A note on objective functions in multistage stochastic nonlinear programming problems

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Abstract

Multistage stochastic programming problems well correspond to many practical situations in which a random element exists and moreover it is reasonable to treat them with respect to some discrete time interval. In particular, this type of problems correspond to practical situations that can be considered with respect to some time interval and simultaneously decomposed with respect to the individual time points. The aim of this paper is to investigate the objective functions corresponding to the individual problems belonging to the one multistage stochastic programming problem. A special attention is paid to the Lipschitz property.

Keywords

multistage stochastic programming problem, discrete time interval, individual time points, objective functions, Lipschitz property

1 INTRODUCTION

Let (Ω, S, P) be a probability space,

$\xi^j = \xi^j(\omega) = [\xi_1^j(\omega), \xi_2^j(\omega), \dots, \xi_s^j(\omega)]$, $j = 0, 1, \dots, M$, $M \geq 1$ be an s -dimensional random vector defined on (Ω, S, P) ,

$\xi^k = \xi^k(\omega) = [\xi_1^k(\omega), \xi_2^k(\omega), \dots, \xi_s^k(\omega)]$, $k = 0, 1, \dots, M$,

$F^{\xi^j}(z^j)$, $F^{\xi^k}(\bar{z}^k)$, $F^{\xi^k|\xi^{k-1}}(z^k|\bar{z}^{k-1})$, $z^j \in E_s$, $j = 0, 1, \dots, M$, $\bar{z}^k \in E_{(k+1)s}$, $k = 0, 1, \dots, M$ denote the distribution functions of the ξ^j , $\xi^k(\omega)$ and the conditional distribution function ($\xi^k(\omega)$ conditioned by $\xi^{k-1}(\omega)$),

$\bar{z}^k = [z^0, z^1, \dots, z^k]$, $z^j \in E_s$, $z^j = [z_1^j, \dots, z_s^j]$, $j = 0, 1, \dots, k$, $k = 0, 1, \dots, M$,

$Z^k = Z^{\xi^k} \subset E_s$ and $Z^{\bar{z}^k} \subset E_{(k+1)s}$, $k = 0, 1, \dots, M$ denote the supports corresponding to the distribution functions $F^{\xi^k}(z^k)$ and $F^{\xi^k}(\bar{z}^k)$, $z^k \in E_s$, $\bar{z}^k \in E_{(k+1)s}$.

Let, moreover, $f_i^k(x^k)$ and $h_i^k(\bar{x}^{k-1}, \bar{z}^{k-1})$, $i = 1, 2, \dots, l_k$, $k = 1, 2, \dots, M$ be real-

valued, continuous functions defined on E_n and $E_{kn} \times E_{ks}$,
 $g_0^M(\bar{x}^M, \bar{z}^M)$ be a real-valued function defined on $E_{(M+1)n} \times E_{(M+1)s}$,
 $\bar{x}^k = [x^0, x^1, \dots, x^k]$, $x^j \in E_n$, $x^j = [x_1^j, \dots, x_n^j]$, $j = 0, 1, \dots, k$, $k = 0, 1, \dots, M$,
 $X^j \subset E_n$, $j = 0, 1, \dots, M$ be nonempty sets,
 $\bar{X}^k = X^0 \times X^1 \times \dots \times X^k$, $\bar{Z}^k = Z^{\xi^0} \times Z^{\xi^1} \times \dots, Z^{\xi^k}$, $k = 0, 1, \dots, M$.
(E_n , $n \geq 1$ denotes an n -dimensional Euclidean space.)

A general $(M+1)$ -stage stochastic programming problem ($M \geq 1$) can be introduced as an optimization problem considered with respect to some rather general space, see e.g. Dupačová (1995), Hartley (1980) or Rockafellar and Wets (1978). It is well-known that such defined optimization problem is very complicated and mostly suitable for some theoretical investigation. However, in the literature there also exists another (perhaps for a practical treatment more suitable) definition of this type of the stochastic optimization problems, see e.g. Birge (1988), Dupačová (1995) or King (1988). We can introduce it in the form:

