Discrete-time Models for Connection Admission Control (CAC) in ATM *

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This paper provides two different discrete-time finite-buffer queueing models of the $G_1^{[X]}+G_2/D/1/s$ -type for modelling CAC in ATM. In the $G_1^{[X]}+G_2$ -input, G_1 and G_2 produce (correlated) interarrival times and the independent r.v. X generates the size of the batch arriving, when G_1 produces an arrival. Thus, this model represents the situation, when a new connection (G_2) is admitted, i.e. superimposed to already existing traffic ($G_1^{[X]}$) in an ATM-switch. The first model -given in detail here- is based on Neuts' M/G/1-type paradigm, and allows the solution of the queueing model for a DBMAP+DMAP input including the $DMAP_1^{[X]}+DMAP_2$ case (also for a *general* service process). *Per-stream probability functions (p.f.'s) of the occupancy and loss probabilities* can be computed. The second model is based on the unfinished work and allows to compute *per-stream loss probabilities, occupancy and waiting-time p.f.'s*, as well as, for stream 2, an exact value of the *conditional loss probability* that given a cell of stream 2 is lost, the next cell of stream 2 is also lost. Since the $DMAP_1^{[X]}+DMAP_2$ -input can be handled by both methods, an *analytical validation* of the results is possible, which typically only differ in the last 4 of 16 figures (using double-precision-numbers in a C-program). Both models allow service times *higher* than the time-unit, which is necessary for a correct description of a multiplexer's behaviour, when the output line is slower than the input lines. Numerical results show that positive correlation in one stream can have a significant negative influence on QOS parameters of both streams so that effective bandwidth does better than sigma rule, since it inherently carries some estimate of correlation.

Keyword Codes: I.6.4, G.2.m, G.3; Keywords: ATM, Finite-Buffer Discrete-time Queues, CAC

1. Introduction

Broadband networks based on ATM conceptually use "preventive congestion control" since there is no flow control in the switches. This function is called *Connection Admission Control* (CAC) and is closely related to the *policing* function needed to limit the bandwidth usage of sources, that have already been admitted to the network, to the requirements agreed upon within the connection set-up phase. The easiest way to perform CAC is that of using the peak-rates of all the sources fed to the same output line: If the sum of the peak-bit-rates of the existing connections and the new one is smaller than the bit-rate of the output line, the admission is possible. This, however wastes bandwidth, since VBR (Variable Bit Rate) sources (esp. video) do not all work on their peak-bit-rate,

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Fig. 1 A three-link-route between subscriber A and B explaining the queueing model parameters and the minimum interarrival time.

simultaneously. In order to profit from statistical multiplexing, CAC can be described from a queueing point of view as explained in section 4. For this, a (discrete-time) queueing model is necessary.

1.1. Discrete-time modelling in ATM

ATM multiplexers are usually modelled by means of discrete-time systems [3, 4, 17], since the input lines are assumed to work synchronously. The service process is deterministic due to the constant cell length for all connections. In most cases the service time is set equal to the time unit, esp. if the bitrate of the output line is higher than that of the input line.

However, we also need **service-times higher than the time unit**: Imagine a route of three links between two subscribers A and B (Fig. 1): The link from A (150Mbit/s) leads to an ATM switch that guides the cells of the connection (e.g. VBR video source) to an output line of 600Mbit/s. Cells of the connection are switched to the last line of 150Mbit/s and then reach the receiving terminal equipment B. For a **qualitative discussion**, assume that there is no traffic besides the single connection from A to B. At A, this source is assumed to produce bits at its peak rate of 50Mbit/s. The time between cells of the video source on the first 150Mbit/s-line is clearly given by the time needed for filling a cell with bits of the source: $\tau = (48 \cdot 8\text{bit}) / (50\text{Mbit/s})$. This time distance does not change when the cells are switched to the 600Mbit/s-link: they need not wait in the buffer, since it was assumed that there is no other traffic on the link. However, the number of cells *between* two consecutive cells of the video source has changed: On the 150Mbit/s-link, there are (approx.) 2 empty cells between two consecutive video cells ($((48/50)/(53/150)) \approx 3$), on the 600Mbit/s-link there are (approx.) 10 empty cells in between ($((48/50)/(53/600)) \approx 11$). Using the duration for transmitting a cell on the 600Mbit/s-link as a time unit, the queue at the entrance of the 600Mbit/s-link sees interarrival times, which are equal to (approx.) 11 time units (or more, if the bitrate of the source becomes less than the peak rate). The queue's service time is equal to the time unit. The queue at the entrance of the last link sees the same interarrival times, whereas, due to the bitrate of 150Mbit/s, the service time is 4 time-units.

For modelling policing mechanisms as well as traffic shapers based on "leaky bucket" [19], higher values of the deterministic service time are *essential*.

In contrast to continuous time systems, arrivals and departures can happen simultaneously with positive probability in the discrete-time case. Therefore two "waiting room policies" have to be distinguished [8]: **Arrival first (AF)**: in case of the simultaneity of an arrival and a departure the arriving customer still sees the departing one. This has the

consequence of loss in a finite capacity queue, if the occupancy at the departure instant, which is also an arrival instant, is equal to the total capacity. **Departure first (DF):** The departing customer leaves immediately before the arriving one enters, if an arrival and a departure happen simultaneously.

1.2. Correlated inputs in ATM

The ATM field has also evoked interest in queues with *correlated* input processes: There are individual sources which have a typically correlated structure, e.g. VBR video sources (due to run-length coding) [9]; on the other hand the superposition of sources, which are renewal but non-Poisson (discrete-time: non-Bernoulli) is known to result in a stochastic process with interarrival times, which are not mutually independent, i.e. non-renewal [23]. Furthermore, the interdeparture times of an ATM MUX are known to be correlated, and the numerical results given here reflect this fact, too. Analytical solutions are, on the other hand, necessary for validating methods for *rare event* simulations, which are being developed for estimating extremely low loss probabilities (10^{-9} for video) in ATM.

This paper analyses the discrete-time DBMAP/G/1/s finite buffer queue for AF and DF, thus extending [4], where the DMAP input was considered for AF, and [8], dealing with Bernoulli inputs for AF and DF, as well as [2]. [2] deals with the DBMAP/G/1/s-case without distinguishing between AF and DF and analyzes AF using a z -transform approach.

A DBMAP incorporates correlations and batches and is the discrete-time version of the Batch Markovian Arrival process of [16]. We use similar techniques as in the continuous time case of the BMAP/G/1 *infinite* buffer case [16]. The analysis yields the probability function of the occupancy and a characterization of the output process as a Semi-Markov process with closed-form expressions for the mean and variance of the interdeparture times, as well as their first-order coefficient of correlation. Since the superposition DBMAP+DMAP is again a DBMAP, it is possible to model CAC from a queueing point of view, see section 4, yielding per-stream QOS parameters. Since the DMAP can represent a general renewal process with bounded interarrival time [4] and a $\text{GEO}^{[X]}$ is DBMAP [3], the solution presented here also includes the case of the $\text{GEO}^{[X]} + \text{GI}/D/1/s$ system considered (for $D = 1$) in [17].

In the appendix, a short introduction to Markov Renewal and Semi-Markov processes ("two points-of-view for the same thing"), which occur in the analysis, and symbols are listed there.

2. The Discrete Batch Markovian Arrival Process (DBMAP)

We recall the definition of the DBMAP (Discrete time analogue of the Batch Markovian Arrival process due to Lucantoni [16]) given in [3]. Consider an m -state Markov chain (MC). A transition from state $S(t) = i$ to state $S(t+1) = j$, $t = 0, 1, \dots$, happens according to the transition probabilities $[\mathbf{D}_w]_{ij}$ ($\mathbf{D}_w$: transition matrix, $[\cdot]_{ij}$: entry in the i -th row and j -th column) $w = 0, 1, 2, \dots, b_{\max}$, $\mathbf{D}_w := \mathbf{0}$ for $w > b_{\max}$, and a transition is interpreted as an arrival of a w -batch (batch of size w), if $w > 0$ and there is no arrival if $w = 0$. Thus the model generates interarrival times with batch arrivals possible and successive interarrival times are not independent. In order to distinguish from other state random variables, the m states of the DBMAP are usually referred to

as *phases*. The MC has *different types* of transitions between two phases i and j , thus extending the conventional definition of a MC. $\widehat{\mathbf{D}} := \sum_{w=0}^{\infty} \mathbf{D}_w$ is a stochastic matrix and represents the (conventional) MC which only considers the transitions between two phases no matter what type of transition. Let π denote the stationary phase probability vector $\pi \widehat{\mathbf{D}} = \pi$. It can be proved that the point process of the arrival instants is Semi-Markovian with the following SM kernel (with T_n denoting the discrete-time instant of the n -th arrival)

$$P\{S(T_{n+1}) = j, T_{n+1} - T_n = k | S(T_n) = i\} = [\mathbf{D}_0^{k-1} \cdot (\widehat{\mathbf{D}} - \mathbf{D}_0)]_{ij}.$$

