The impact of the reactive functions on the LAN interconnection by a Frame-Relay Net

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Abstract

This article proposes the use of a diffusion approximation to model the effect of preventive and reactive functions implemented in a Frame-Relay network interconnecting LAN networks. These functions are based on two mechanisms: discard eligibility (DE) and explicit congestion notification (ECN). Diffusion models applied in this paper allow us to define how the input stream issued by LANs is changed as a result of the DE mechanism and to estimate the losses due to queue overflow when either a push-out or a threshold policy is used to manage the queue. The transient solution to the diffusion equation serves to estimate the dynamics of the evolution of queues during the changes of input flow issued by LANs and moderated by the ECN mechanism. It is the basis of a closed-loop, control theory model describing the influence of the ECN mechanism on FR network performance.

Keywords

Performance evaluation, Frame-Relay network, diffusion approximation.

1 INTRODUCTION

Frame Relay (FR) is a new fast packet technique that has been defined as a packet mode bearer service for the ISDN (CCITT,1991). It is widely seen as a solution for LAN-to-LAN interconnection. FR promises to combine circuit and packet switching and to achieve better response times than existing packet-switch networks can provide. It has been conceived to satisfy the requirements of emerging high speed data applications, to minimize transit delay or maximize throughput.

Like most high speed networks, FR networks require effective congestion control mechanisms to cope with unanticipated network component failures and overloads. Consequently, effective and efficient alternative congestion controls are particularly important in the architectural design of ISDN FR networks. In fast packet networks, alternative techniques based on congestion notification have been proposed to operate at the source level. These techniques use *Discard Eligibility* (DE) and *Explicit Congestion Notification* (ECN) mechanisms. They include the preventive and reactive control mechanisms.

First, each time permission is asked by a source to send additional traffic (a stream of frames), *Connection Admission Control* (CAC) procedures estimate whether this additional traffic will not deteriorate the quality of service, measured in frame loss probability due to overflow of buffers in transmission nodes. If the loss probability of frames does not overpass a predefined level, the new traffic is allowed but the agreed parameters are monitored constantly. The admission control mechanism makes use of four parameters:

- T_c – *Committed Rate Measurement Interval*, the reference interval for B_c and B_e which are defined below,
- B_c – *Committed Burst Size*: maximum amount of data which can be sent with high priority during the interval T_c ,
- B_e – *Excess Burst Size*: maximum amount of data which can be sent in addition to B_c with low priority during the interval T_c ,
- CIR – *Committed Information Rate*: average transmission rate guaranteed, $CIR = \frac{B_c}{T_c}$. T_c is the basic interval of control performed at the *interface* between LAN and FR networks: as long as the volume of traffic received in the interval T_c remains below the level B_c , all frames are marked as “high priority” (DE bit set to 0). When the threshold B_c is reached, frames are marked as “low priority” (DE bit set to 1). All frames received in excess of $B_c + B_e$ are discarded. Frames with low priority are first to be discarded in FR nodes when the danger of congestion arises.

Apart from the *preventive functions* sketched above, *reactive functions* watch if the current situation in the network does not indicate congestion and, if necessary, inform the source that it should limit its activity. It is done with the use of ECN messages which are sent forward by network nodes (FECN bit in frames is set to 1) and/or sent backward (BECN bit is set and a special control packet is sent back to the source) to notify the receiver and/or the sender that the node is entering a rejection rate (e.g. the loss of frames exceeds a certain threshold or the queue length is greater than a determined value). According to CCITT recommendations (CCITT,1991) this function is optional and the details are not specified. The source will subsequently reduce its traffic input rate to the network. If control packets are not received during a certain time interval, then the source can start to increase its traffic rate.

In this paper, we consider a model of LAN interconnection through a Frame Relay network. The model is shown in Figure 1; Figure 2 represents the queueing model. The

Figure 1 Model of LANs interconnected by a FR network.

