	28.0 ± 0.24	23.7 ± 0.22	57.5 ± 0.41	51.8 ± 0.16

two approximations. In the first one we assume that the reattempt time is zero and in the second one we assume that it is identically distributed to the OFF time. To calculate the stationary state probabilities with the first approach the balanced global equations have to be solved, while in the second approach they have a simple product form solution. We have also calculated the mean blocking time assuming the first approach.

The numerical study shows that there are not big differences between the blocking probabilities obtained with both approximations. The approximation of identical reattempt and OFF time distribution gives a much simple way to compute the blocking probabilities and can be easily extended to more than one htype source or even more than two types of sources.

The results also show that when multiplexing different type of sources, blocking probability and blocking time depend on the source parameters. This can be interpreted as a lack of fairness, in the sense that they will have a different burst access. It is actually seen that an increase on the bitrate or the mean burst duration of a connection can result in a considerably increase of the blocking probability and blocking time of the other connections.

Recently the ATM Block Transfer Capability (ABT) with two variants ABT/DT and ABT/IT based on the FRP/DT and FRP/IT respectively have been defined, ITU (1995). In this recommendation a Block level QoS commitment is defined in which a reservation request should be accepted by the network within finite time limits (ABT/DT), or with a specified block discard probability (ABT/IT), as long as blocks of the connection are conforming to the specified Sustainable Cell Rate. These QoS parameters can be easily derived form the blocking probability and blocking time parameters we have measured (we note that such ITU recommendation is subsequent to the study carried out in this paper).

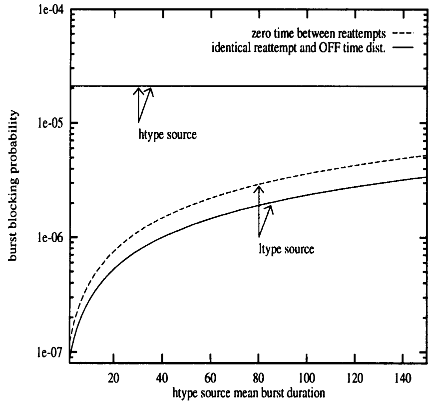

Figure 3 Influence of the htype source mean burst duration on the blocking probability

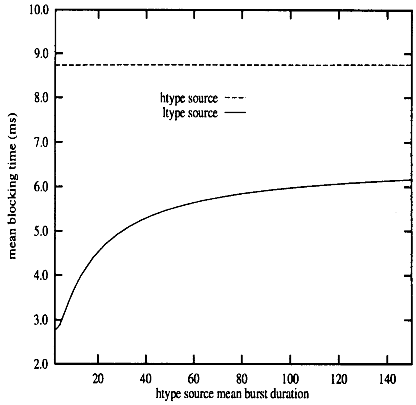

Figure 4 Influence of the htype source mean burst duration on the mean blocking time

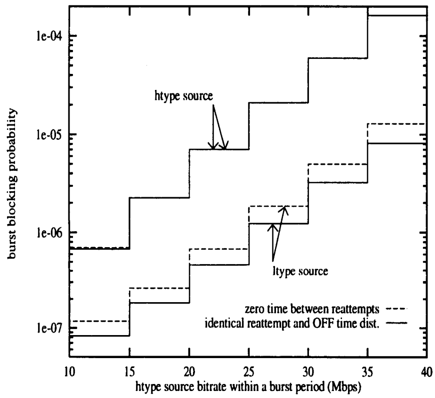

Figure 5 Influence of the htype source bitrate within a burst period on the blocking probability

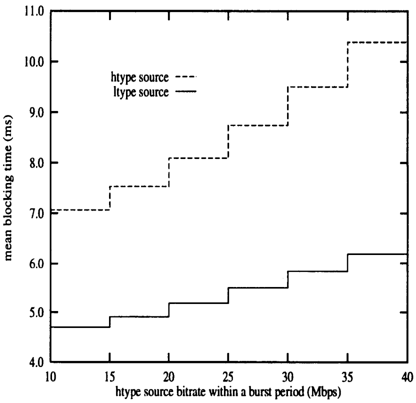

Figure 6 Influence of the htype source bitrate within a burst period on the mean blocking time

Table 2 Source parameters

link	htype source				ltype source				
capacity 150 Mbps	bitrate (Mbps)	t_h^{on} (ms)	t_h^{off} (ms)	burst-iness	bitrate (Mbps)	t_l^{on} (ms)	t_l^{off} (ms)	burst-iness	num. of sources
fig. 3 & 4	30	0~150	1400	$\infty \sim 10.3$	5	100	1400	15	150
fig. 5 & 6	10~40	50	1400	29	5	100	1400	15	150
table 1	30	50	900	19	5	100	900	10	150