The embedded MC has the transition matrix $(\mathbf{I} - \mathbf{D}_0)^{-1}(\widehat{\mathbf{D}} - \mathbf{D}_0)$. With $\underline{P} = \pi(\widehat{\mathbf{D}} - \mathbf{D}_0)/[\pi(\mathbf{I} - \mathbf{D}_0)\underline{e}]$ denoting its stationary phase probability vector at arrival instants², we get for the mean value of the time between successive arrival instants³ $E[A] = \underline{P} \sum_{k=1}^{\infty} k \mathbf{D}_0^{k-1}(\widehat{\mathbf{D}} - \mathbf{D}_0)\underline{e} = \underline{P}(\mathbf{I} - \mathbf{D}_0)^{-1}\underline{e}$. $\underline{e}$ denotes the column vector of 1's, $\underline{e} = (1, 1, \dots, 1)^T$, m components. Note that $E[A]^{-1} \neq \lambda := \pi \sum_{w=1}^{\infty} w \mathbf{D}_w \underline{e}$, which is the arrival rate. In the special case of the DMAP ($b_{max} = 1$) it is the custom to write: $\mathbf{C} := \mathbf{D}_0$, and $\mathbf{D} := \mathbf{D}_1$.

2.1. The counting process of a DBMAP

According to [3] the counting process of a DBMAP is a discrete-time two-dimensional MC $\{(N(0, t], S(t)), t \in \mathbb{N}_0\}$, where $N(0, t]$ denotes the number of arrivals in the intervall $(0, t]$, $S(t) \in \{1, 2, \dots, m\}$ the phase of the process, with the following state transition matrix $\mathbf{T}$:

$$\mathbf{T} = \begin{pmatrix} \mathbf{D}_0 & \mathbf{D}_1 & \mathbf{D}_2 & \mathbf{D}_3 & \cdots \\ \mathbf{0} & \mathbf{D}_0 & \mathbf{D}_1 & \mathbf{D}_2 & \cdots \\ \mathbf{0} & \mathbf{0} & \mathbf{D}_0 & \mathbf{D}_1 & \cdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{D}_0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

A transition from state (v, i) to (w, j) corresponds to an arrival of size $w - v$ and a change of the phase of the input process from i to j . The corresponding state transition probability is

$$P\{N(0; t+1] = w, S(t+1) = j \mid N(0; t] = v, S(t) = i\} =$$

$$[\mathbf{T}]_{vw} = \begin{cases} [\mathbf{D}_{w-v}]_{ij} & \text{for } w \geq v \\ 0 & \text{else} \end{cases}$$

We define: $P\{N(0; 0] = v, S(0) = i\} = \delta_{v,0} \cdot P\{S(0) = i\}$, $\delta_{v,0}$ Kronecker symbol.

In $\mathbf{T}$ the w -th main row results from the first one by shifting it by $w - 1$ main columns and filling the remaining main columns by $\mathbf{0}$ -matrices. Any power of $\mathbf{T}$ also shows this property, formally: $[\mathbf{T}_{vw}^{t_0}]_{ij} = [\mathbf{T}_{0w-v}^{t_0}]_{ij}$ for $w \geq v$. Due to the Markov property we therefore get:

$$P\{N(0; t+t_0] = w, S(t+t_0) = j \mid N(0; t] = v, S(t) = i\} = [\mathbf{T}^{t_0}]_{vw}]_{ij} =$$

² $\underline{P}\underline{e} = 1$ and $\underline{P}(\mathbf{I} - \mathbf{D}_0)^{-1}(\widehat{\mathbf{D}} - \mathbf{D}_0) = \underline{P}$ are easily verified.

³ $A_n = T_{n+1} - T_n$ denoting the n -th interarrival time, $A = \{A_n, n \in \mathbb{N}_0\}$ the stochastic process of the interarrival times

$$= \left[[\mathbf{T}^{t_0}]_{0w-v} \right]_{ij} = P\{N(0; t_0) = w - v, S(t_0) = j \mid N(0; 0) = 0, S(0) = i\} \quad (1)$$

and it suffices only to compute the first main row of $\mathbf{T}^t$ in order to know $\mathbf{T}^t$ completely. In the following we use the abbreviation ($\mathbf{I}$: $m \times m$ -matrix of unity) for $w \geq 0$:

$$[\mathbf{P}(w, t_0)]_{ij} := P\{N(0; t_0) = w, S(t_0) = j \mid N(0; 0) = 0, S(0) = i\}, \quad \mathbf{P}(w, 0) = \mathbf{1}_{w=0} \cdot \mathbf{I}$$

For $w < 0$ it is defined: $\mathbf{P}(w, t_0) := \mathbf{0}$. By considering the product $\mathbf{T}^{t+1} = \mathbf{T}^t \cdot \mathbf{T}$ we find:

$$\mathbf{P}(w, t+1) = [\mathbf{T}^{t+1}]_{0w} = \begin{cases} [\mathbf{T}^t]_{00} \cdot \mathbf{D}_0 = \mathbf{P}(0, t) \cdot \mathbf{D}_0 & \text{for } w = 0 \\ \sum_{v=0}^w [\mathbf{T}^t]_{0v} \mathbf{D}_{w-v} = \sum_{v=0}^w \mathbf{P}(v, t) \mathbf{D}_{w-v} & \text{for } w \geq 1 \end{cases} \quad (2)$$

2.2. DBMAPs representing real sources

[3] presents the matrices $\mathbf{D}_w$ for VBR video sources (both individual sources and superpositions thereof) of uniform and multiple activity levels, using the mean value, mean deviation and the autocovariance function of the bitrate as traffic descriptors. In [12], the DMAP^[X] is used for representing periodic correlation functions as measured for VBR video without frame-buffering. The DMAP covers the burst-silence model used for data and voice, as well as the Markov Modulated deterministic process [4] and the General Modulated Deterministic Process. However, a general method for adjusting the parameters of a DBMAP to an arbitrary source seems not yet to exist.

3. The analysis of the queueing system

We consider a FIFO queue with a buffer of s places (total capacity: $s + 1$). The inter-arrival times are produced according to a DBMAP, the service times are i.i.d. (renewal) with the probability function $h(t)$. Due to the properties of the DBMAP, the occupancy at successive time instants immediately after a departure results in an *embedded* Markov Renewal process. (It is an embedded MRP, since the state the MRP rests between two departures is not equal to the state of the queueing system in terms of the occupancy between departure epochs, since in between arrivals can happen.)

The resulting Semi-Markov kernel $\tilde{\mathbf{q}}(t)$ is of M/G/1 type [18]. By $\tilde{\mathbf{q}} := \sum_{t=1}^{\infty} \tilde{\mathbf{q}}(t)$ we denote the embedded Markov chain (MC) of the MRP. The stationary state probability of this MC are needed for computing the probability function of the occupancy at arbitrary and arrival instants. Furthermore the MRP characterizes the output process in terms of the interdeparture times completely. The computation of the joint probability functions of successive interdeparture times is done by using standard techniques of Semi-Markov processes [5, 11].

3.1. The embedded Markov Renewal process (MRP) at departure instants

The Semi-Markov kernel $\tilde{\mathbf{q}}(t)$ of the MRP states about the probability $\left[[\tilde{\mathbf{q}}(t)]_{k\ell} \right]_{ij}$ ³ that, given a departure leaves k customers in the system with the input process in phase i , the next departure happens after t time units and leaves ℓ customers in the system

³ $[[\tilde{\mathbf{q}}(t)]_{k\ell}]_{ij}$ means the submatrix in the k -th main row and the ℓ -th column of $\tilde{\mathbf{q}}(t)$ and in this submatrix the entry in the i -th row and the j -th column.

