Modelling the Reassembly Buffer in a Connectionless Server

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One way of providing connectionless services over an ATM network is by means of Connectionless Servers (CLSs), routing and forwarding cells on a per packet basis (message mode). Packets are sent to a CLS over ATM connections (either virtual channel or virtual path connections). These packets come from other CLSs, interworking units, or end systems. The CLS stores cells of a packet in a reassembly buffer until the last one has been received, and forwards all cells of the packet at once. We present a performance model of the reassembly buffer in a CLS operating in message mode.

We model the arrival process to the reassembly buffer as a Discrete-time Markovian Arrival Process which is modulated by the number of active incoming ATM connections. We assume that at most m connections can be active at a certain moment, and that an active connection generates a cell in a certain time slot with probability $1/m$. Furthermore, we assume that a cell is the last one of a packet with probability p .

For the system model, we assume a finite reassembly buffer with a length of n cells. The number of cells in the buffer after a cell arrival depends on the number of cells in the buffer before the arrival and the number of active connections. We obtain a discrete-time Markov process $\{A_t, N_t\}$, where A_t is the number of active connections and N_t is the number of cells in the reassembly buffer at the beginning of time-slot t . We solve the Markov process for the stationary probability distribution, and derive expressions for performance measures such as packet loss probability.

In order to reduce the computational complexity of the model, we apply model decomposition techniques. We are able to approximate performance measures accurately for systems of a realistic size.

Numerical results show the accuracy of the analysis, and its applicability to reassembly buffer dimensioning.

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1. INTRODUCTION

Communications in networks based on the Asynchronous Transfer Mode (ATM) has a connection-oriented nature. However, many present and future applications require a connectionless network service, e.g., TCP/IP based applications. In order to support these applications, extra entities, called Connectionless Servers (CLSs), can be placed in the network ([1], [2], [3]). These entities route and forward cells belonging to packets from a connectionless network protocol from an incoming to an outgoing ATM connection. The forwarding can be done either on a per cell basis (streaming mode) or on a per packet basis (message mode). We concentrate on the latter case.

The CLS maintains a number of ATM connections (either virtual path, or virtual channel connections) from and to other CLSs, interworking units, and end systems. On the incoming connections, packets, consisting of several ATM cells, arrive. We assume that at most one packet is being sent per connection at a time, i.e., no interleaving takes place on a connection. The arriving cells are stored in the reassembly buffer until a packet has been completely received. As soon as the last cell of the packet arrives, all cells of that packet are forwarded and removed from the reassembly buffer.

Other functions such as routing and traffic shaping for the outgoing ATM connection have to be performed in the CLS as well. Routing can be performed in parallel with the packet reassembly since the routing information will be carried in the first cell of the packet. The traffic shaping function assures that the agreed bandwidth of the outgoing ATM connection is not exceeded. Traffic shaping can only start as soon as the entire packet is ready for transmission on the outgoing ATM connection, i.e., at the moment it is forwarded from the reassembly buffer.

In this paper we only consider the behaviour of the reassembly buffer. In order to be able to dimension the reassembly buffer properly, we model the buffer in such a way that we can obtain the packet loss probability given a certain buffer size, and a certain load on the buffer. This allows us to dimension the reassembly buffer properly, depending on the load to the buffer.

Related work on message reassembly ([4], [5]) has a more general nature, and is not oriented towards the application to the packet reassembly problem in a CLS. Therefore, we are able to come with more detailed and accurate results.

This paper is structured as follows. First, in Section 2 we model the arrival process to the CLS. In Section 3 we model the reassembly buffer, using the arrival model. Next, in Section 4 we derive performance measures from our model. In Section 5, we apply model decomposition techniques to the proposed model in order to decrease its computational complexity. In Section 6 we demonstrate the accuracy of our model and give numerical results. Finally, in Section 7 we draw some conclusions.

2. THE ARRIVAL PROCESS

We assume that the arrival process to the reassembly buffer is slotted. Time is divided into equal sized slots. Since cells arrive from an ATM link, at most one cell arrives in each slot. The arrival is assumed to be completed at the end of the slot. Furthermore, we only consider the arrival stream at slot boundaries, i.e., we assume discrete time. The cell arrival process that we will describe in this section is also known as a Discrete-time Markovian Arrival Process (D-MAP) ([6]).

The cells arriving on a connection constitute the packets that should be reassembled. We assume that the packet length is geometrically distributed with a mean of $1/p$ cells, i.e., an arriving cell is with probability p the last one of a packet.