APPENDIX 1

In this appendix we derive $P(T_h \leq x|B_{2j})$, $j \in \{K_1+1,\ldots,N_l\}$ of expression 9. If an htype and ltype sources are blocked, the ltype source will be accepted first (i.e. the htype source does not see a FIFO queue), so $P(T_h \leq x|B_{2j})$ is the distribution of the first passage time from the blocking state B_{2j} to the non blocking state $(1, K_1)$. To calculate this probability we follow the method described in Neuts (1989). We are only concerned about the states $(1,K_1),(2,K_1+1),\ldots,(2,N_l)$ so to simplify the notation we will refer to them as E_j, $j = K_1, K_1+1,\ldots,N_l$. We define the following events

$T(j, j-r)$ = first passage time from state E_j to state E_{j-r}
$V(j, j-r)$ = number of transitions involved in $T(j, j-r)$

and their joint probability $G_j^{(r)}(x,k) = P\{T(j,j-r) \leq x, V(j,j-r) = k\}$. The probability we are looking for is given by

$$P(T_h \leq x|B_{2j}) = \sum_{k=1}^{\infty} G_j^{(j-K_1)}(x,k) \tag{13}$$

We now compute $G_j^{(r)}(x,k)$. To simplify the notation, in case of one state transition we will write $G_j(x,k) = G_j^{(1)}(x,k)$. Let q_j^+ be the transition rate from the state E_j to the state E_{j+1}; q_j^- the transition rate from the state E_j to the state E_{j-1}; and q_j the self state transition rate (cfr. figure 1).

$$\begin{aligned} q_j^+ &= (N_l - j)\,\alpha_l \\ q_j^- &= \begin{cases} j\,\mu_l\,, & j \leq K_2 \\ K_2\,\mu_l\,, & j > K_2 \end{cases} \\ q_j &= q_j^+ + q_j^- \end{aligned}$$

We define the one state forward and backward transition probability $A_j^-(x) = P\{T(j,j-1) \leq x, V(j,j-1) = 1\}$ and $A_j^+(x) = P\{T(j,j+1) \leq x, V(j,j+1) = 1\}$. We have

$$\begin{aligned} A_j^-(x) &= (1 - e^{-q_j x})\frac{q_j^-}{q_j}\,,\ j = K_1+1,\ldots,N_l \\ A_j^+(x) &= (1 - e^{-q_j x})\frac{q_j^+}{q_j}\,,\ j = K_1,\ldots,N_l - 1 \end{aligned} \tag{14}$$

yielding

$$G_j(x,k) = \begin{cases} A_j^-(x)\,, & k = 1 \\ 0\,, & k = 2n \\ \sum_{l=1}^{n} A_j^+(\cdot) * G_{j+1}(\cdot, 2(n-l)+1) * G_j(x, 2l-1)\,, & k = 2n+1 \end{cases} \tag{15}$$

and

$$G_j^{(r)}(x,k) = \sum_{k_1+\cdots+k_r=k} G_j(\cdot,k_1) * G_{j-1}(\cdot,k_1) * \cdots * G_{j-r+1}(x,k_r) \tag{16}$$

where $*$ is the convolution of the distribution functions (i.e. $F_1(\cdot) * F_2(x) = \int_{-\infty}^{\infty} F_1(x-\lambda)\, dF_2(\lambda)$)
From 15 we derive the following recursive equation for the joint transform $\tilde{G}_j(s,z) = \sum_{k=0}^{\infty} \int_0^{\infty} e^{-sx} z^k\, dG_j(x,k)$

$$\tilde{G}_j(s,z) = \frac{z\, A_j^-(s)}{1 - z\, A_j^+(s)\, \tilde{G}_{j+1}(s,z)} \tag{17}$$

and

$$\tilde{G}_j^{(r)}(s,z) = \sum_{k=0}^{\infty} z^k \sum_{k_1+\cdots+k_r=k} G_j(s,k_1)\, G_{j-1}(s,k_1) \cdots G_{j-r+1}(s,k_r) = \tilde{G}_j(s,z)\, \tilde{G}_{j-1}(s,z) \cdots \tilde{G}_{j-r+1}(s,z) \tag{18}$$

where $A_j^-(s)$, $A_j^+(s)$ and $G_j(s,k)$ are the Laplace-Stieltjes transform of the distribution functions (i.e. $F(s) = \int_0^{\infty} e^{-sx}\, dF(x)$). From 14 we obtain

$$\begin{aligned} A_j^-(s) &= \frac{q_j^-}{s+q_j},\ j = K_1+1,\ldots,N_l \\ A_j^+(s) &= \frac{q_j^+}{s+q_j},\ j = K_1,\ldots,N_l-1 \end{aligned} \tag{19}$$