Figure 2 Queueing model of FR network.

network nodes can perform both access and transit functions and are interconnected with transmission links. The interface between the LAN and FR network is used to perform the CAC procedures and to mark the frames with two priority levels with the use of the DE bit. The network nodes have a single finite buffer shared by all the connections using that link. The buffers are managed according to FIFO and, in the case of congestion, they discard low priority the frames.

To date, most studies of the impact of control functions on the performance of a network have used to simulation, (Castelli,1993, Fratta,1993). In this paper, we propose an analytical queueing model based, a on diffusion approximation. We assume that a FR node sends a ECN signal to the source. This feedback signal is a request to lower the LAN activity and it is transmitted to the LAN with a certain delay.

The article is organised as follows. Section 2 presents a model of the admission control performed by the interface and studies the influence of this procedure on the parameters of the stream of high and low priority frames. Section 3 introduces the model of a FR transmission node with push-out and threshold policies. Section 4 presents a transient analysis of the behaviour of a FR node. With these results, we are able to propose in Section 5 a closed-loop model including the ECN mechanism. Section 6 presents our conclusion.

Diffusion approximation which is used here developes models proposed in (Gelenbe,75,76): the solution of diffusion equation

$$\frac{\partial f(x, t; x_0)}{\partial t} = \frac{\alpha}{2} \frac{\partial^2 f(x, t; x_0)}{\partial x^2} - \beta \frac{\partial f(x, t; x_0)}{\partial x} \quad (1)$$

with properly chosen boundary conditions (they have the form of instantaneous return processes) and parameters α, β yields the density function $f(x)$ which approximates the queue distribution: $f(n, t; n_0) \approx p(n, t; n_0)$, in steady state $f(n) \approx p(n)$.

2 INTERFACE NODE MODEL

In our model, the input streams from LANs are characterized by two parameters, namely the mean and variance of interarrival times. Suppose that the frame arrival rate is λ and the squared coefficient of the interarrival time distribution $A(x)$ is C_A^2 . The values of these parameteres are modified by the admission policies performed by the interface described in the introduction. In the interface, the stream of frames is observed in an interval T_c . This implies a type of Jumping Window process. The interval is divided into three periods: T_1, T_2 and T_3 . Their length is random and depends on the traffic intensity. During T_1 , B_c volume of information is guaranteed to transmit within T_c and the frames are marked as high priority. During T_2 , the volume of information in excess of B_c (until B_e) is transmitted but the frames are marked with low priority. If the B_e level is reached, the period T_3 starts and lasts until the end of T_c . During T_3 , the arriving frames are discarded.

The queueing model determines the parameters of streams for two classes of customers leaving the interface and representing high and low priority frames. Denote their rates by $\lambda^{(1)}$ and $\lambda^{(2)}$, respectively. During the period T_1 , $\lambda^{(1)} = \lambda$ and outside it $\lambda^{(1)} = 0$; during T_2 , $\lambda^{(2)} = \lambda$ and $\lambda^{(2)} = 0$, otherwise. Let $h_1(x)$, $h_2(x)$ and $h_3(x)$ denote the densities of the duration of the three mentioned periods within T_c . In terms of the diffusion model $h_1(x)$ is the density of the first passage time of the diffusion process between $x = 0$ and

$x = B_c$; $h_2(x)$ is the density of the first passage time of the process between $x = B_c$ and $x = B_c + B_e$. Naturally, T_2 occurs only if the duration of T_1 is inferior to T_c . Note that we replace the volume of information by the number of frames.

In a $G/G/1$ or $G/G/1/N$ diffusion model, diffusion process $X(t)$ represents the number of customers $N(t)$ present in the system, hence coefficients of the diffusion equation take in account the arrivals and departures of customers: $\beta = \lambda - \mu$, $\alpha = C_B^2 \mu$, where $1/\mu$ and C_B^2 are the mean and squared coefficient of variation of the service time distribution $B(x)$.