The total number of incoming connections in the CLS is denoted as m . Let the number of active connections at the beginning of time-slot t be defined by the stochastic process $\{A_t, t=(0, 1, 2, \dots), A_t \in \{0, 1, \dots, m\}\}$. We assume that A_t is a Discrete-Time Markov Chain (DTMC) with the following transition matrix (D):

$$D = [d_{ij}]_{(m+1) \times (m+1)} \quad (1)$$

Clearly, d_{ij} denotes the probability that j connections are active at the end of a slot[‡], given that i connections were active at the beginning of the slot. The elements of the transition matrix are defined as follows:

$$D = \begin{bmatrix} 1 - mr & mr & 0 & \dots & 0 & 0 & 0 \\ s & 1 - (m-1)r - s & (m-1)r & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & (m-1)s & 1 - r - (m-1)s & r \\ 0 & 0 & 0 & \dots & 0 & ms & 1 - ms \end{bmatrix} \quad (2)$$

Now we define d_{ij}^0 as the probability that a time-slot is not filled and at the end j connections are active, given that at the beginning of the time-slot, i connections were active. Similarly, we define d_{ij}^1 as the probability that a time-slot is filled with a cell and at the end of the time-slot j connections are active, given that at the beginning of the slot i connections were active. We can also put this in matrix form:

$$D^0 = [d_{ij}^0]_{(m+1) \times (m+1)}, \text{ and} \quad (3)$$

[‡] Since the end of a slot coincides with the beginning of the next slot, we use both terms to refer to this slot boundary.

$$D^1 = [d_{ij}^1]_{(m+1) \times (m+1)} . \quad (4)$$

It can easily be seen that

$$D = D^0 + D^1 . \quad (5)$$

In general, an arriving cell will originate from one of the active connections. However, it could also originate from one of the silent connections that becomes active during the slot. We furthermore distinguish between cells that are the last one of a packet and cells that are not. We thus define matrices

$$D^{al} = [d_{ij}^{al}]_{(m+1) \times (m+1)} , \text{ and} \quad (6)$$

$$D^{an} = [d_{ij}^{an}]_{(m+1) \times (m+1)} , \quad (7)$$

where the (i, j) -th element is the probability that a cell from one of the active connections arrives that is (*al*) or is *not* (*an*) the last one of a packet respectively, while the modulating Markov process makes a transition from state i to state j . Furthermore, we define

$$D^{sl} = [d_{ij}^{sl}]_{(m+1) \times (m+1)} , \text{ and} \quad (8)$$

$$D^{sn} = [d_{ij}^{sn}]_{(m+1) \times (m+1)} , \quad (9)$$

where the (i, j) -th element is the probability that a cell from one of the silent connections arrives that is (*sl*) or is *not* (*sn*) the last one of a packet respectively, while the modulating Markov process makes a transition from state i to state j . Clearly, we have

$$D^1 = D^{al} + D^{an} + D^{sl} + D^{sn} . \quad (10)$$

Let us now define the elements of these matrices.

D^0 : We first observe that a change of the state of a connection coincides always with the arrival of a cell for that connection. A connection that becomes active starts sending a packet immediately. A connection becoming silent is just finishing sending a packet, i.e., the last cell of the packet arrives in the CLS. This implies that the state of the arrival process will not change if no cell arrives, i.e.,

$$d_{ij}^0 = 0 \quad (i \neq j, 0 \leq i, j \leq m) . \quad (11)$$

If i connections are active, the probability that one of them sends a cell equals i/m . Furthermore, we define the probability that one of the silent connections becomes active and hence sends a cell as δ_i . Thus we can state for the probability that no cell arrives while i connections are active:

$$d_{ii}^0 = 1 - \frac{i}{m} - \delta_i \quad (0 \leq i \leq m) . \quad (12)$$

D^{al} : From Eq. 2, it can be seen that a connection becomes silent with probability is . This can only occur if a last cell from one of the active connections arrives, i.e.,

$$d_{ij}^{al} = is \quad (0 \leq j = i - 1 \leq m - 1) . \quad (13)$$

The probability of a cell arrival from one of the active connections is i/m . Recalling that the probability of a cell being the last one of a packet has been defined as p , and taking Eq. 13 into account, we define

$$d_{ii}^{al} = p \frac{i}{m} - is \quad (0 \leq i \leq m) . \quad (14)$$

D^{an} : Similarly, we can define for the probability of the arrival of a cell which is not the last one of a packet:

$$d_{ii}^{an} = (1 - p) \frac{i}{m} \quad (0 \leq i \leq m) . \quad (15)$$

D^{sl} : As stated before, the probability that a cell from one of the silent connections arrives is defined as δ_i . With probability ms the new connection becomes immediately silent again, i.e.,

$$d_{ii}^{sl} = ms\delta_i \quad (0 \leq i \leq m) . \quad (16)$$

The probability that a silent connection becomes active because of the arrival of a cell which is also the last (and only) one of a packet is

$$d_{ij}^{sl} = (p - ms)\delta_i \quad (1 \leq j = i + 1 \leq m) . \quad (17)$$

D^{sn} : Finally, the probability that a silent connection becomes active because of the arrival of a cell which is not the last one of a packet is

$$d_{ij}^{sn} = (1-p)\delta_i \quad (1 \leq j = i+1 \leq m) . \quad (18)$$

All other elements of D^{al} , D^{an} , D^{sl} , and D^{sn} are zero.

We can derive the probability of a connection becoming active, δ_i , by recognizing that it equals the probability of a connection becoming active and staying active plus the probability of a connection becoming active and becoming silent again. These probabilities have been given in Eq. 2 and Eq. 16 respectively, so that

$$\delta_i = (m-i)r + \delta_i ms . \quad (19)$$

Consequently, we can derive

$$\delta_i = \frac{(m-i)r}{1-ms} \quad (0 \leq i \leq m) . \quad (20)$$

Now we have completely specified the arrival process. We have specified the transition probabilities between the states of the modulating DTMC, and the probabilities of the events that can occur at these transitions. Such an event is the arrival of a cell, which is or is not the last one of a packet.

The arrival process can be interpreted as follows. A certain time-slot is assigned randomly to one of the m incoming connections. If this connection is active at the beginning of the slot, it fills the slot with a cell. Furthermore, it can become silent during the time-slot with probability ms . If the connection is silent at the beginning of the time-slot, it can become active during the time-slot, stay active, and fill the time-slot with a cell with probability mr . Hence, a silent connection has a probability r to become active and stay active in an arbitrary time-slot, because it can only become active when the slot has been assigned to it, which happens with probability $1/m$. Similarly, an active connection has a probability s to become silent in an arbitrary time-slot. Finally, an active connection has a probability $1/m$ to send a cell in an arbitrary time-slot.

Let π^D be the stationary state probability vector of the underlying DTMC of the arrival process, i.e.,

$$\pi^D D = \pi^D , \text{ and} \quad (21)$$

$$\pi^D \bar{e} = 1 , \quad (22)$$

where $\bar{e}$ is a column vector of ones. The i -th element of π^D , π_i^D , is similar to the stationary probability for the continuous time M/M/ ∞ /M queue (see [7]: Section 3.9):

$$\pi_i^D = \frac{\left(\frac{r}{s}\right)^i \binom{m}{i}}{(1 + r/s)^m} . \quad (23)$$

Finally, we define the fundamental cell arrival probability p_{arrival} as the probability that a cell arrives in an arbitrary slot. It is given by

$$p_{\text{arrival}} = \pi^D D^1 \bar{e} . \quad (24)$$

Eq. 24 can be interpreted as the sum of the probabilities of all transitions that correspond to a cell arrival. It can not be easily simplified, because of the terms for arrivals from silent connections that become active.

3. THE SYSTEM MODEL

In the previous section, we have defined the probabilities of making a certain transition in the modulating arrival process, and having a cell arrival of a certain type (a cell from one of the active, or one of the silent connections, which is, or is not the last one of a packet) or having no arrival. In this section we describe what happens to the reassembly buffer if such a transition occurs. We assume that the buffer can contain at most n cells.

The state of the buffer can exactly be defined by the number of cells that are stored for each of the m connections. For silent connections, this number will be 0, for active connections, this number corresponds to the number of cells received from the packet currently being reassembled. Indeed, we have modelled the system in this way. However, we have specified and analysed this so-called *overall model*, using Stochastic Activity Networks, and the software tool UltraSAN ([8]), which allows for a description at a higher level. The problem of the overall model is its state space, which has a size of order $O(m^n)$.

In an attempt to reduce the state space of our model, we lump all those states that have the same total number of cells in the buffer. By this lumping, we loose the Markovian property of our model, i.e., the probability of a certain transition is no longer totally independent of previous states. However, we approximate the model as if this property was still valid. In fact only few transitions in the model do depend on previous states the system was in. In Section 6, we will compare results from this so-called *lumped model* with results from the overall model, to investigate the accuracy of the approximation.