Substitution into 17 yields

$$\tilde{G}_j(s,z) = \frac{z\, q_j^-}{s + q_j^+ - z\, q_j^- \tilde{G}_{j+1}(s,z)},\ j = K_1+1,\ldots,N_l-1 \tag{20}$$

$$\tilde{G}_{N_l}(s,z) = z\, \frac{q_{N_l}^-}{s + q_{N_l}^-} \tag{21}$$

Substituing recursively 21 into 20, and then into 18 we obtain $\tilde{G}_j^{(r)}(s,z)$, $j = K_1 + 1,\ldots,N_l$. Finally, from 13 we see that $G_j^{(j-K_1)}(s,z)_{z=1}$ is the Laplace-Stieltjes transform of $P(T_h \le x | B_{2j})$. Inverting it and substituting into 9 we obtain the distribution of the blocking time T_h. This is rather arduous, but from the previous equations we can derive a straightforward formula for the mean blocking time $\bar{T}_h$.

Let us define $\bar{T}_j^{(j-K_1)} = \mathrm{E}[x|B_j]$, i.e. the mean first passage time from the state E_j to the state E_{k1}. We also define $\bar{T}_j$ as the mean first passage time from the state E_j to the state E_{j-1}. Clearly $\bar{T}_j^{(j-K_1)} = -\frac{\partial}{\partial s}\tilde{G}_j^{(j-K_1)}(s,z)\Big|_{s=0,\,z=1}$, $\bar{T}_j = -\frac{\partial}{\partial s}\tilde{G}_j(s,z)\Big|_{s=0,\,z=1}$. From 20, 21 and 18, and since $\tilde{G}_j(s,z)_{s=0,z=1} = 1$ we obtain

$$\bar{T}_j = \frac{1 + q_j^+ \bar{T}_{j+1}}{q_j^-} \tag{22}$$

$$\bar{T}_{N_l} = \frac{1}{q_{N_l}^-} \tag{23}$$

$$\bar{T}_j^{(j-K_1)} = \sum_{k=K_1+1}^{j} \bar{T}_k \tag{24}$$

Substituing recursively 23 into 22, and then into 24 we compute $\bar{T}_j^{(j-K_1)}$ and finally from 9 we obtain the mean blocking time

$$\bar{T}_h = \sum_{j=K_1+1}^{N_l} \bar{T}_j^{(j-K_1)} P(B_{2j}) \tag{25}$$

APPENDIX 2

In this appendix we derive the $P(T_l \leq x|B_{ij})$ of the expression 11. To calculate this probability we consider the following cases:

1. $B_{ij} \in \{(2, K_2+1), \ldots, (2, N_l), (0, K_2+1), \ldots, (0, N_l)\}$

In this case the ltype source is blocked while the htype source is silent or blocked. Being $B_{ij} = (i,j)$ the state resulting from the blocking transition, the source will find $j - K_2 - 1$ ltype sources already blocked and it will have to wait until $j - K_2$ ltype sources are served (we say that a source is served when one of the K_2 bursts being transferred ends). Let $S_l^{(n)}$ be the service time of n ltype sources and $F_{S_l}^{(n)}(x)$ its distribution. Clearly $F_{S_l}^{(1)}(x) = 1 - e^{-K_2\mu_l x}$ and $F_{S_l}^{(n)}(x) = F_{S_l}^{(1)}(\cdot) * \overset{n}{\ldots} * F_{S_l}^{(1)}(x)$. Let $F_{T_l|B_{ij}}(s)$ be the Laplace-Stieltjes transform of $P(T_l \leq x|B_{ij})$. We have

$$F_{T_l|B_{ij}}(s) = F_{S_l}^{(j-K_2)}(s) = \frac{(K_2\mu_l)^{j-K_2}}{(s + K_2\mu_l)^{j-K_2}} \tag{26}$$