Here, the diffusion process represents the number of arrivals to the interface, hence we choose the diffusion parameters $\beta = \lambda$, $\alpha = \sigma_A^2 \lambda^3 = C_A^2 \lambda$ and we model the arrival process as a diffusion process initiated at $t = 0$ at the point $x_0 = 0$ and having an absorbing barrier at $x = B_c$. Once the process reaches $x = B_c$, it remains there. It does not seem necessary to bound the process at $x = 0$, due to the constant growing of the process ($\beta > 0$). Hence we omit it for simplicity. The pdf of such a process is (Cox, 1965)

$$p(x, t) = \frac{1}{\sqrt{2\alpha\pi t}} \left[\exp \left\{ -\frac{(x - \beta t)^2}{2\alpha t} \right\} - \exp \left\{ \frac{2\beta N}{\alpha} - \frac{(x - 2N - \beta t)^2}{2\alpha t} \right\} \right]. \quad (2)$$

The density of the first passage time from $x_0 = 0$ to the barrier at $x = B_c$ is

$$h_1(t) = -\frac{d}{dt} \int_{-\infty}^{B_c} p(x, t) dx = \frac{B_c}{\sqrt{2\alpha\pi t^3}} \exp \left[-\frac{(B_c - \beta t)^2}{2\alpha t} \right] \quad (3)$$

Similarly, the density $h_2(t)$ of the second period is expressed as

$$h_2(t) = \frac{B_e}{\sqrt{2\alpha\pi t^3}} \exp \left[-\frac{(B_e - \beta t)^2}{2\alpha t} \right]. \quad (4)$$

The n -th moments of the duration of T_1 and T_2 are

$$E[T_1^n] = \int_0^{T_c} h_1(x) x^n dx + \left[1 - \int_0^{T_c} h_1(x) dx \right] T_c^n \quad (5)$$

$$E[T_2^n] = \int_0^{T_c} h_1(x) \left\{ \int_0^{T_c-x} \xi^n h_2(\xi) d\xi + \left[1 - \int_0^{T_c-x} h_2(\xi) d\xi \right] (T_c - x)^n \right\} dx. \quad (6)$$

The sum of three periods equals T_c . Hence $h_1(x) * h_2(x) * h_3(x) = \delta(x - T_c)$, or

$$\bar{h}_3(s) = \frac{e^{-T_c s}}{\bar{h}_1(s) \bar{h}_2(s)} = e^{-T_c s - \frac{B_c + B_e}{\alpha} (\beta - \sqrt{\beta^2 - 2\alpha s})}$$

and the departure rates of high and low priority frames are $\lambda^{(1)} = \frac{E[T_1]}{T_c} \lambda$, $\lambda^{(2)} = \frac{E[T_2]}{T_c} \lambda$. An estimate of the variation of the output streams of both classes is

$$C_A^{(1)2} \approx C_A^2 \left(\frac{E[T_1]}{T_c} \right)^2, \quad C_A^{(2)2} \approx C_A^2 \left(\frac{E[T_1]}{T_c} \right)^2. \quad (7)$$

3 FR TRANSMISSION NODE MODEL

We consider two types of queue policy at a FR node: push-out and threshold policies. The FR node is represented by a multiclass $G/G/1/N$ diffusion model (Gelenbe, 1975, 1976) adapted to include either push-out or threshold policy of selection of frames to be discarded.

3.1 $G/G/1/N$ diffusion model with push-out policy

The model was proposed and validated in (Czachórski, 1992) for the case of ATM networks. We recall here its principles making minor but necessary changes regarding FR networks.

As long as the number of customers n in the $G/G/1/N$ queue with push-out policy is less than N , it acts as a standard $G/G/1/N$ queue with two classes of customers. During non-saturation periods, we obtain from the $G/G/1/N$ model the function $f(x)$ and probabilities p_0, p_N that the process is at lower or upper boundary, hence the distribution $p(n)$ of n customers of both classes taken together. The conditional distribution $p(n|n < N)$, which corresponds to the non-saturation period, can be easily obtained.