The lumped model is described by the DTMC $\{A_t, N_t\}$, where A_t is defined as in the previous section, and $\{N_t, t=(0, 1, 2, \dots)\}$ gives the number of cells in the buffer at the beginning of time-slot t . Furthermore, $N_t = 0$ if $A_t = 0$, and $N_t \in \{0, 1, \dots, n\}$ if $A_t > 0$. The number of states in the lumped model equals $m(n+1) + 1$, i.e., it is of order $O(mn)$.

Let the states of this DTMC be ordered lexicographically $((0,0), (1,0), (1,1), \dots, (1,n), \dots, (m,0), (m,1), \dots, (m,n))$. Then the transition matrix T of the model can be written as follows:

$$T = \begin{bmatrix} t_{00} & t_{01} & \dots & t_{0j} & \dots & t_{0m} \\ i_{10} & T_{11} & \dots & T_{1j} & \dots & T_{1m} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ i_{i0} & T_{i1} & \dots & T_{ij} & \dots & T_{im} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ i_{m0} & T_{m1} & \dots & T_{mj} & \dots & T_{mm} \end{bmatrix}. \quad (25)$$

Here, t_{00} is the probability that the buffer is empty at the end of a slot with all connections silent, given that the buffer was empty at the beginning of the slot and that all connections were silent. It can be expressed in the transitions of the arrival process (Section 2) as follows:

$$t_{00} = d_{00}^0 + d_{00}^{sl}. \quad (26)$$

t_{0j} is a row vector of which the l -th element ($0 \leq l \leq n$) gives the probability that at the end of a slot l cells are in the buffer and j connections are active, given an empty buffer and no active connections at the beginning of the slot. This can only happen if a cell from one of the silent connections arrives. If the cell is the last one of a packet, the buffer remains empty. Otherwise the buffer will contain a single cell at the end of the slot, i.e.,

$$t_{0j} = d_{0j}^{sl} [1 \ 0 \ 0 \ \dots \ 0] + d_{0j}^{sn} [0 \ 1 \ 0 \ \dots \ 0]. \quad (27)$$

i_{i0} is a column vector of which the k -th element ($0 \leq k \leq n$) defines the probability that at the end of a slot the buffer is empty and no connections are active, given k cells in the buffer and i active connections at the beginning of the slot. Since this can only happen upon arrival of a cell from one of the active connections which is the last one of a packet, the column vector can be defined as

$$i_{i0} = d_{i0}^{al} \bar{e}, \quad (28)$$

where $\bar{e}$ is again a column vector of ones.

T_{ij} is an $(n+1) \times (n+1)$ matrix of which the (k,l) -th element defines the probability that the buffer contains l cells and j connections are active at the end of a slot, given k cells in the buffer and i active connections at the beginning of the slot. We define this matrix by

$$T_{ij} = d_{ij}^0 B_i^0 + d_{ij}^{al} B_i^{al} + d_{ij}^{an} B_i^{an} + d_{ij}^{sl} B_i^{sl} + d_{ij}^{sn} B_i^{sn} , \quad (29)$$

where B_i^0 is a probability matrix that describes the transition probabilities of the buffer contents (N_i), given that no cell arrives in a slot, and i connections are active at the beginning of the slot. Similarly B_i^{al} , B_i^{an} , B_i^{sl} , and B_i^{sn} describe the transition probabilities of the buffer contents given the arrival of a cell of the corresponding type and i active connections at the beginning of the slot. In conclusion, T_{ij} can be interpreted as the sum of a number of terms, where each term consists of two factors; the first one to express the probability of a certain event (e.g., the arrival of a cell of a certain type), and the second one to express the impact of that event on the buffer contents.

Let us now define the elements of the B -matrices.

B_i^0 : Since the buffer contents do not change if no cell arrives, we define

$$B_i^0 = I \quad (1 \leq i \leq m) , \quad (30)$$

where I is the $(n+1) \times (n+1)$ identity matrix.