2. $B_{ij} \in \{(1, K_1+1), \ldots, (1, K_2)\}$

In this case the ltype source is blocked while the htype source is transferring a burst, but the maximun of active ltype sources is K_2. Being $B_{ij} = (i,j)$ the state resulting from the blocking transition, the source will find $j - K_1 - 1$ ltype sources already blocked. So to be accepted it will have to wait until the htype source is served or until $j - K_1$ ltype sources are served. Let S_h be the service time of the htype source and $S_l^{(n)}$ the service time of n ltype sources. Let $F_{S_h}(x)$ and $F_{S_l}^{(n)}(x)$ be their distribution. Clearly $F_{S_h}(x) = 1 - e^{-\mu_h x}$ and $F_{S_l}^{(n)}(x)$ is the same as in the previous case, but changing K_2 for K_1. We have

$$\begin{aligned} &P(T_l \leq x|B_{ij}) = 1 - P(S_h > x)\, P(S_l^{(j-K_1)} > x) = \\ &1 - (1 - F_{S_h}(x))(1 - F_{S_l}^{(j-K_1)}(x)) = 1 - (1 - F_{S_l}^{(j-K_1)}(x))\, e^{-\mu_h x} \end{aligned} \tag{27}$$

the Laplace-Stieltjes of the previous equation is

$$F_{T_l|B_{ij}}(s) = s\int_0^\infty \left(e^{-sx} - (1 - F_{S_l}^{(j-K_1)}(x))e^{-(s+\mu_l)x}\right) dx =$$
$$1 - \frac{s}{s+\mu_h}\left[1 - \left(\frac{K_1\mu_l}{s+\mu_h+K_1\mu_l}\right)^{j-K_1}\right] \tag{28}$$

3. $B_{ij} \in \{(1, K_2+1), \ldots, (1, N_l)\}$

In this case the ltype source is blocked while the htype source is transferring a burst, but there are more than K_2 active ltype sources. So although the htype source is served, the ltype source can still remain blocked. Let S_h be the service time of the htype source and $S_l^{(n)}$ the service time of n ltype sources while the htype is being served. Let us consider the density of the blocking time. For convenience of notation we define $P_{ij}(T_l = x) = P(T_l = x|B_{ij})$. Clearly

$$P_{ij}(T_l = x) = P_{ij}(T_l = x,\ S_h < S_l^{(1)}) +$$
$$\sum_{k=1}^{j-K_2-1} P_{ij}(T_l = x,\ S_l^{(k)} < S_h < S_l^{(k+1)}) + P_{ij}(T_l = x,\ S_h > S_l^{(j-K_2)}) \tag{29}$$

After some computation, the Laplace transform of the previous expression yields:

$$F_{T_l|B_{ij}}(s) = \sum_{k=0}^{j-K_2-1} \mu_h \left(\frac{K_2\mu_l}{s+K_2\mu_l}\right)^{j-k-K_2} \frac{(K_1\mu_l)^k}{(s+\mu_h+K_1\mu_l)^{k+1}} +$$
$$\left[1 - \frac{s}{s+\mu_h}\left[1 - \left(\frac{K_1\mu_l}{s+\mu_h+K_1\mu_l}\right)^{K_2-K_1}\right]\right]\left(\frac{K_1\mu_l}{s+\mu_h+K_1\mu_l}\right)^{j-K_2} \tag{30}$$

Inversion of 26, 28 and 30, and substitution into 11 yields the distribution of the blocking time T_l. Differentiating these equations we calculate the mean blocking time

$$\bar{T}_l = \sum_{\forall B_{ij}} -\frac{d}{ds} F_{T_l|B_{ij}}(s)\Big|_{s=0} P(B_{ij}) \tag{31}$$

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BIOGRAPHY

Llorenç Cerdà graduated in Telecommunications Engineering from Polytechnic University of Catalonia (UPC) in 1993. He joined UPC in 1994 and currently he is Assistant Professor and a PhD student in the Computer Architecture Department.

Jorge García graduated and received his PhD in Telecommunications Engineering from UPC in 1988 and 1992, respectively. He joined UPC in 1988 and currently is Associate Professor of the Computer Architecture department. In 1992-93 he was Visiting Scientist at the Systems and Industrial Department of the University of Arizona with a NATO fellowship. He has been involved in several RACE projects and currently he is involved in COST-242 project.

Olga Casals graduated and received her PhD from UPC in 1983 and 1986 respectively, both in Telecommunications Engineering. She joined UPC in 1983 where she became Full Professor in 1994 and she is head of a research group on traffic in B-ISDN communications systems. She has been working on ATM networks since 1988 with her participation in the RACE project R1022. She has been also involved in the RACE projects EXPLOIT and BAF and currently she is involved in ACTS project EXPERT and in COST-242.