$$\begin{array}{c}
\begin{array}{cccccc}
1 & 2 & 3 & \cdots & s & s+1 & s+2
\end{array} \\
\left(\begin{array}{c|c|c|c|c|c|c}
\mathbf{B}_0(t) & \mathbf{B}_1(t) & \mathbf{B}_2(t) & \cdots & \mathbf{B}_{s-1}(t) & \mathbf{B}_s(t) + \sum_{k=s+1}^{\infty} \mathbf{B}'_k(t) & \sum_{k=s+1}^{\infty} \mathbf{B}''_k(t) \\
\mathbf{A}_0(t) & \mathbf{A}_1(t) & \mathbf{A}_2(t) & \cdots & \mathbf{A}_{s-1}(t) & \mathbf{A}_s(t) + \sum_{k=s+1}^{\infty} \mathbf{A}'_k(t) & \sum_{k=s+1}^{\infty} \mathbf{A}''_k(t) \\
\mathbf{0} & \mathbf{A}_0(t) & \mathbf{A}_1(t) & \cdots & \mathbf{A}_{s-2}(t) & \mathbf{A}_{s-1}(t) + \sum_{k=s}^{\infty} \mathbf{A}'_k(t) & \sum_{k=s}^{\infty} \mathbf{A}''_k(t) \\
\mathbf{0} & \mathbf{0} & \mathbf{A}_0(t) & \cdots & \mathbf{A}_{s-3}(t) & \mathbf{A}_{s-2}(t) + \sum_{k=s-1}^{\infty} \mathbf{A}'_k(t) & \sum_{k=s-1}^{\infty} \mathbf{A}''_k(t) \\
\vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{A}_0(t) & \mathbf{A}_1(t) + \sum_{k=2}^{\infty} \mathbf{A}'_k(t) & \sum_{k=2}^{\infty} \mathbf{A}''_k(t) \\
\hline
\mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{A}_0(t) + \sum_{k=1}^{\infty} \mathbf{A}'_k(t) & \sum_{k=1}^{\infty} \mathbf{A}''_k(t)
\end{array} \right) \left[\begin{array}{c}
\sum_{k=s}^{\infty} \mathbf{B}_k(t) \\
\sum_{k=s}^{\infty} \mathbf{A}_k(t) \\
\sum_{k=s-1}^{\infty} \mathbf{A}_k(t) \\
\sum_{k=s-2}^{\infty} \mathbf{A}_k(t) \\
\vdots \\
\sum_{k=1}^{\infty} \mathbf{A}_k(t)
\end{array} \right] \cdot
\end{array}$$

(*)

Fig. 2 The SM kernel $\tilde{\mathbf{q}}(t)$ for DF with $s+2$ main rows and columns. The kernel for AF is obtained by omitting, in the SM kernel for DF, the last main row and column and replacing the $(s+1)$ -th main column by (*).

with the input process in phase j . In case of both AF and DF "leave behind" means "immediately after a departure" so that an arrival simultaneous with a departure is taken into account.

3.2. Arrival First

The case AF leads to a Semi-Markov kernel $\tilde{\mathbf{q}}(t)$ similar to that of the continuous-time case. It is described by the following matrices:

- $\mathbf{A}_k(t)$, where $[\mathbf{A}_k(t)]_{ij}$ denotes the probability, that given a departure, which leaves at least one customer in the system with the input process in phase i , there are k arrivals within the following service time lasting t time units with the input process in phase j at the next departure. In the present case it is: $\mathbf{A}_k(t) = \mathbf{P}(k, t)h(t)$.
- $\mathbf{B}_k(t)$, where $[\mathbf{B}_k(t)]_{ij}$ denotes the conditional probability, that given a departure, which leaves the system empty with the input process in phase i , the next departure happens after t time units with the input process in phase j , and there were k arrivals during the service time of that departure. In the present case it is

$$\mathbf{B}_k(t) = \sum_{w=1}^{\infty} \sum_{v=1}^t \mathbf{D}_0^{v-1} \mathbf{D}_w \mathbf{P}(k-w+1, t-v) h(t-v) = \sum_{w=1}^{\infty} \sum_{v=1}^t \mathbf{D}_0^{v-1} \mathbf{D}_w \mathbf{A}_{k-w+1}(t-v)$$

With AF a departure can leave at most s customers in the system (as it is in the continuous-time case) so that the state space of the MRP is $\{0, 1, \dots, s\} \times \{1, 2, \dots, m\}$. The resulting Semi-Markov kernel is shown in Fig. 2 together with that of DF. The transition probability matrix of the embedded MC $\tilde{\mathbf{q}}$ is composed of the entries $\mathbf{B}_k := \sum_{t=1}^{\infty} \mathbf{B}_k(t)$

and $\mathbf{A}_k := \sum_{t=1}^{\infty} \mathbf{A}_k(t)$ among which there is the following relation:

$$\mathbf{B}_k = \sum_{w=1}^{\infty} \sum_{v=1}^{\infty} \mathbf{D}_0^{v-1} \mathbf{D}_w \sum_{t=1}^{\infty} \mathbf{A}_{k-w+1}(t-v) = (\mathbf{I} - \mathbf{D}_0)^{-1} \sum_{w=0}^k \mathbf{D}_{w+1} \mathbf{A}_{k-w}. \quad (3)$$

3.3. Departure First

The matrices $\mathbf{A}_k(t)$ and $\mathbf{B}_k(t)$ are not sufficient for DF, since with DF a departure can leave $s+1$ customers in the system iff an arrival and a departure happen simultaneously and this arrival makes the system full. This is taken into account by partitioning $\mathbf{A}_k(t) = \mathbf{A}'_k(t) + \mathbf{A}''_k(t)$ and $\mathbf{B}_k(t) = \mathbf{B}'_k(t) + \mathbf{B}''_k(t)$, where $\mathbf{A}'_k(t)$ and $\mathbf{B}'_k(t)$ account for the cases that none of the arrivals within the service time (lasting t time units) happens simultaneously with the departure at the end of this service time. Formally we have:

$$\mathbf{A}''_k(t) := \sum_{\mu=1}^k \mathbf{P}(k-\mu, t-1) \mathbf{D}_{\mu} h(t), \quad \mathbf{A}'_k(t) := \mathbf{P}(k, t-1) \mathbf{D}_0 h(t), \quad (4)$$

$$\mathbf{B}''_k(t) := \sum_{w=1}^{\infty} \sum_{v=1}^t \mathbf{D}_0^{v-1} \mathbf{D}_w \mathbf{A}''_{k-w+1}(t-v), \quad \mathbf{B}'_k(t) := \sum_{w=1}^{\infty} \sum_{v=1}^t \mathbf{D}_0^{v-1} \mathbf{D}_w \mathbf{A}'_{k-w+1}(t-v) \quad (5)$$

$\tilde{\mathbf{q}}$ is composed of

$$\mathbf{A}''_k := \sum_{t=1}^{\infty} \mathbf{A}''_k(t), \quad \mathbf{A}'_k := \sum_{t=1}^{\infty} \mathbf{A}'_k(t), \quad \mathbf{B}''_k := \sum_{t=1}^{\infty} \mathbf{B}''_k(t), \quad \mathbf{B}'_k := \sum_{t=1}^{\infty} \mathbf{B}'_k(t). \quad (6)$$

The state space of the MRP is $\{0, 1, \dots, s, s+1\} \times \{1, 2, \dots, m\}$. The SM kernel is given in Fig. 2.

3.4. Stationary probabilities for the occupancy at departure instants

For determining the stationary state probabilities $[\underline{x}_r]_i$, $r = 0, 1, \dots, s$ ($s+1$ for DF), that in the equilibrium situation a departure leaves r customers in the system with the input process in phase i , the algorithms of [15], [7] should be used. They work on the state transition matrix producing only positive numbers in each iteration, so that they are numerically very stable. They use the stochastic properties of the matrices to advantage.

3.5. The output process

Since the Semi-Markov kernel $\tilde{\mathbf{q}}(t)$ is known, we can compute (joint) probability functions of the interdeparture times E_n using eq. (27) in the Appendix. They are invariant towards an arbitrary shift of the index n , because $\underline{\mathbf{x}}$ is a stationary solution of the embedded MC.

$$P\{E_n = t_n, \dots, E_{n+\nu} = t_{n+\nu}\} = \underline{\mathbf{x}} \cdot \tilde{\mathbf{q}}(t_n) \cdot \dots \cdot \tilde{\mathbf{q}}(t_{n+\nu}) \cdot \underline{\mathbf{e}}, \quad \nu = 0, 1, 2, \dots \quad (7)$$