B_i^{sl} : Since a cell arriving from one of the silent connections is always the first one of a packet, the buffer contents does not change if it is also the last one (a single-cell packet),

$$B_i^{sl} = I \quad (1 \leq i \leq m) . \quad (31)$$

B_i^{sn} : If the cell arriving from one of the silent connections is not the last one of a packet, it is added to the buffer, unless the buffer is already full, i.e.,

$$B_i^{sn} = \begin{bmatrix} 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & 0 & \dots & 1 \end{bmatrix} \quad (1 \leq i \leq m) . \quad (32)$$

B_i^{al} : For the arrival of a cell of one of the active connections, things are a bit more complicated. Let us first consider the case of only one active connection at the beginning of the slot. If a cell arrives, which is the last one of a packet, the buffer will be emptied, because all the cells in the buffer belonged to that single packet, i.e.,

$$B_1^{al} = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 1 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & \dots & 0 \end{bmatrix}. \quad (33)$$

If several connections are active, it is not known from the state of our model how many cells are removed from the buffer upon arrival of the last cell of a packet. All other cells belonging to that packet, which are already in the buffer, will be removed upon the arrival. However, we lost the information concerning the size of the packet by lumping all states which had the same total number of cells in the buffer. In the sequel we will determine the probability distribution for this size, and consequently for the number of cells to be removed from the buffer.

For each of the i active connections, the number of received cells since (including) the last one of the previous packet has a geometric distribution with parameter p . So, assuming that no cells have been lost, for each active connection, the number of cells in the buffer plus one has this distribution. Consequently, the total number of cells in the buffer plus i has a Pascal distribution with parameters p and i , since the sum of a number of geometrically distributed random variables has a Pascal distribution (see [9]: Section 29.14). Let us define the random variable denoting the number of cells in the buffer that are to be removed because of the arrival of the last cell of a packet as X . Furthermore, let us define the random variable denoting the number of cells in the buffer originated by i connections as Y_i . Now we can state for the probability that l cells remain in the buffer after the arrival of a last cell from one of the active connections, given k cells in the buffer before the arrival and i active connections, i.e., for the (k,l) -th element of B_i^{al} :

$$b_{ikl}^{al} = P\{X = k-l | Y_i = k\} = \frac{P\{X = k-l, Y_i = k\}}{P\{Y_i = k\}} \quad (0 \leq l \leq k \leq n). \quad (34)$$

By conditioning on X , we get

$$b_{ikl}^{al} = \frac{P\{Y_i = k | X = k-l\} P\{X = k-l\}}{P\{Y_i = k\}}. \quad (35)$$

By recognizing that Y_i is the sum of X and the number of cells in the buffer generated by $i - 1$ other connections, i.e., $Y_i = Y_{i-1} + X$, we can derive

$$b_{ikl}^{al} = \frac{P\{Y_i - X = l\} P\{X = k - l\}}{P\{Y_i = k\}} = \frac{P\{Y_{i-1} = l\} P\{X = k - l\}}{P\{Y_i = k\}}. \quad (36)$$

Let us now modify this expression by adding constants to both sides of the equalities:

$$b_{ikl}^{al} = \frac{P\{Y_{i-1} + (i - 1) = l + (i - 1)\} P\{X + 1 = k - l + 1\}}{P\{Y_i + i = k + i\}}. \quad (37)$$

By recalling that $X + 1$ has a geometric distribution with parameter p , $Y_i + i$ has a Pascal distribution with parameters p and i , and $Y_{i-1} + (i - 1)$ has a Pascal distribution with parameters p and $i - 1$, we can derive from Eq. 37:

$$b_{ikl}^{al} = \frac{\left(\binom{l+i-2}{i-2} p^{i-1} (1-p)^l\right) ((1-p)^{k-l} p)}{\left(\binom{k+i-1}{i-1} p^i (1-p)^k\right)} = \frac{\binom{l+i-2}{i-2}}{\binom{k+i-1}{i-1}}. \quad (38)$$

Of course, the number of cells in the buffer will never increase upon arrival of a cell which is the last one of a packet, i.e.,

$$b_{ikl}^{al} = 0 \quad (0 \leq k < l \leq n). \quad (39)$$

Now we are able to define the matrix B_i^{al} in terms of its elements:

$$B_i^{al} = [b_{ikl}^{al}]_{(n+1) \times (n+1)} \quad (2 \leq i \leq m, 0 \leq k, l \leq n). \quad (40)$$

B_i^{an} : For the specification of B_i^{an} , we need to observe that an arriving cell is stored in the buffer if it is not the last one of a packet, and the buffer is not already full. If the arriving cell finds the buffer full, all cells belonging to the same packet are removed from the buffer. Because of the geometric, and hence memoryless, distribution of the packet length, this implies that the distribution of the number of cells to be removed from the buffer is in this case as if the arriving cell was the last one of a packet, i.e.,

$$B_1^{an} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ 1 & 0 & 0 & \dots & 0 \end{bmatrix}, \text{ and} \quad (41)$$

$$B_i^{an} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ b_{in0}^{al} & b_{in1}^{al} & b_{an2}^{al} & \dots & b_{inn}^{al} \end{bmatrix} \quad (2 \leq i \leq m) . \quad (42)$$

This concludes the specification of the DTMC that describes the lumped model. Let us now continue with the analysis of the model.