The process enters the saturation period with probability $p(N)$. The conditional distribution of the number of class- k customers calculated without replacements is as follows:

$$p^{(k)}(n^{(k)}|N) = \binom{N}{n^{(k)}} \left(\frac{\lambda^{(k)}}{\lambda} \right)^{n^{(k)}} \left(1 - \frac{\lambda^{(k)}}{\lambda} \right)^{N-n^{(k)}}, \quad k = 1, 2. \quad (8)$$

Now we study the policy of replacement. If we approximate the stream of class-1 customers during a saturation period by a Poisson process with parameter $\lambda^{(1)}$, the probability of n arrivals of class-1 customers within a single saturation period is

$$p_{\text{arriv}}(n) = \int_0^\infty \frac{(\lambda^{(1)}x)^n}{n!} \exp^{-\lambda^{(1)}x} b(x) dx, \quad n = 0, 1, \dots$$

where $b(x)$ is the density of service time distribution and the probability $p_{\text{rep}}(n)$ of n replacements is

$$p_{\text{rep}}(n) = p_{\text{arriv}}(n) \sum_{n_2=n+1}^N p^{(2)}(n^{(2)}|N) + p^{(2)}(n^{(2)}|N) \sum_{i=n}^\infty p_{\text{arriv}}(i).$$

The first sum corresponds to situations where there are n arrivals and at least $n+1$ class-2 customers which could be replaced. The second sum corresponds to situations where there are n class-2 customers and at least n class-1 arrivals.

In the case of a non-Poisson input stream, the pdf $a(x)$ or distribution function $A(x)$ of interarrival times is required to determine the probability of n arrivals:

$$p_{\text{arriv}}(n) = \int_0^\infty b(x) \int_0^x a^{*n}(t) [1 - A(x-t)] dt dx,$$

where a^{*n} denotes the n -fold convolution.

Due to the policy of replacements, the effective throughputs are

$$\lambda_{\text{eff}}^{(1)} = \lambda^{(1)}(1 - p_N) + p_N \lambda^{(1)} \varepsilon, \quad \lambda_{\text{eff}}^{(2)} = \lambda^{(2)}(1 - p_N) - p_N \lambda^{(1)} \varepsilon, \quad (9)$$

where ε is the probability that a class-1 customer arriving at a saturation period may replace a class-2 customer, that is the ratio of mean number of replacements to mean number of arrivals in this period:

$$\varepsilon = \frac{\sum_{k=1}^N p^{(2)}(k|N) [\sum_{i=0}^{k-1} i p_{\text{arriv}}(i) + k \sum_{i=k}^{\infty} p_{\text{arriv}}(i)]}{\sum_{k=1}^{\infty} k p_{\text{arriv}}(k)}. \quad (10)$$

Taking these throughputs into account, we recalculate new $f(x)$, p_0 , p_N in the $G/G/1/N$ model and we iterate until convergence is achieved. The loss ratios of class-1 and class-2 customers are

$$L^{(1)} = \frac{\lambda^{(1)} - \lambda_{\text{eff}}^{(1)}}{\lambda^{(1)}} = p_N(1 - \varepsilon), \quad L^{(2)} = \frac{\lambda^{(2)} - \lambda_{\text{eff}}^{(2)}}{\lambda^{(2)}} = p_N \left(1 + \frac{\lambda^{(1)}}{\lambda^{(2)}} \varepsilon\right). \quad (11)$$

In the case where the input varies with the time, the above steady-state model uses transient solution of $G/G/1/N$ queue presented in the next section. In order to correct the values of $\lambda_{\text{eff}}^{(1)}$ and $\lambda_{\text{eff}}^{(2)}$, the algorithm reflecting the push-out mechanism should be restarted every fixed time-interval chosen sufficiently small with respect to the time-scale of changes of input parameters.