$\underline{\mathbf{e}} = (1, 1, \dots, 1)^T$, $m \cdot (s+1)$ resp. $m \cdot (s+2)$ components for AF resp. DF. $\underline{\mathbf{x}} = (\underline{x}_0, \dots, \underline{x}_s, [\underline{x}_{s+1}])$. The mean value and mean deviation of the interdeparture time is determined by $\underline{\mathbf{x}} \cdot \tilde{\mathbf{q}}(t) \cdot \underline{\mathbf{e}}$ and $\text{Cov}[E_n, E_{n+1}]$ in addition by $\underline{\mathbf{x}} \cdot \tilde{\mathbf{q}}(t_1) \tilde{\mathbf{q}}(t_2) \cdot \underline{\mathbf{e}}$. For AF and DF they only differ in that for DF summation goes up to $s+1$, whereas in AF only up to s . By $E^* = \mathbb{E}[E_n]$ we denote the "fundamental mean of the embedded MRP" [18]. We have

for AF and DF with $\boxed{\mathcal{S} := s \text{ for AF}}$ and $\boxed{\mathcal{S} := s + 1 \text{ for DF}}$ $\bar{h}_1 := \sum_{t=1}^{\infty} t h(t)$, $\bar{h}_2 := \sum_{t=1}^{\infty} t^2 h(t)$:

$$\begin{aligned} E^* &= \underline{x} \cdot \sum_{t=1}^{\infty} t \tilde{\mathbf{q}}(t) \cdot \underline{e} = \sum_{t=1}^{\infty} t \left[\underline{x}_0 \cdot \sum_{k=0}^{\infty} \mathbf{B}_k(t) \cdot \underline{e} + \left(\sum_{r=1}^S \underline{x}_r \right) \cdot \sum_{k=0}^{\infty} \mathbf{A}_k(t) \cdot \underline{e} \right] \\ &= \underline{x}_0 (\mathbf{I} - \mathbf{D}_0)^{-1} \underline{e} + \bar{h}_1 \end{aligned} \quad (8)$$

$$\text{Var}[E_n] = \bar{h}_2 + \underline{x}_0 \left[2\bar{h}_1 (\mathbf{I} - \mathbf{D}_0)^{-1} \underline{e} + 2(\mathbf{I} - \mathbf{D}_0)^{-2} \underline{e} - (\mathbf{I} - \mathbf{D}_0)^{-1} \underline{e} \right] - (E^*)^2 \quad (9)$$

$$\begin{aligned} \text{Cov}[E_n, E_{n+1}] &= \left[\underline{x}_0 (\mathbf{I} - \mathbf{D}_0)^{-2} \mathbf{D}_1 \mathbf{A}_0 + \right. \\ &\quad \left. + \left(\underline{x}_0 (\mathbf{I} - \mathbf{D}_0)^{-1} \mathbf{D}_1 + \underline{x}_1 \right) \sum_{t=1}^{\infty} t \mathbf{A}_0(t) - E^* \underline{x}_0 \right] \cdot (\mathbf{I} - \mathbf{D}_0)^{-1} \underline{e} \end{aligned} \quad (10)$$

3.6. The probability function of the occupancy

3.6.1. Queue length at an arbitrary time instant

As explained in the case of the M/G/1 queue in [18] we first consider $y_{kr}^{ij}(t) := P\{\text{occupancy at } t \text{ is } r, S(t) = j \text{ given that } 0 \text{ is a departure instant leaving } k \text{ customers in the system, } S(0) = i\}$ and calculate the expressions. By the occupancy at t we mean the occupancy with taking possible arrivals at t into account. With t going to infinity we get the stationary probability function of the occupancy at an arbitrary instant t , for details see [13]. The occupancy in the system at an arbitrary time instant t in the equilibrium situation results from t going to infinity. For calculating the limits we use Theorem (10.4.17) in [5], p. 334 ("aperiodic case", since the state sojourn time distribution functions of a discrete-time MRP are lattice with span 1); note that due to the Theorem the limits are no longer depending on k and i , we therefore drop them and write: $[\underline{y}_r]_j := \lim_{t \rightarrow \infty} y_{kr}^{ij}(t)$. So $[\underline{y}_r]_j$ denotes the probability that in the equilibrium situation there are r customers in the system at an arbitrary time instant with the input process in phase j . With $H^c(t) := 1 - H(t)$ denoting the complementary distribution function of the service time we get for AF and DF:

$$\underline{y}_0 = \frac{1}{E^*} \underline{x}_0 (\mathbf{I} - \mathbf{D}_0)^{-1} \quad (11)$$

$$\underline{y}_r = \frac{1}{E^*} \sum_{\ell=1}^r \left[\left(\underline{x}_0 (\mathbf{I} - \mathbf{D}_0)^{-1} \mathbf{D}_{\ell} + \underline{x}_{\ell} \right) \sum_{\tau=0}^{\infty} \mathbf{P}(r - \ell, \tau) H^c(\tau) \right] \quad \text{for } 1 \leq r \leq s. \quad (12)$$

$$\underline{y}_{s+1} = \underline{\pi} - \sum_{r=0}^s \underline{y}_r \quad (13)$$

Using the identity $\mathbf{P}(r, \tau) (\mathbf{I} - \mathbf{D}_0) - \sum_{r'=1}^r \mathbf{P}(r - r', \tau) \mathbf{D}_{r'} = \mathbf{P}(r, \tau) - \mathbf{P}(r, \tau + 1)$ for $r \geq 1$ it can be shown (see [13]) that there is the following relation between $\underline{y}_{r+1}$ and $\underline{y}_r$ for $0 \leq r \leq s - 1$:

$$\underline{y}_{r+1} = \left[\frac{1}{E^*} (\underline{x}_{r+1} - \underline{x}_r) + \sum_{k=0}^r \underline{y}_k \mathbf{D}_{r+1-k} \right] (\mathbf{I} - \mathbf{D}_0)^{-1} \quad (14)$$

3.6.2. Occupancy at stationary arrival instants (with ignoring the arrival)

The vectors $\underline{y}_r$ refer to an arbitrary time instant, which can be an arrival instant, a departure instant, both an arrival and a departure instant, or none of them. The occupancy immediately before an arrival instant (or "with ignoring the arrival") decides about whether an arrival is lost or not. For AF it is easily related to the vectors $\underline{y}_r$, whereas for DF one has to consider simultaneity of an arrival and a departure, separately.

Theorem 1 Let $\underline{y}_r$, $r = 0, 1, \dots, s+1$ be the occupancy probability vectors at an arbitrary time instant in the equilibrium situation. They are clearly different for AF and DF. Let $[\underline{z}_r(w)]_i$ denote $P\{a\ w\text{-batch arrives and sees } r \text{ customers with the input process in phase } i\}$. $\underline{z}_r(w)$ is related to $\underline{y}_r$ as follows:

For AF:

For DF:

$$\underline{z}_r(w) = \underline{y}_r \mathbf{D}_w, \quad 0 \leq r \leq s+1 \quad \underline{z}_r(w) = \begin{cases} [\underline{y}_0 + \underline{\Delta}_0] \mathbf{D}_w & r = 0 \\ [\underline{y}_r - \underline{\Delta}_{r-1} + \underline{\Delta}_r] \mathbf{D}_w & 1 \leq r \leq s \\ [\underline{y}_{s+1} - \underline{\Delta}_s] \mathbf{D}_w & r = s+1 \end{cases} \quad (15)$$

where it is $[\underline{\Delta}_r \mathbf{D}_w]_i = P\{a\ w\text{-batch arrives simultaneously with a departure and sees } r \text{ customers with the input process in phase } i\}$.

For DF, $\underline{\Delta}_r$ is given by

$$\underline{\Delta}_r = \begin{cases} \frac{1}{E^*} \left(\underline{x}_0 \mathbf{B}_r''' + \sum_{\ell=1}^{r+1} \underline{x}_\ell \mathbf{A}_{r+1-\ell}''' \right) & \text{for } 0 \leq r \leq s-1 \\ \frac{1}{E^*} \left(\underline{x}_0 \sum_{r'=s}^{\infty} \mathbf{B}_{r'}''' + \sum_{\ell=1}^{s+1} \underline{x}_\ell \sum_{r'=s+2-\ell}^{\infty} \mathbf{A}_{r'}''' \right) & \text{for } r = s \\ \underline{0} & \text{for } r = s+1 \end{cases} \quad (16)$$

It is $\mathbf{A}_r''' := \sum_{\sigma=0}^{\infty} \mathbf{P}(r, \sigma) h(\sigma+1)$ and $\mathbf{B}_r''' := (\mathbf{I} - \mathbf{D}_0)^{-1} \sum_{\ell=1}^{r+1} \mathbf{D}_\ell \mathbf{A}_{r+1-\ell}'''$, and there are the relations for $\mathbf{A}_r'$ and $\mathbf{B}_r'$ according to eqs. (6), (4) and (5): $\mathbf{A}_r' = \mathbf{A}_r''' \mathbf{D}_0$ and $\mathbf{B}_r' = \mathbf{B}_r''' \mathbf{D}_0$.

Proof: By $[\underline{y}_r \mathbf{D}_w]_i$ one has the probability that - for stationarity - there are r customers in the system at some time instant and *one time unit later* a w -batch arrives with the input process in phase i . At this arrival instant, there can be a departure or not. In case of AF, the probability that the arrival sees an occupancy of r with the input process in state i is equal to $[\underline{y}_r \mathbf{D}_w]_i$ no matter whether there is a departure or not, since a possible departure happens immediately after the arrival.