Find

$$\inf \{E_{F^{\xi^0}} g_0^0(x^0, \xi^0) \mid x^0 \in X^0\}, \quad (1)$$

where the function $g_0^0(x^0, z^0)$ is given by the recurrent system of the equations

$$g_0^k(\bar{x}^k, \bar{z}^k) = \inf \{E_{F^{\xi^{k+1}} | \bar{\xi}^k = \bar{z}^k} g_0^{k+1}(\bar{x}^{k+1}, \xi^{k+1}) \mid x^{k+1} \in X^{k+1} : \\ f_i^{k+1}(x^{k+1}) \leq h_i^{k+1}(\bar{x}^k, \bar{z}^k), i = 1, 2, \dots, l_{k+1}\}, \quad k = 0, 1, \dots, M-1. \quad (2)$$

(The symbols $E_{F^{\xi^k}}$, $E_{F^{\xi^k} | \bar{\xi}^{k-1}}$ denote the operators of the mathematical expectation considered with respect to the distribution functions F^{ξ^k} and $F^{\xi^k | \bar{\xi}^{k-1}}$, $k = 0, 1, \dots, M$, $E_{F^{\xi^0} | \bar{\xi}^{-1}} = E_{F^{\xi^0}}$.)

The problems given by the relation (2) are parametric problems and they can be considered as inner optimization problems connected with the stage $k+1$, $k = 0, 1, \dots, M-1$. Moreover, the problems given by (2) can be, under some assumptions, considered as a decomposition of the corresponding problem considered with respect to a general space.

In this paper, we shall follow the definition given by (1) and (2). In particular, we shall deal with the behaviour of the objective functions $g_0^k(\bar{x}^k, \bar{z}^k)$, $k = 0, \dots, M-1$. We focus mostly our investigation to determine assumptions under which these functions are Lipschitz. Namely, this property give the possibility to generalize the results achieved for one and two-stage stochastic programming problems in Kaňková (1980b, 1994a, 1994b) to the multistage case. In detail, the new results enable to investigate successively stability of the multistage problems, to construct the approximative problems with approximation error not greater than some prescribed value as well as to plan a random sample size to be the empirical estimates of the optimal value and the optimal solution enough "accurate" with the high probability. Moreover, we expect that the new results will be useful for the choice of scenarios in the corresponding approach.

Remark 1 In general, it may happen that some symbols in (1) and (2) are not well defined. However, this situation cannot appear under the assumptions considered in this paper.

2 SOME DEFINITIONS AND AUXILIARY ASSERTIONS

Evidently, to investigate the behaviour of the functions $g_0^k(\bar{x}^k, \bar{z}^k)$, $k = 0, \dots, M-1$ it is necessary to investigate mostly the behaviour of the multifunctions $\mathcal{K}^{k+1}(\bar{x}^k, \bar{z}^k)$, $\bar{x}^k \in E_{(k+1)n}$, $\bar{z}^k \in E_{(k+1)s}$ $k = 0, \dots, M-1$ given by

$$\mathcal{K}^{k+1}(\bar{x}^k, \bar{z}^k) = \{x^{k+1} \in X^{k+1} : f_i^{k+1}(x^{k+1}) \leq h_i^{k+1}(\bar{x}^k, \bar{z}^k), i = 1, 2, \dots, l_{k+1}\}. \quad (3)$$

To this end, we substitute

$$y_i^k = h_i^{k+1}(\bar{x}^k, \bar{z}^k), i = 1, 2, \dots, l_{k+1}, k = 0, 1, \dots, M-1, \bar{x}^k \in E_{(k+1)n}, \bar{z}^k \in E_{(k+1)s} \quad (4)$$

and define for $k = 0, 1, \dots, M-1$, $y^k \in E_{l_{k+1}}$ the multifunctions $\bar{\mathcal{K}}^{k+1}(y^k)$ by the relation

$$\bar{\mathcal{K}}^{k+1}(y^k) = \{x^{k+1} \in X^{k+1} : f_i^{k+1}(x^{k+1}) \leq y_i^k, i = 1, 2, \dots, l_{k+1}\}. \quad (5)$$

Evidently

$$\bar{\mathcal{K}}^{k+1}(y^k) = \mathcal{K}^{k+1}(\bar{x}^k, \bar{z}^k) \quad \text{for } y^k = (y_1^k, \dots, y_{l_{k+1}}^k) \quad \text{fulfilling relation (4)}. \quad (6)$$