Figures 3 – 4 present some numerical results obtained with the model described above. They refer to the loss ratios $L^{(1)}$ and $L^{(2)}$ of priority and non-priority frames. The service time is exponential and its mean value is equal to the time unit; the input stream is not Poisson, $C_A^2 = 5$. The loss ratio is much higher than that in the case of $C_A^2 = 1$. Note that the estimation of such small losses, of the order 10^{-15} would be difficult to obtain with other methods, e.g. simulation.

In (Gravey,1991) the authors studied a $M_1 + M_2/G/1/N$ queue with non-preemptive HOL and pushout priorities. The behaviour of the system is similar to the one of our system in term of loss probabilities for pushout cells.

3.2 The $G/G/1/N$ diffusion model with threshold

If the number $N(t)$ of frames at the moment t is less than a defined threshold N_1 , both classes of frames are queued and served on a FIFO. When the number of frames is equal or greater than N_1 , only priority frames are admitted and ordinary ones are lost. Hence, the arrival stream depends on the state of the queue and the diffusion parameters α , β should reflect this fact and depend on the value of x : $\alpha(x)$, $\beta(x)$. We assume that these parameters are piecewise constant. A natural choice is as follows:

$$\beta(x) = \begin{cases} \beta_1 = \lambda^{(1)} + \lambda^{(2)} - \mu & \text{for } 0 < x \leq N_1, \\ \beta_2 = \lambda^{(1)} - \mu & \text{for } N_1 < x < N \end{cases} \quad (12)$$

and

$$\alpha(x) = \begin{cases} \alpha_1 = \lambda^{(1)} C_A^{(1)2} + \lambda^{(2)} C_A^{(2)2} + \mu C_B^2 & \text{for } 0 < x \leq N_1, \\ \alpha_2 = \lambda^{(1)} C_A^{(1)2} + \mu C_B^2 & \text{for } N_1 < x < N. \end{cases} \quad (13)$$

Let $f_1(x)$ and $f_2(x)$ denote the pdf function of the diffusion process in intervals $x \in (0, N_1]$ and $x \in [N_1, N)$. We suppose that:

- $\lim_{x \rightarrow 0} f_1(x, t; x_0) = \lim_{x \rightarrow N} f_2(x, t; x_0) = 0$,

Figure 3 Loss ratio $L^{(1)}$ as a function of $\lambda^{(2)}$, ($\lambda^{(1)}=0.2$); buffer length $N = 10, 15, 20$; $C_A^2 = 5$; push-out mechanism.

Figure 4 Loss ratio $L^{(2)}$ as a function of $\lambda^{(2)}$, ($\lambda^{(1)}=0.2$); buffer length $N = 10, 15, 20$; $C_A^2 = 5$; push-out mechanism.

- $f_1(x)$ and $f_2(x)$ functions have the same value at the point N_1 : $f_1(N_1) = f_2(N_1)$,
- there is no probability mass flow within the interval $x \in (1, N-1)$:

$$\frac{\alpha_n}{2} \frac{d f_n(x)}{d x} - \beta_n f_n(x) = 0, \quad x \in (1, N-1), \quad n = 1, 2,$$

and we obtain the solution of the diffusion equations:

$$f_1(x) = \begin{cases} \frac{\lambda_0 p_0}{-\beta_1} (1 - e^{z_1 x}) & \text{for } 0 < x \leq 1, \\ \frac{\lambda_0 p_0}{-\beta_1} (1 - e^{z_1}) e^{z_1(x-1)} & \text{for } 1 \leq x \leq N_1, \end{cases} \quad (14)$$

$$f_2(x) = \begin{cases} f_1(N_1) e^{z_2(x-N_1)} & \text{for } N_1 \leq x \leq N-1, \\ \frac{\mu p_N}{-\beta_2} [1 - e^{z_2(x-N)}] & \text{for } N-1 \leq x < N, \end{cases} \quad (15)$$

where $z_n = \frac{2\beta_n}{\alpha_n}$, $n = 1, 2$. Probabilities p_0 , p_N are obtained with the use of the normalization condition. The loss ratio $L^{(1)}$ is expressed by the probability p_N , the loss ratio $L^{(2)}$ is determined by the probability $P[x > N_1] = \int_{N_1}^N f_2(x) dx + p_N$. We omit here numerical examples but they are easily obtained from Eqs. (15, 14).