For DF, a departure at the arrival instant happens before the arrival so that it has to be considered, if there is a departure simultaneous with the arrival or not. $[\underline{y}_r \mathbf{D}_w]_i$ for $1 \leq r \leq s$ is equal to the probability that a w -batch arrives and sees r customers in the system and there is *no* departure simultaneous with the arrival *or* that a w -batch arrives simultaneously with a departure and sees $r-1$ customers. The event, the probability of which is $[\underline{y}_0 \mathbf{D}_w]_i$, cannot contain a departure simultaneous with the arriving w -batch. Concerning $[\underline{y}_{s+1} \mathbf{D}_w]_i$ it has to be observed that an arrival simultaneous with a departure cannot see an occupancy greater than s , since the departure happens immediately before the arrival. The expressions for $\underline{\Delta}_r$ are again found by applying arguments similar to those for obtaining the vectors $\underline{y}_r$. For details see [13]. $\square$

In the implementation of computer programs for DF the matrices $\mathbf{A}_r'''$ and $\mathbf{B}_r'''$ have to be computed anyway, see eqs. (4) and (5), when generating the entries of the transition matrix $\mathbf{q} = \sum_{t=1}^{\infty} \mathbf{q}(t)$ resulting from the SM kernel for DF as given in Fig. 2. They have to be stored until $\underline{z}_r(w)$ is computed.

Confining to arrival instants only ($P\{\text{an arrival happens}\} = \pi(\widehat{\mathbf{D}} - \mathbf{D}_0)\underline{e} = 1 - \pi\mathbf{D}_0\underline{e}$), yields

$$P\{w\text{-batch sees } r \text{ customers in the system}\} = \frac{1}{1 - \pi\mathbf{D}_0\underline{e}} \underline{z}_r(w)\underline{e} \quad (17)$$

For the actual waiting time a closed form of the z -transform of the probability function is not available, since it needs a decomposition of the matrix inverse $(\mathbf{I}z_1 - \sum_{w=0}^{b_{\max}} \mathbf{D}_w z_2^w)^{-1}$ into a series of powers of z_2 [1].

4. The DBMAP₁+DMP₂/D/1/s queue for modelling CAC

From a queueing point of view, Connection Admission Control (CAC) can be described as follows (Fig. 3): Among the topologically possible ways between two subscribers, who are to be connected, those routes must be found, which allow guaranteeing the QOS (Quality of Service: delay, delay jitter, cell loss probability) of *both* the existing traffic *and* the new connection, if the new one is admitted; and this needs to be known *in advance*. This can be done -approximately- on each link of such a route, between neighbouring switches separately by superimposing the existing traffic (which involves batches and - as a traffic superposition - is naturally correlated) and the new connection (which can also be correlated: VBR video), both represented by stochastic processes generating the cell interarrival times. This gives rise, in each link, to a queueing system, the input of which is a superposition of those two streams. If QOS parameters can be calculated for both streams separately, a decision can be taken based on these values, which take the *individual* QOS requirements of the new connection into account. The approximation of the approach lies in the fact, that the cell stream of the new connection will change when it enters the network since each switch adds some delay and delay jitter, whereas the existing traffic can be measured in each switch. However, the approximation is quite good, since the changes must not be large. Otherwise the original signal would be disturbed, and limiting the changes is enforced by sufficiently small QOS parameters. (Note that the procedures for CAC, that use simple traffic descriptors (mean and peak bitrate or efficient bitrate) also ignore the changes imposed by a switch.)

A suitable model is the DBMAP₁+DMP₂ input, since it incorporates in one stream batches and in both streams correlations. It also includes the Geo^[X]+GI input of [17].

Let DBMAP₁ be given by $\mathbf{D}_w^{(1)}$, $w \in \mathbb{N}_0$, $b_{\max}^{(1)}$, $\widehat{\mathbf{D}}^{(1)} := \sum_{w=0}^{b_{\max}^{(1)}} \mathbf{D}_w^{(1)}$, and DMP₂ by $\mathbf{C}^{(2)}$ and $\mathbf{D}^{(2)}$. Since they both are generated by a MC (with different types of phase transitions) the superposition, which is still a DBMAP (given by $\widehat{\mathbf{D}}_w$, $w \in \mathbb{N}_0$, $\sum_{w=0}^{\infty} \widehat{\mathbf{D}}_w = \widehat{\mathbf{D}}$) can be described by means of the Kronecker product $\otimes$:

$$\widehat{\mathbf{D}}_w = \begin{cases} \mathbf{D}_0^{(1)} \otimes \mathbf{C}^{(2)} & \text{for } w = 0 \\ \mathbf{D}_w^{(1)} \otimes \mathbf{C}^{(2)} + \mathbf{D}_{w-1}^{(1)} \otimes \mathbf{D}^{(2)} & \text{for } 1 \leq w \leq b_{\max}^{(1)} \\ \mathbf{D}_w^{(1)} \otimes \mathbf{D}^{(2)} & \text{for } w = b_{\max}^{(1)} + 1. \end{cases}$$

Fig. 3 One topologically possible way between two subscribers and the resulting queueing system in each switch of this way.

Whenever two or more cells arrive simultaneously, we assume that the sequence of selecting the cells of the same w -batch for admission to the buffer is chosen randomly, i.e. the probability of being chosen ζ -th (or "in ζ -th position") is $\frac{1}{w}$.

Per-stream loss probabilities. For DBMAP₁, which produces batches, we refer to the loss probability as that of a *test-cell* in the arriving batch. Of an arriving w -batch seeing an occupancy L , only the first $s + 1 - L$ cells can be admitted, and there are $w - (s + 1 - L)$ choosing positions for the test-cell to be lost. Thus, the following expressions of the loss probabilities P_{lossi} , $i = 1, 2$ and $P_{loss[1+2]}$ of the full stream for **AF** are obtained:

$$P_{loss1} = \frac{1}{\mathcal{A}_1} \sum_{\mu=0}^{s+1} \sum_{w=s+2-\mu}^{\infty} \frac{w - (s + 1 - \mu)}{w} \underline{y}_{\mu} \left[\mathbf{D}_w^{(1)} \otimes \mathbf{C}^{(2)} + \mathbf{1}_{w \geq 2} \mathbf{D}_{w-1} \otimes \mathbf{D}^{(2)} \right] \underline{e} \quad (18)$$

$$P_{loss2} = \frac{1}{\mathcal{A}_2} \left\{ \underline{y}_{s+1} \left[\mathbf{D}_0^{(1)} \otimes \mathbf{D}^{(2)} \right] \underline{e} + \sum_{\mu=0}^{s+1} \sum_{w=s+2-\mu}^{\infty} \frac{w - (s + 1 - \mu)}{w} \underline{y}_{\mu} \left[\mathbf{1}_{w \geq 2} \mathbf{D}_{w-1} \otimes \mathbf{D}^{(2)} \right] \underline{e} \right\} \quad (19)$$

$$P_{loss[1+2]} = \frac{1}{\mathcal{A}_{[1+2]}} \left\{ \underline{y}_{s+1} \left[\mathbf{D}_0^{(1)} \otimes \mathbf{D}^{(2)} \right] \underline{e} + \mathcal{A}_1 \cdot P_{loss1} \right\} \quad (20)$$

Note that there are *different* denominators for $P_{loss\ i}, i = 1, 2, [1 + 2]$:

$\mathcal{A}_1 := \pi(\widehat{\mathbf{D}}^{(1)} - \mathbf{D}_0^{(1)}) \otimes (\mathbf{C}^{(2)} + \mathbf{D}^{(2)})_{\underline{e}}$ denotes the probability that the arrival in mind contains a batch of stream 1 (and therefore a test-cell),

$\mathcal{A}_2 := \pi(\widehat{\mathbf{D}}^{(1)} \otimes \mathbf{D}^{(2)})_{\underline{e}}$ the probability that the arrival contains a cell of stream 2,

$\mathcal{A}_{[1+2]} := \pi(\widehat{\mathbf{D}}^{(1)} \otimes (\mathbf{C}^{(2)} + \mathbf{D}^{(2)}) - \mathbf{D}_0^{(1)} \otimes \mathbf{C})_{\underline{e}}$ represents the probability that there is an arrival due to one of the two streams, i.e. a test-cell of the *full* stream arrives.

This corresponds to the fact that *cell loss rate* is equal to the quotient of the count of lost cells of stream i divided by the count of cells generated by stream i ; it is the reason, why $P_{loss1} + P_{loss2} \neq P_{loss[1+2]}$! For DF $\underline{y}_r$ has to be replaced suitably according to eqs. (16), (15), (17).