The assumptions under which $\bar{\mathcal{K}}^{k+1}(y^k)$, for every given $k = 0, 1, \dots, M-1$, is a uniformly continuous (with respect to the Hausdorff distance – for definition see e.g. Kaňková (1980a)) multifunction are introduced, for example in Kaňková (1980a). By the technique of that paper we obtain also sufficient assumptions under which the functions $g_0^k(\bar{x}^k, \bar{z}^k)$, $k = 0, 1, \dots, M-1$ are uniformly continuous. To investigate the assumptions, under which the multifunctions $\bar{\mathcal{K}}^{k+1}(y^k)$, $k = 1, 2, \dots, M-1$ are Lipschitz (with respect to the Hausdorff distance), we denote by Y_0^{k+1} the convex hull of the set Y^{k+1}

$$Y^{k+1} = \{y^k \in E_{l_{k+1}}, y^k = (y_1^k, \dots, y_{l_{k+1}}^k) : \text{there exists } \hat{x}^k \in \hat{X}^k, \bar{z}^k \in \bar{Z}^k \text{ such that } y_i = h_i^{k+1}(\hat{x}^k, \bar{z}^k) \text{ for every } i \in \{1, 2, \dots, l_{k+1}\}\}.$$

Furthermore, for $\varepsilon' > 0$ we denote by $Y(\varepsilon')$ the ε' neighbourhood of the nonempty set $Y \subset E_l$, $l \geq 1$.

We shall introduce the following system of the assumptions.

i.1 For every $k \in \{0, 1, \dots, M-1\}$

- a. $f_i^{k+1}(x^{k+1})$, $i = 1, 2, \dots, l_{k+1}$ are linear functions, $X^{k+1} = E_n$, without loss of generality, we can consider in this case the constraints in (2) (consequently in (3)) to be in the form of equalities,
- b. for every $y^k \in Y^{k+1}$, $\bar{\mathcal{K}}^{k+1}(y^k)$ is a nonempty, compact set,
- c. the matrix A^{k+1} of the type $(l_{k+1} \times n)$, $l_{k+1} \leq n$ fulfils the relation

$$\bar{\mathcal{K}}^{k+1}(y^k) = \{x^{k+1} \in X^{k+1} : A^{k+1}x^{k+1} = y^k\}, y^k \in Y^{k+1}$$

and, moreover, all its submatrices of the types $(l_{k+1} \times l_{k+1})$, $A^{k+1}(1)$, $A^{k+1}(2)$, $\dots$, $A^{k+1}(m)$ are nonsingular,

- d. $C(k+1) = l_{k+1} \max_{i \in \{1, \dots, m\}} a_{i \tau}^{k+1}(s)$, where $a_{i \tau}^{k+1}(s)$ for $s \in \{1, 2, \dots, m\}$ denote the elements of the inverse matrix to $A^{k+1}(s)$,

i.2 For every $k \in \{0, 1, \dots, M-1\}$

- a. X^{k+1} is a convex, compact set, $M_1^{k+1} = \sup_{x^1, x^2 \in X^{k+1}} \|x^1 - x^2\|$,
- b. $f_i^{k+1}(x^{k+1})$, $i = 1, 2, \dots, l_{k+1}$ are convex functions on X^{k+1} ,
- c. $\bar{K}^{k+1}(y^k)$ is a nonempty set for every $y^k \in Y^{k+1}(\varepsilon)$ and some $\varepsilon > 0$,
- d. $C(k+1) = \frac{M_1^{k+1}}{\varepsilon_0}$ for an $\varepsilon_0 \in (0, \varepsilon)$,

i.3 For every $k \in \{0, 1, \dots, M-1\}$ there exists real-valued constants d_1^{k+1} , $\gamma_2^{k+1} > 0$ such that

- a. if $x^{k+1} \in X^{k+1}$, $y^k \in Y_0^{k+1}(d_1^{k+1} \gamma_2^{k+1})$ fulfil the relations $f_j^{k+1}(x^{k+1}) \leq y_j^k$, $j = 1, \dots, l_{k+1}$ and simultaneously $f_i^{k+1}(x^{k+1}) = y_i^k$ for at least one $i \in \{1, \dots, l_{k+1}\}$, then there exists a vector $x^{k+1}(0) = x^{k+1}(0, x^{k+1}) \in E_n$ fulfilling for every $d \in (0, d_1^{k+1})$ and every $i = 1, 2, \dots, l_{k+1}$ the relations

$$\|x^{k+1}(0)\| = 1, \quad x^{k+1} + dx^{k+1}(0) \in X^{k+1},$$

$$f_i^{k+1}(x^{k+1}) - f_i^{k+1}(x^{k+1} + dx^{k+1}(0)) \geq \gamma_2^{k+1} d,$$

- b. for every $y^k \in Y_0^{k+1}(\gamma_2^{k+1} d_1)$, $\bar{K}^{k+1}(y^k)$ is a nonempty set,

$$c. C(k+1) = \frac{1}{\gamma_2^{k+1}}.$$

($\|\cdot\|$ denotes the Euclidean norm in E_n .)