4 TRANSIENT BEHAVIOUR OF THE NODE

In the transient solution of the $G/G/1/N$ model, we consider a diffusion process with two absorbing barriers at $x = 0$ and $x = N$, starting at $t = 0$ from $x = x_0$. Its probability density function $\phi(x, t; x_0)$ has the following form (Cox, 1965)

$$\phi(x, t; x_0) = \begin{cases} \delta(x - x_0) & \text{for } t = 0 \\ \frac{1}{\sqrt{2\pi\alpha t}} \sum_{n=-\infty}^{\infty} \left\{ \exp \left[\frac{\beta x'_n}{\alpha} - \frac{(x - x_0 - x'_n - \beta t)^2}{2\alpha t} \right] - \exp \left[\frac{\beta x''_n}{\alpha} - \frac{(x - x_0 - x''_n - \beta t)^2}{2\alpha t} \right] \right\} & \text{for } t > 0, \end{cases} \quad (16)$$

where $x'_n = 2nN$, $x''_n = -2x_0 - x'_n$.

If the initial condition is defined by a function $\psi(x)$, $x \in (0, N)$, $\lim_{x \rightarrow 0} \psi(x) = \lim_{x \rightarrow N} \psi(x) = 0$, then the pdf of the process has the form $\phi(x, t; \psi) = \int_0^N \phi(x, t; \xi) \psi(\xi) d\xi$.

The probability density function $f(x, t; \psi)$ of the diffusion process with barriers and jumps from $x = 0$ to $x = 1$ and from $x = N$ to $x = N-1$ is composed of the function $\phi(x, t; \psi)$ which represents the influence of the initial conditions and of a spectrum of functions $\phi(x, t - \tau; 1)$, $\phi(x, t - \tau; N-1)$ which are pd functions of diffusion processes with absorbing barriers at $x = 0$ and $x = N$, starting at time $\tau < t$ at points $x = 1$ and $x = N-1$ with densities $g_1(\tau)$ and $g_{N-1}(\tau)$, cf. (Czachòrski, 1993):

$$f(x, t; \psi) = \phi(x, t; \psi) + \int_0^t g_1(\tau) \phi(x, t - \tau; 1) d\tau + \int_0^t g_{N-1}(\tau) \phi(x, t - \tau; N-1) d\tau. \quad (17)$$

Densities $\gamma_0(t)$, $\gamma_N(t)$ of probability that at time t the process enters to $x = 0$ or $x = N$ are

$$\begin{aligned}\gamma_0(t) &= p_0(0)\delta(t) + [1 - p_0(0) - p_N(0)]\gamma_{\psi,0}(t) + \int_0^t g_1(\tau)\gamma_{1,0}(t-\tau)d\tau \\ &\quad + \int_0^t g_{N-1}(\tau)\gamma_{N-1,0}(t-\tau)d\tau, \\ \gamma_N(t) &= p_N(0)\delta(t) + [1 - p_0(0) - p_N(0)]\gamma_{\psi,N}(t) + \int_0^t g_1(\tau)\gamma_{1,N}(t-\tau)d\tau \\ &\quad + \int_0^t g_{N-1}(\tau)\gamma_{N-1,N}(t-\tau)d\tau,\end{aligned}\tag{18}$$

where $\gamma_{1,0}(t)$, $\gamma_{1,N}(t)$, $\gamma_{N-1,0}(t)$, $\gamma_{N-1,N}(t)$ are densities of the first passage time between corresponding points. The functions $\gamma_{\psi,0}(t)$, $\gamma_{\psi,N}(t)$ denote densities of probabilities that the initial process, starting at $t = 0$ at the point ξ with density $\psi(\xi)$ will end at time t by entering respectively $x = 0$ or $x = N$.