Per-stream waiting times. Since a closed form of the probability function (or its z -transform) of the unfinished work at arrival instants is not available in the finite buffer case, we use the occupancy multiplied by the deterministic service time D . The moments of the resulting p.f. are *upper bounds* for the moments of the waiting-time. A test-cell of a w -batch, chosen ζ -th and not lost, sees a waiting time which is equal to the sum of the unfinished work at the arrival instant and the service times of those cells of the same batch, which were admitted before the test-cell. $N_i = \sum_{\ell=0}^s P\{W_i = \ell \cdot D\}$, $i = 1, 2$ are normalizing constants. For AF it is

$$P\{W_1 = \ell \cdot D\} = \frac{\mathbf{1}_{\ell=0}^s}{N_1} \left[\sum_{w=1}^{b_{max}^{(1)}} \frac{1}{w} \sum_{\zeta=1}^w \underline{y}_{\ell-\zeta+1} \cdot (\mathbf{D}_w^{(1)} \otimes \mathbf{C}^{(2)})_{\underline{e}} + \sum_{w=2}^{b_{max}^{(1)}} \frac{1}{w} \sum_{\zeta=1}^w \underline{y}_{\ell-\zeta+1} \cdot (\mathbf{D}_{w-1}^{(1)} \otimes \mathbf{D}^{(2)})_{\underline{e}} \right] \quad (21)$$

$$P\{W_2 = \ell \cdot D\} = \frac{\mathbf{1}_{\ell=0}^s}{N_2} \left[\sum_{w=1}^{b_{max}^{(1)}} \frac{1}{w} \sum_{\zeta=1}^w \underline{y}_{\ell-\zeta+1} \cdot (\mathbf{D}_{w-1}^{(1)} \otimes \mathbf{D}^{(2)})_{\underline{e}} \right] \quad (22)$$

and for DF $\underline{y}_{\ell-\zeta+1}$ has to be replaced suitably according to eqs. (16), (15), (17). For the indicator function $\mathbf{1}$ see appendix, list of symbols.

4.1. Modelling priorities

The ATM cell header contains a *single* bit for two different priorities (0 and 1). In case of congestion, cells with priority 1 are protected against loss. Such priority can be modelled for stream 2 in the queue considered above, in that the position of admission of the cell of stream 2 is not chosen according a uniform distribution, but by a distribution that reflects the ratio of cells with priority 1 of stream 2. It is, however, more difficult to incorporate in the model the pushing out of cells of lower priority, that have already joined the queue.

5. The $G_1^{[X]} + G_2/D/1/s$ -queue solved by the unfinished work

Let G_1 and G_2 produce (correlated) interarrival times. If there is a Semi-Markovian representation of the superposition $G_1 + G_2$, it was shown in [11] that the $G_1^{[X]} + G_2/D/1/s$ -queue can also be solved for $D \geq 1$ by looking at the n -th arrival instant and using the

unfinished work U_n seen by this arrival. G_1 produces the arrival instants of the batch stream $G_1^{[X]}$, and the process X of independent identically distributed r.v.'s X_n determines the batch size at the n -th arrival instant. In the superposition $G_1^{[X]} + G_2$, the value of X_n is ignored, if G_2 produces an arrival alone. In order to compute per-stream QOS parameters, the state space E of $G_1 + G_2$ must be given as a decomposition $E = E_1 \cup E_2 \cup E_3$, the states of E_1 [E_2] corresponding to arrival epochs due to G_1 [G_2] and those of E_3 to arrival epochs due to both. $S_n \in E$ denotes the state of the input process at the n -th arrival instant. It is shown in [13] (for AF and DF) that the compound process (U_n, S_n, X_n) is a Markov chain, and its state transition matrix is given. A method is outlined for computing - by means of a *reduced* state space - the probabilities $P\{U_n = k, S_n = i\}$ that the n -th arrival instant sees an unfinished work of k time units with the input process in phase i . Due to the constant service time, $\lceil U_n/D \rceil$ represents the occupancy at this arrival instant, and U_n also determines the actual waiting-time, if the arrival is not lost. Thus both per-stream loss probabilities and waiting-time p.f.'s can be computed. With the SM representation of $\text{DMAP}_1 + \text{DMAP}_2$ with state space decomposition given in [10], it is thus possible to compute for the $\text{DMAP}_1^{[X]} + \text{DMAP}_2/D/1/s$ queue per-stream loss probabilities with *two* completely different methods. The numerical results of the following section were obtained by means of algorithms implemented in C language with **double precision** numbers. The probabilities obtained for both methods in exponential representation only differed in the last four figures of sixteen. Finally, due to the MC (U_n, S_n, X_n) , it is possible to compute a conceptually exact probability that "given stream 2 loses a cell, its next cell is also lost" as outlined in [13]. We do not go into details in order to avoid an overloading of this paper with theory.

6. Numerical results

6.1. Correlation effect on per-stream QOS parameters

First, the influence of correlation in a *single* stream on QOS parameters of *both* streams in the superposition is investigated numerically for a $\text{DMAP}_1^{[X]} + \text{DMAP}_2$ -input. According to [3] $\text{DMAP}_1^{[X]}$ is a DBMAP₁ with $\mathbf{D}_v^{(1)} = x(v)\mathbf{D}^{(1)}$, $\mathbf{D}_0 = \mathbf{C}^{(1)}$, where $\mathbf{C}^{(1)}$, $\mathbf{D}^{(1)}$ determine DMAP₁ and $x(v)$ is the batch size p.f., which is chosen here to be truncated geometrical: $x(v) = \frac{1-0.4}{1-0.4^{b_{\max}}} 0.4^{v-1}$, $1 \leq v \leq b_{\max}$ with $b_{\max} = 8$ (4) representing 8 (4) input lines, switched to the same output line (existing traffic). This p.f. is justified, since the cells of a batch stem from different input lines so that the p.f. $x(v)$ should decrease with growing v , and have a maximum for $v = 1$. For both DMAPs, we use the matrices

$$\mathbf{C}^{(i)} = \begin{pmatrix} 1 - q_1^{(i)} & 0 \\ 0 & 1 - q_2^{(i)} \end{pmatrix} \quad \text{and} \quad \mathbf{D}^{(i)} = \begin{pmatrix} q_1^{(i)} & 0 \\ 0 & q_2^{(i)} \end{pmatrix} \cdot \mathbf{p}^{(i)} \quad \text{for } i = 1, 2$$

since in this case $\mathbf{p}^{(i)} = (\mathbf{I} - \mathbf{C}^{(i)})^{-1}\mathbf{D}^{(i)}$, $i = 1, 2$ is the transition matrix of the MC embedded at arrival instants, and its coefficient of correlation $\kappa^{(i)}$ can be varied as explained in the following. It is $\kappa^{(i)} = 1 - [\mathbf{p}^{(i)}]_{21} - [\mathbf{p}^{(i)}]_{12}$, and $\kappa^{(i)}$ is *proportional* to the coefficient of correlation of the interarrival times, which is by a factor 10 lower than $\kappa^{(i)}$ for the examples here. Using $\underline{P}^{(i)} = (P_1^{(i)}, P_2^{(i)}) = (0.3, 0.7)$ for the state probability vector of the MC, for a given $\kappa^{(i)}$, $\mathbf{p}^{(i)}$ is completely fixed due to $P_1^{(i)}[\mathbf{p}^{(i)}]_{12} = P_2^{(i)}[\mathbf{p}^{(i)}]_{21}$. For further

Fig. 4 Per-stream and full stream loss probabilities over coefficient of correlation of stream 2 for two values of b_{max} , $\eta_1 = 0.3$, $\eta_2 = 0.2$, $s = 30$, $D = 4$ and uncorrelated stream 2.

Fig. 5 Influence of the load ratio on per-stream loss probabilities. $\eta_{1+2} = 0.8$, $D = 4$, $\kappa^{(1)} = \kappa^{(2)} = 0.2$, $s = 30$, $\eta_1 > \eta_2$.

details, see [11]. Fig. 4 shows the influence of correlation in stream 1 (stream 2 uncorrelated). We see that positive correlation in one stream influences loss probabilities of both streams in the superposition negatively. This influence is quite strong for stream 1 (Fig. 4), since it carries batches. For stream 2 the influence was found remarkable only if the values $\eta_i = 1, 2$ of the offered traffic of both streams are similar, but this situation could be given, when a big correlated video source is admitted. For the mean value and mean deviation of the waiting-time, a similar, but weaker influence of correlation was found. The absolute values of loss probabilities of both streams do not differ very much. The coefficient of correlation of the interdeparture times computed with (9), (10), was by a factor of 4 higher than that of the interarrival times.

6.2. Per-stream loss probabilities in dependence of the load ratio

Now, the influence of the ratio of the offered traffic of both streams is investigated. For this, a $\text{DMAP}_1 + \text{DMAP}_2$ -input is used. The offered traffic of the superposition is set to 0.8, and the ratio η_1/η_2 is varied, Fig. 5. It can be seen that loss probabilities do not differ very much whether one stream is stronger than the other or not. With growing ratio η_1/η_2 , loss probability of stream 1 equals that of the full stream.