We introduce the next auxiliary assertion.

Lemma 1 *If at least one of the assumptions (i.1), (i.2), (i.3) is fulfilled, then*

$$\Delta[\bar{K}^{k+1}(y^k(1)), \bar{K}^{k+1}(y^k(2))] \leq C(k+1) \|y^k(1) - y^k(2)\|$$

for every $y^k(1), y^k(2) \in Y^{k+1}$, $k = 0, 1, \dots, M-1$.

($\Delta[\cdot, \cdot]$ denotes the Hausdorff distance of the nonempty, closed subsets of E_n .)

Proof. If the assumptions (i.1) or (i.2) are fulfilled, then we refer the reader to the papers Kaňková (1980b, 1994a). Consequently, it remains to consider only the assumptions (i.3). To this end, let $k \in \{0, 1, \dots, M-1\}$, $y^k(1), y^k(2) \in Y_0^{k+1}$, $y^k(1) = (y_1^k(1), \dots, y_{l_{k+1}}^k(1))$, $y^k(2) = (y_1^k(2), \dots, y_{l_{k+1}}^k(2))$ be arbitrary given. We distinguish two cases

- a. $\|y^k(1) - y^k(2)\| < \gamma_2^{k+1} d_1$, b. $\|y^k(1) - y^k(2)\| \geq \gamma_2^{k+1} d_1$.

First, let us consider the case a. If $\underline{y}, \bar{y} \in E_{l_{k+1}}$, $\underline{y} = (\underline{y}_1, \underline{y}_2, \dots, \underline{y}_{l_{k+1}})$, $\bar{y} = (\bar{y}_1, \bar{y}_2, \dots, \bar{y}_{l_{k+1}})$ are defined by the relations

$$\underline{y}_i = \min(y_i^k(1), y_i^k(2)), \quad \bar{y}_i = \max(y_i^k(1), y_i^k(2)), \quad i = 1, 2, \dots, l_{k+1},$$

then

$$\|y^k(1) - y^k(2)\| = \|\underline{y} - \bar{y}\|, \quad \underline{y}, \bar{y} \in Y_0^{k+1}(\gamma_2^{k+1} d_1).$$

If, furthermore, we define the set X_0^{k+1} by the relation

$$\begin{aligned} X_0^{k+1} = \{ & x^{k+1} \in X^{k+1} : \text{there exists } y^k \in Y_0^{k+1}(\gamma_2^{k+1} d_1) \text{ such that } f_j(x^{k+1}) \leq y_j^k, \\ & \text{for every } j \in \{1, 2, \dots, l_{k+1}\} \text{ and simultaneously } f_i(x^{k+1}) = y_i^k \text{ for} \\ & \text{at least one } i \in \{1, 2, \dots, l_{k+1}\}, y^k = (y_1^k, \dots, y_{l_{k+1}}^k)\}, \end{aligned}$$

then we can formulate the next implication

$$\begin{aligned} x \in \bar{\mathcal{K}}^{k+1}(\bar{y}), \quad x \in X_0^{k+1} \implies & \text{there exists } x' \in \bar{\mathcal{K}}^{k+1}(\underline{y}) \text{ such that} \\ \|x - x'\| \leq & \frac{1}{\gamma_2^{k+1}} \|\underline{y} - \bar{y}\|. \end{aligned} \quad (7)$$

If we define the point x' by the relation

$$x' = x + \frac{\|\underline{y} - \bar{y}\|}{\gamma_2^{k+1}} x^{k+1}(0),$$

then it follows from the assumptions of Lemma 1 that

$$\|x - x'\| = \frac{\|\underline{y} - \bar{y}\|}{\gamma_2^{k+1}} < d_1 \quad (8)$$

and simultaneously

$$f_i^{k+1}(x) - f_i^{k+1}(x') \geq \|\underline{y} - \bar{y}\| \geq \bar{y}_i - \underline{y}_i, \quad i = 1, 2, \dots, l_{k+1}.$$