We may express $g_1(t)$ and $g_N(t)$ with the use of functions $\gamma_0(t)$ and $\gamma_N(t)$:

$$g_1(\tau) = \int_0^\tau \gamma_0(t)l_0(\tau-t)dt, \quad g_{N-1}(\tau) = \int_0^\tau \gamma_N(t)l_N(\tau-t)dt, \tag{19}$$

where $l_0(x)$, $l_N(x)$ are the densities of sojourn times in $x = 0$ and $x = N$; note that the distributions of these times are not restricted to exponential ones. Laplace transforms of Eqs. (18,19) give us $\bar{g}_1(s)$ and $\bar{g}_{N-1}(s)$. The Laplace transform of the density function $f(x, t; \psi)$ is obtained as

$$\bar{f}(x, s; \psi) = \bar{\phi}(x, s; \psi) + \bar{g}_1(s) \bar{\phi}(x, s; 1) + \bar{g}_{N-1}(s) \bar{\phi}(x, s; N-1). \tag{20}$$

Probabilities that at the moment t the process has the value $x = 0$ or $x = N$ are

$$\bar{p}_0(s) = \frac{1}{s} [\bar{\gamma}_0(s) - \bar{g}_1(s)], \quad \bar{p}_N(s) = \frac{1}{s} [\bar{\gamma}_N(s) - \bar{g}_{N-1}(s)]. \tag{21}$$

Numerical inversions of these functions have been carried out using Stehfest's algorithm (Stehfest, 1970).

Figs. 5 and 6 present the dynamics of mean queue changes studied by the model of this section. In the case of Figure 5, at $t = 0$ frames begin to arrive with constant intensity λ to a $G/G/1/N$ queue, empty previously. The mean queue $E[N(t)]$ begins to increase and finally reaches its steady state value $E[N(\infty)]$. Figure 6 presents the opposite case: at time $t = 0$ a $G/G/1/N$ queue is in steady state and has the mean value $E[N(0)]$. There are no arrivals to the queue after $t = 0$ and its mean value decreases. The shapes of the curves are very similar to the exponential functions $1 - e^{-t/T}$ and $e^{-t/T}$. The time-constant T defines the length of the transient period. We see how the speed of changes depends on the value of $C^2 = C_A^2 = C_B^2$. Another factor that strongly influences the length of transient period is the utilisation ρ . Figure 7 presents how the value of T depends on C^2 (linear dependance) and on ρ (nonlinear dependance) during the growth of the queue. The character of dependance $T(C_A^2, C_B^2, \rho)$ is the same when the decrease of the queue is considered, but the values of T are different (smaller), hence we distinguish T_{load} and T_{unload} .

Figure 5 Loading of a queue.

Figure 6 Unloading of a queue.

Figure 7 Constant T_{load} during growing of the queue as a function C^2 .

The above transient solution of a $G/G/1/N$ model assumes that the parameters of the model are constant. In a network of queues however the output flows of stations change continuously, hence the input parameters of each station are also changing during a transient period. We are obliged to discretize these changes and keep the parameters constant within relatively small time-interval Δt . The transient solution at the end of each interval Δt allows one to determine $\rho(t)$ and then $\lambda(t)$ and C_D^2 . This solution serves also as the initial condition for the solution in the next interval: for the n -th interval, $t \in [(n-1)\Delta t, n\Delta t]$, the density function of the diffusion process of station i is $f(x, t; \psi_n(x))$, where $\psi_n(x) = f[x, t = (n-1)\Delta t; \psi_{n-1}(x)]$. This method has been already successfully applied in the analysis of a single ATM node and of the virtual path composed of such nodes (Czachórski 1992, 1994).