6.3. Effective Bandwidth

The numerical results of the last subsection make transparent that positive correlation in one of the streams can affect QOS parameters of both streams in the superposition. Therefore a procedure for connection admission should take into consideration correlation. The concept of the so-called "effective bandwidth" [14, 20, 6] takes correlation into account *per definition*: According to [20], $C_{\text{eff}}(P_{\text{loss}}, s)$ of an input process is defined by the service

α	D	P_{loss}
0.006	100	1.13e-10
0.003	200	1.14e-10
0.0015	400	1.16e-10
0.00075	800	1.20e-10
0.0006	1000	1.09e-10
0.0003	2000	1.13e-10

Tab. 1 Loss probabilities for a service rate of $1/D$ for stream 1. $q_1 = 0.9$, $q_2 = 0.3$, $\text{Corr}[A_n, A_{n+1}] = 0.03$, $\sqrt{\text{Var}[A]/E[A]} = 1.55$, $b_{max} = 4$, $E[X] = 1.56$, $s_0 = 30$.

α	D	P_{loss}
0.025	80	1.2e-10
0.02	100	1.3e-10
0.01	200	1.34e-10
0.0025	800	1.39e-10
0.002	1000	1.39e-10
0.001	2000	1.40e-10

Tab. 2 Loss probabilities for a service rate of $1/D$ for stream 2. $q_1 = 0.9$, $q_2 = 0.3$, $\text{Corr}[A_n, A_{n+1}] = 0.03$, $\sqrt{\text{Var}[A]/E[A]} = 1.55$, $s_0 = 30$.

rate of a queue with deterministic service time D and buffer size s , for which the input process suffers from a loss probability of P_{loss} . This service rate $1/D$ can also be expressed in terms of the load $\eta = D/E[A_n]$.

Now, a multiplexer at the entrance of a link between neighbouring switches within a whole route is considered. The multiplexer has a buffer of s_0 places. Using an imaginary queue with s_0 buffer places, the effective bandwidths of both a (given) new connection ($C_{eff}^{(2)}(P_{loss}, s_0)$) and the existing traffic ($C_{eff}^{(1)}(P_{loss}, s_0)$) are determined. If $C_{eff}^{(1)}(P_{loss}, s_0) + C_{eff}^{(2)}(P_{loss}, s_0)$ is not greater than the bitrate of the output line, the new connection can be admitted.

The correctness of this approach is now verified by means of two given streams $\text{DMAP}_1^{[X]}$ and DMAP_2 . By means of the queues $\text{DMAP}_1^{[X]}/D/1/s_0$ and $\text{DMAP}_2/D/1/s_0$ for $P_{loss} < 10^{-9}$ $C_{eff}^{(1)}(P_{loss}, s_0)$ and $C_{eff}^{(2)}(P_{loss}, s_0)$ are determined. Then the superposition feeds the $\text{DMAP}_1^{[X]} + \text{DMAP}_2/D/1/s_0$ -queue and the per-stream loss probabilities are computed, this time with a service rate, that equals $C_{eff}^{(1)}(P_{loss}, s_0) + C_{eff}^{(2)}(P_{loss}, s_0)$. We use a discrete-time system. Let $D = 1$ represent the maximum bitrate of all lines, i.e. 600 Mbit/s. Thus, e.g. $D = 100$ models a bitrate of 6 Mbit/s.

The DMAPs are given by

$$\mathbf{C}_\alpha = \begin{pmatrix} 1 - \alpha q_1 & 0 \\ 0 & 1 - \alpha q_2 \end{pmatrix} \quad \text{and} \quad \mathbf{D}_\alpha = \begin{pmatrix} \alpha q_1 & 0 \\ 0 & \alpha q_2 \end{pmatrix} \cdot \mathbf{p}$$

with $q_1 = 0.9$, $q_2 = 0.3$, and $\mathbf{p}$ was chosen to yield $\text{Corr}(A_n, A_{n+1}) = 0.03$ for both streams. DMAP_1 is modulated by a batch size r.v. X with $b_{max} = 4$ and the same truncated geometrical distribution as in the last section. Varying the parameter α , one gets input streams that differ (only) in their arrival rates. For a given α , the service rate $1/D$ is determined in order to achieve a loss probability smaller than 10^{-9} given the buffer size $s_0 = 30$. Tables 1 and 2 show for given values of α the resulting service rates and loss probabilities.

Table 3 shows the resulting per-stream loss probabilities, if both streams feed the queue with a service rate, that equals the sum of the effective bandwidths of both streams. The results show that per-stream loss probabilities are sometimes even smaller than those that have been demanded for, see Tables 1, 2, 3.

effective bandwidth $1/D$			P_{loss1}	P_{loss2}
Stream 1	Stream 2	Stream1+2		
1/100	1/100	1/50	9.14e-11	3.82e-11
1/200	1/100	1/66	8.16e-11	3.55e-11
		1/67	1.14e-10	5.01e-11
1/100	1/200	1/66	7.22e-11	2.87e-11
		1/67	9.42e-11	3.77e-11
1/400	1/80	1/66	8.87e-11	4.06e-11
		1/67	1.29e-10	6.0e-11
1/400	1/200	1/133	9.99e-11	4.43e-11
		1/134	1.18e-10	5.24e-11
1/400	1/800	1/266	8.49e-11	3.35e-11
		1/267	9.08e-11	3.59e-11
1/800	1/400	1/266	1.01e-10	4.45e-11
		1/267	1.09e-10	4.85e-11
1/1000	1/800	1/444	9.53e-11	3.93e-11
		1/445	9.99e-11	4.13e-11
1/1000	1/1000	1/500	9.39e-11	3.76e-11
1/2000	1/800	1/571	1.09e-10	4.89e-11
		1/572	1.14e-10	5.10e-11

Tab. 3 Per-stream loss probabilities for both streams in the superposition for the $DMAP_1^{(X)} + DMAP_2/D/1/s_0$ -queue with a service rate of $C_{eff}^{(1)} + C_{eff}^{(2)}$, $s_0 = 30$.

For this numerical example, it is verified that per-stream loss probabilities of two superimposed-streams (representing a superposition, here $DMAP_1^{(X)}$, or an individual source, here $DMAP_2$) do not exceed the loss probabilities, that characterize the effective bandwidth of each stream alone, if only the service rate of the queue fed by the superposition of stream 1 and 2 is not smaller than the sum of the effective bandwidths. Enlarging correlation in one stream yields a higher effective bandwidth of that particular source, and the criterion for admission that $C_{eff}^{(1)}(P_{loss}, s_0) + C_{eff}^{(2)}(P_{loss}, s_0)$ must not exceed the rate of the multiplexer's output line, *does* take into account this higher correlation.

Results similar to those of Tables 1, 2, 3 were also found for higher loss probabilities (10^{-4}). This indicates that the effective bandwidth actually represents a simple *additive* estimate that provides a correct decision. So far, work on effective bandwidth only considers infinite queues and very low loss probabilities in the region of 10^{-9} [20, 6].

7. Conclusions

We have analysed here the DBMAP/G/1/s queue for the waiting room policies AF and DF, thus extending several results known from the literature [8][4][17]. By this queueing model per-stream QOS parameters for an arbitrary DBMAP+DMAP input, which is also

a DBMAP can be computed. The G-service process is necessary in order to model service times higher than the time unit; this is necessary, if

- the output bitrate of an ATM MUX (usually next to the receiving terminal equipment) is smaller than that of the input lines,
- the effective bandwidth of a single source or a superposition of sources has to be computed [20],
- the leaky bucket policing mechanism is modelled.

By means of a completely different approach for computing the per-stream-p.f.'s of the occupancy at arrival instants for the $\text{DMAP}_1^{[X]} + \text{DMP}_2/D/1/s$ -queue using *the unfinished work*, the implemented algorithms for computing loss probabilities have been validated numerically, yielding very good conformity.

The model's output stream is shown to be a Semi-Markov process, a closed form expression of the coefficient of correlation of the interdeparture times is presented. This model supports the comparison of different methods for CAC, such as effective bandwidth [14, 20, 6] and the "sigma rule" [22], esp. with respect to loss probability of video sources, which are very difficult to investigate by simulation due to the extremely rare event of loss (cell-loss-rate 10^{-9}). The simulation study in [21] only considers loss probabilities down to 10^{-5} . Work on effective bandwidth serving for CAC [14, 20, 6] so far only considers **infinite** buffer queues, whereas the model presented here deals with a finite system. Effective bandwidth seems to be suitable, since it inherently contains some estimate of the correlation of the interarrival times, the influence of which is illustrated by the numerical results presented here, Fig. 4, whereas the "sigma rule" only considers peak- and mean-bitrates. Finally, Fig. 5 reveals that guaranteeing a loss probability of 10^{-9} for one stream in a superposition *without priorities* implies approximately the same loss probability for all streams in the superposition, unless there is some priority management using VPIs and VCIs for distinguishing different classes.