However, it follows immediately from the last inequality and from the implication assumption that $x' \in \bar{\mathcal{K}}^{k+1}(\underline{y})$. We have proven the validity of the implication (7). Since evidently $\underline{y} \leq y^k(1) \leq \bar{y}$, $\underline{y} \leq y^k(2) \leq \bar{y}$ we get

$$\bar{\mathcal{K}}^{k+1}(\underline{y}) \subset \bar{\mathcal{K}}^{k+1}(y^k(1)) \subset \bar{\mathcal{K}}^{k+1}(\bar{y}), \quad \bar{\mathcal{K}}^{k+1}(\underline{y}) \subset \bar{\mathcal{K}}^{k+1}(y^k(2)) \subset \bar{\mathcal{K}}^{k+1}(\bar{y}),$$

It follows from the last relation, the relation (8), the definition of the multifunction $\bar{\mathcal{K}}^{k+1}(y^k)$ and from the properties of the Hausdorff distance that also

$$\Delta[\bar{\mathcal{K}}^{k+1}(y^k(1)), \bar{\mathcal{K}}^{k+1}(y^k(2))] \leq C(k+1) \|y^k(1) - y^k(2)\|.$$

We have proven the validity of the assertion of Lemma 1 for $y^k(1), y^k(2)$ fulfilling the condition a. It remains to consider the case b. However, the assertion of Lemma 1 follows, in this case, from the results already proven in the case a., from the convex behaviour of the set Y_0^{k+1} and from the triangular inequality due to the Hausdorff distance (Salinetti and Wets 1979) in the space of the closed subsets of E_n .

Since $k \in \{0, 1, \dots, M-1\}$ was arbitrary we have finished the proof of Lemma 1. $\square$

3 THE MAIN RESULTS

Combining the achieved auxiliary result and the appropriate behaviour of the functions $h_i^k(\bar{x}^{k-1}, \bar{z}^{k-1}), g_0^M(\bar{x}^M, \bar{z}^M), i = 1, 2, \dots, l_{k+1}, k = 1, 2, \dots, M$ we can obtain the behaviour of the functions $g_0^k(\bar{x}^k, \bar{z}^k), k = 0, 1, \dots, M-1$. We shall focus our investigation mostly to the Lipschitz behaviour. To this end let us introduce the following system of assumptions.

- L.2 a. For every $k = 0, 1, \dots, M-1, i = 1, 2, \dots, l_{k+1}, h_i^{k+1}(\bar{x}^k, \bar{z}^k)$ are Lipschitz functions on $\bar{X}^k \times \bar{Z}^k$ with the Lipschitz constants $\bar{L}_i(k)$,
- b. $g_0^M(\bar{x}^M, \bar{z}^M)$ is a Lipschitz function on $\bar{X}^M \times \bar{Z}^M$ with the Lipschitz constant $\bar{L}_0(M)$.

We shall prove the next theorem.

Theorem 1 If

1. at least one of the assumptions (i.1), (i.2), (i.3) is fulfilled,
2. the assumptions (L.2) are fulfilled,
3. for every $k \in \{0, 1, \dots, M-1\}$ at least one of the following assumptions is fulfilled
 - a. $K^{k+1}(\bar{x}^k, \bar{z}^k)$ is a compact set,
 - b. $g_0^M(\bar{x}^M, \bar{z}^M)$ is a bounded function,
4. the vectors $\xi^j(\omega), j = 0, 1, \dots, M$ are stochastically independent and moreover a finite $E_{F^k M} \bar{\xi}^M$ exists,

then for every $k = 0, 1, \dots, M$

1. there exists a finite

$$E_{F^k} g_0^k(\bar{x}^k, \bar{\xi}^k), \bar{x}^k \in \bar{X}^k,$$

2. $g_0^k(\bar{x}^k, \bar{z}^k)$ is a Lipschitz function on $\bar{X}^k \times \bar{Z}^k$ with the Lipschitz constant

$$\bar{L}_0(k) = \bar{L}_0(M) \prod_{r=k+1}^M [1 + C(r) \sum_{i=1}^{l_{r+1}} \bar{L}_i(r-1)].$$