5 CLOSED-LOOP MODEL

The conclusion of the study performed in the previous section may be as follows. We can approximately analyse the dynamics of a node seeing it as a first order (inertia) system. We apply here a notion frequently used in control theory; inertia is a system which transforms an input signal having the form of the unit step function $\mathbf{1}(t)$ ($\mathbf{1}(t) = 0$ for $t < 0$, $\mathbf{1}(t) = 1$ for $t \geq 0$) into the function $k(1 - e^{-t/T})$ where constant k denotes amplification of the system and constant T characterizes the speed of the output changes. It means that if we consider the time-varying intensity $\lambda(t)$ of traffic as an input signal and the mean queue $E[N(t)]$ as an output signal, the dynamics of the output given by a node is similar to the answer of a first order system. The same applies to the loss ratio $L^{(1)}(t)$ or $L^{(2)}(t)$ considered as the output signal.

If $\bar{X}_i(s)$ is the Laplace transform of the input signal (traffic intensity changing in time) at the node i of a network and $\bar{Y}_i(s)$ is the Laplace transform of the output signal (mean

Input signal $X_0(t)$:

λ of traffic generated by LAN

Figure 8 Block diagram of feedback control system using BECN bit.

queue length or the loss ratio changing in time), the transition function of the system is $\bar{K}_i(s) = \frac{k_i}{1+sT_i}$, i.e. the output signal is given by

$$\bar{Y}_i(s) = \bar{K}_i(s)\bar{X}_i(s) = \frac{k_i}{1+sT_i}\bar{X}_i(s). \quad (22)$$

Parameters k_{1i} and T_i depend on the utilisation of the queue i and variances of interarrival and service time distributions: $k_{1i} = k_{1i}(\rho_i, C_{A_i}^2, C_{B_i}^2)$, $T_i = T_i(\rho_i, C_{A_i}^2, C_{B_i}^2)$ in the linear and nonlinear manner presented in the previous section. Naturally, the outputs $E[N(t)]$, $L^{(1)}(t)$ and $L^{(2)}(t)$ demand different values for the coefficients k_{1i} .

The interface station changes only the parameters of the input stream, hence its transition function in the model is a constant k_0 .

The warning about congestion which is sent to the source with the use of the BECN is received with a fixed delay D_i . This can be modelled by a transition function $k_{2i}e^{-sD_i}$.

The whole model is represented in Figure 8. A simple example of the time-evolution of signals is given in Figure 9. At $t = 0$, the level of traffic increases and the output of the node increases until it reaches a given level Y_u . At this moment, a message demanding the diminution of traffic by the given value $\Delta\lambda$ is issued; it reaches its destination after time D_i ; meanwhile the output still rises. Then the input is limited and the output starts to decrease: periodic oscillations are seen in the figure. This is only a simple example; once the transfer function is determined, we can consider any form of input signal.

6 CONCLUSIONS

Diffusion approximations seem to be a proper tool to model the behaviour of high speed networks where we can observe a large number of entities in the network. It allows us (a)

Figure 9 An example of evolution of traffic and congestion in the feedback control system using BECN bit.

to treat transient states and (b) regard second moments of traffic.

FR networks adopt the Explicit Congestion Notification (ECN) mechanism to implement flow control at the source level and to avoid the congestion in the network. In this paper, we have presented an analytical model to study the impact of the reactive functions on the system performance. We have analysed the FR node under two policies: push-out and threshold. The ECN mechanism is modelled as a closed-loop scheme using signal theory.

We have also presented a transient behaviour study of the FR node in order to estimate the queue length changes in time. This model serves us to construct the feed-back scheme which predicts the effect of the ECN mechanism on the network performance.

Preventive control is used at the interface between the LAN and FR networks to make sure that the transmission resources can support a new connection and whether the agreed traffic parameters are not violated. At the interface, the frames receive high or low priority, or are discarded, depending whether the transmitted amounts of data during an interval T_c exceed defined levels.

Numerical results show that the presented method can easily be used to study cases where the loss ratios of frames are small, e.g. less than 10^{-12} for higher priority frames and less than 10^{-7} for lower priority frames.

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