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Appendix

A. Markov Renewal and Semi-Markov processes

According to Cinlar [5], p.316 a Semi-Markov process (SMP) $\{Y_t; t \in \mathbb{R}_0^+\}$ can be regarded as "another convenient picture in describing" a Markov Renewal process (MEP) $\{(S_n, T_n); n \in \mathbb{N}_0\}$, S_n taking values in the state space $\mathcal{IE} = \{1, \dots, m\}$, T_n representing a random variable of the time instants of changing the state ($T_n \leq T_{n+1}$), where S_n, T_n fulfill the central defining Markov Renewal property:

$$\begin{aligned} & P\{S_{n+1} = j, T_{n+1} - T_n \leq t | S_n, \dots, S_0, T_n, \dots, T_0\} \\ &= P\{S_{n+1} = j, T_{n+1} - T_n \leq t | S_n\}. \end{aligned} \quad (23)$$

The relationship between an MEP and the associated SMP is given by:

$$Y_t = S_n \quad \text{for} \quad T_n \leq t < T_{n+1}.$$

Time homogeneity is supposed, which means:

$$P\{S_{n+1} = j, T_{n+1} - T_n \leq t | S_n = i\} =: Q_{ij}(t) \quad (24)$$

is independent of n . The set of probabilities $\{Q_{ij}(t) : i, j \in \mathbb{I}, t \in \mathbb{R}_0^+\}$ is called a **Semi-Markov kernel** over $\mathbb{I}$. $\{S_n; n \in \mathbb{N}_0\}$ is a Markov chain (MC) with the transition probabilities $p_{ij} = \lim_{t \rightarrow \infty} Q_{ij}(t)$,

$$F_{ij}(t) := \frac{Q_{ij}(t)}{p_{ij}} = P\{T_{n+1} - T_n \leq t | S_{n+1} = j, S_n = i\} \quad (25)$$

is the distribution function of the sojourn time in the state i until changing to state j . (As $p_{ij} = 0$ implies $Q_{ij}(t) = 0$, it is usually defined $F_{ij}(t) = 1$ for $p_{ij} = 0$.) For further details see [5]. Since it is dealt here only with discrete-time MEPs, we use state sojourn time probability functions (p.f.'s)

$$f_{ij}(t) = P\{A_{n-1} = t | S_n = j, S_{n-1} = i\},$$

where $A_{n-1} := T_n - T_{n-1}$. There is the relation

$$q_{ij}(t) := P\{S_{n+1} = j, T_{n+1} - T_n = t | S_n = i\} = p_{ij} f_{ij}(t) \quad (26)$$

We use the $(m \times m)$ -matrices $\mathbf{p} = (p_{ij})_{1 \leq i, j \leq m}$, $\mathbf{q}(t) = (q_{ij}(t))_{1 \leq i, j \leq m}$, and $\mathbf{Q}(t) = (Q_{ij}(t))_{1 \leq i, j \leq m}$. Both $\mathbf{Q}(t)$ and $\mathbf{q}(t)$ are identified with the SM kernel.

Because of the MC, the state sojourn times are, in general, not independent. Joint p.f.'s of state sojourn times $A_n, A_{n+1}, \dots, A_{n+\nu}$, $\nu = 0, 1, 2, \dots$ are given by a matrix product

$$P\{A_n = t_n, \dots, A_{n+\nu} = t_{n+\nu}\} = \underline{P}^{(n)} \cdot \prod_{\mu=0}^{\nu} \mathbf{q}(t_{n+\mu}) \cdot \underline{e} \quad (27)$$

$\underline{P}^{(n)}$ denoting the n -step state probability vector of the embedded MC, $\underline{e} = (1, 1, \dots, 1)^T$ meaning a column vector of 1's with m components. Eq. (27) is a consequence of the "conditional independence" property for MEPs

$$\begin{aligned} P\{A_n = t_n, \dots, A_{n+\nu} = t_{n+\nu} | S_n = i_n, \dots, S_{n+\nu+1} = i_{n+\nu+1}\} \\ = f_{i_n i_{n+1}}(t_n) \cdot \dots \cdot f_{i_{n+\nu} i_{n+\nu+1}}(t_{n+\nu}) \end{aligned} \quad (28)$$

(discrete-time analogue of proposition (10.1.10) in [5], p. 314). In general, (27) is different from the product $\prod_{\mu=0}^{\nu} P\{A_{n+\mu} = t_{n+\mu}\}$ so that the stochastic process of the time epochs of changing the state is non-renewal, i.e. the state sojourn times are not mutually independent.

In order to get joint p.f.'s of the state sojourn times, which are invariant to an arbitrary shift of the index n , stationary state probabilities are assumed for the embedded MC. Whenever the SMP enters a state i , the next state j to be visited is chosen according to p_{ij} and the sojourn time in state i until changing to j according to $f_{ij}(t)$.

If the SMP is used as an input process of a queue, the time instant of entering a next (possibly the same) state is interpreted as an arrival instant. For a DMAP given by the matrices $\mathbf{C}$ and $\mathbf{D}$, one has $\mathbf{q}(t) = \mathbf{C}^{t-1} \mathbf{D}$.

In the MEP of the DBMAP/G/1/ s -queue embedded at departure instants, the time instants of changing the state are represented by the departure instants, and the states are pairs (k, i) , with k denoting the occupancy of the queue at the departure instant and i denoting the phase of the input process at that time instant.

B. List of symbols

DBMAP MEP r.v.; p.f.; d.f.; MC	Discrete Batch Markovian Arrival process Markov Renewal process random variable; probability function; distribution function; Markov chain
$\ell = \lceil x \rceil$ $\mathbf{1}_{\ell=a}^b$; $\mathbf{1}_{\ell=a}$ $[matrix]_{ij}$; $[vector]_i$ $\underline{e}$, resp. $\underline{e}$ $E[\cdot]$; $\text{Var}[\cdot]$	smallest integer ℓ greater than or equal to x Indicator function: $\mathbf{1}_{\ell=a}^b = 1$ for $a \leq \ell \leq b$ and $= 0$ else; $\mathbf{1}_{\ell=a} := \mathbf{1}_{\ell=a}^a$ matrix element of the i -th row and j -th column; i -th vector component $(1, 1, \dots, 1)^T$, m components, resp. as many as necessary from the context mean value; variance
T_n A_n ; A $\mathbf{Q}(t)$ $\mathbf{q}(t)$ $f_{ij}(t)$, resp. $F_{ij}(t)$ $\mathbf{p}$ $\underline{P}$ S_n ; S U_n	time instant of the n -th arrival $A_n := T_{n+1} - T_n$, n -th interarrival time; $A = \{A_n; n \in \mathbb{N}_0\}$ Semi-Markov kernel $[\mathbf{Q}(t)]_{ij} := Q_{ij}(t) = P\{S_{n+1} = j, T_{n+1} - T_n \leq t S_n = i\}$ $[\mathbf{q}(t)]_{ij} = q_{ij}(t) := P\{S_{n+1} = j, T_{n+1} - T_n = t S_n = i\} = p_{ij}f_{ij}(t)$ p.f., resp. d.f. of the sojourn time in state i until changing to state j $= \sum_{t=0}^{\infty} \mathbf{q}(t)$, MC embedded at arrival instants vector of stationary state probabilities at arrival instants $= S(T_n)$ state r.v. of an SMP at the n -th arrival instant; $S = \{S_n; n \in \mathbb{N}_0\}$ unfinished work at the n -th arrival ignoring the arrival
$\tilde{\mathbf{q}}(t)$ $\tilde{\mathbf{q}}$ $h(t)$ $\mathbf{A}_k(t), \mathbf{B}_k(t), \mathbf{C}, \mathbf{D}, \mathbf{D}_v, \hat{\mathbf{D}}$ $[\underline{x}]_i$ π $\underline{x}$ $\mathbf{P}(w, t)$ E_n ; E phase	SM kernel of the MRP embedded at departure epochs (DBMAP/G/1/s queue) $= \sum_{t=1}^{\infty} \tilde{\mathbf{q}}(t)$ probability function of the service time $m \times m$ matrices describing the DBMAP/G/1/s queue $= P\{\text{departure leaves } k \text{ customers in the system, input process in phase } i\}$ stationary phase probability vector $\pi \hat{\mathbf{D}} = \pi$ AF: $= (\underline{x}_0, \dots, \underline{x}_s)$, DF: $= (\underline{x}_0, \dots, \underline{x}_{s+1})$ counting function of the DBMAP n -th interdeparture time; $E = \{E_n; n \in \mathbb{N}_0\}$ in case of the DBMAP equal to state
m ; $\mathcal{IE}$ s $\mathcal{S}$ X_n ; $x(v)$; b_{max} $D \geq 1$	count of states of an SMP or phases of a DBMAP; $\mathcal{IE} = \{1, 2, \dots, m\}$ buffer size without server $= s$ for AF and $= s + 1$ for DF batch size r.v.; p.f. of the batch size; maximum number of cells of a batch deterministic service time in time units

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