Proof. Since it follows from the assumptions of Theorem 1 that

$$|g_0^M(\bar{x}^M, \bar{z}^M(1)) - g_0^M(\bar{x}^M, \bar{z}^M(2))| \leq \bar{L}_0(M) \|\bar{z}^M(1) - \bar{z}^M(2)\|,$$

for every $\bar{z}^M(1), \bar{z}^M(2) \in \bar{Z}^M$, $\bar{x}^M \in \bar{X}^M$, we can see that under the assumption 3 of Theorem 1 a finite $E_{F_{\bar{\xi}}^M} g_0^M(\bar{x}^M, \bar{\xi}^M)$ exists. Moreover, since it follows from the assumptions and from the assertions of Lemma 1 that

$$\Delta[\bar{\mathcal{K}}^M(y^{M-1}(1)), \bar{\mathcal{K}}^M(y^{M-1}(2))] \leq C(M) \|y^{M-1}(1) - y^{M-1}(2)\|,$$

for every $y^{M-1}(1), y^{M-1}(2) \in Y^M$, we can obtain by the assumptions L.2.a and the relation (6) that

$$\begin{aligned} \Delta [\mathcal{K}^M(\bar{x}^{M-1}(1), \bar{z}^{M-1}(1)), \mathcal{K}^M(\bar{x}^{M-1}(2), \bar{z}^{M-1}(2))] \leq \\ C(M) \sum_{i=1}^{l_M} \bar{L}_i(M-1) \|(\bar{x}^{M-1}(1), \bar{z}^{M-1}(1)) - (\bar{x}^{M-1}(2), \bar{z}^{M-1}(2))\| \end{aligned}$$

for every $\bar{x}^{M-1}(1), \bar{x}^{M-1}(2) \in X^{M-1}$, $\bar{z}^{M-1}(1), \bar{z}^{M-1}(2) \in \bar{Z}^{M-1}$.

Employing the technique of the proof of Lemma 2 in Kaňková (1978) we see that $g_0^{M-1}(\bar{x}^{M-1}, \bar{z}^{M-1})$ is a Lipschitz function on $\bar{X}^{M-1} \times \bar{Z}^{M-1}$ with the Lipschitz constant

$$\bar{L}_0(M-1) = L_0(M)[1 + C(M) \sum_{i=1}^{l_M} \bar{L}_i(M-1)].$$

Evidently, if we substitute successively $M =: k$, $k = M-1, M-2, \dots, 1$ and repeat the proof for $M = k$ we can see that the assertion of Theorem 1 holds. $\square$

Remark 2 *The assumption on the independence of the vectors $\xi^i(\omega)$, $i = 1, 2, \dots, M$ can be replaced by another assumptions. For example, if the assumption 3.b is fulfilled and a bounded, conditioned probability density (with respect to the Lebesgue measure) corresponding to $F_{\xi^k|\bar{\xi}^{k-1}}(z^k|\bar{z}^{k-1})$, $k = 1, 2, \dots, M$ exists such that it is a Lipschitz function on $\bar{Z}^k$. Of course, the value of the corresponding Lipschitz constants to $\bar{L}_0(k)$, $k = 0, 1, \dots, M-1$ depend then also on the characteristics of the probability densities.*

Theorem 1 introduces the assumptions under which the objective functions defined by (2) are Lipschitz. Some special results for the two-stage stochastic programming problems have been already presented in Kaňková (1994a). To obtain sufficient assumptions under which the objective functions in (2) are convex or strongly convex (for definition see e.g. Kaňková (1994a)) we can employ usual techniques of the convex analysis, cf. Rockafellar (1970) and Pšeničnyi and Danilin (1975) (see also Kaňková (1978), (1994)). Moreover, employing the results of the convex analysis we can obtain also other assumptions under which the objective functions in (2) are Lipschitz.

4 CONCLUSION

The aim of the paper is to investigate the behaviour of the objective functions in the problems defined by the relation (1) and (2). We focused our consideration mostly on the Lipschitz property of the objective functions in the inner problems (2). Namely it is known from the literature that just this property give the possibility to investigate the convergence rate in the case of the empirical estimates as well as to investigate the stability or to construct the approximative problems more suitable for the numerical treatment. Moreover, sufficient assumptions of differentiability can be obtained utilizing the result of this paper and the proof technique similar to that used in Kaňková (1978). However, the corresponding investigation of these problems cannot be included in the framework of this paper.

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