Let π^T be the stationary state probability vector of the DTMC. It can be found by solving for

$$\pi^T T = \pi^T , \quad (43)$$

and

$$\pi^T \bar{e} = 1 . \quad (44)$$

π^T has the following form:

$$\pi^T = \left[\pi_0^T \ \pi_1^T \ \dots \ \pi_i^T \ \dots \ \pi_m^T \right] , \quad (45)$$

where π_0^T is a scalar, denoting the probability that all connections are silent, and hence the buffer is empty. π_i^T is a vector of length $n + 1$, of which the k -th element (π_{ik}^T) denotes the probability that i connections are active and the buffer contains k cells.

4. ANALYSIS

Having obtained the stationary state probability vector of the model, we are able to derive performance measures from our model. The distribution of the buffer occupation can easily be obtained. This distribution (π^B) can be defined as follows:

$$\underline{\pi}^B = [\pi_0^B \pi_1^B \dots \pi_k^B \dots \pi_n^B], \text{ with} \quad (46)$$

$$\pi_0^B = \pi_0^T + \sum_{i=1}^m \pi_{i0}^T, \text{ and} \quad (47)$$

$$\pi_k^B = \sum_{i=1}^m \pi_{ik}^T \quad (1 \leq k \leq n) . \quad (48)$$

The measure, we aim at in the modelling and analysis of the reassembly buffer is the packet loss probability. It allows us to dimension the reassembly buffer in a proper way. Packet loss occurs when an arriving cell of the packet cannot be stored any more because the buffer is full. The other cells of the packet in the buffer are then also removed.

If a cell which is not the last one of a packet, arrives when the buffer is full, a batch of cells is removed from the buffer, i.e., is lost. In our model, after the loss of a batch of cells, which will actually result in packet loss, new cells of the packet will be stored in the buffer until the last cell of the packet arrives. This implies that another batch of cells of the same packet can be lost.

Let the random variable L denote the number of lost batches for an arbitrary arriving packet. We determine the expected value of L by conditioning on the number of active connections (A_t):

$$\begin{aligned} E[L|A_t = i] &= E\left[\frac{\text{nr. of batch losses per cell arrival}}{\text{nr. of packet arrivals per cell arrival}} \middle| A_t = i\right] \\ &= \frac{E[\text{nr. of batch losses per cell arrival}|A_t = i]}{E[\text{nr. of packet arrivals per cell arrival}|A_t = i]} . \end{aligned} \quad (49)$$

Since both the number of batch losses and the number of packet arrivals per cell arrival are either 0 or 1, Eq. 49 can be rewritten as

$$E[L|A_t = i] = \frac{P\{\text{batch loss upon cell arrival}|A_t = i\}}{P\{\text{packet arrival upon cell arrival}|A_t = i\}} . \quad (50)$$

As said above, a batch is lost if a cell which is not the last one of a packet arrives when the buffer is full. Hence, Eq. 50 can be rewritten as

$$\begin{aligned}
 E[L|A_t = i] &= \frac{P\{(\text{cell finds buffer full} \wedge \text{cell is not the last one of a packet})|A_t = i\}}{P\{\text{packet arrival upon cell arrival}|A_t = i\}} \\
 &= \frac{P\{\text{cell finds buffer full}|A_t = i\} P\{\text{cell is not the last one of a packet}|A_t = i\}}{P\{\text{packet arrival upon cell arrival}|A_t = i\}} . \quad (51)
 \end{aligned}$$

Recall that the probability of a cell being the first or last one of a packet equals p , and is independent of the number of active connections. This implies that Eq. 51 can be rewritten as

$$E[L|A_t = i] = \frac{1-p}{p} P\{\text{cell finds buffer full}|A_t = i\} . \quad (52)$$

If no connections become active or silent, cells arrive according to a Bernoulli process. It is known that Bernoulli arrivals see time averages (BASTA), so that Eq. 52 can be written as

$$E[L|A_t = i] = \frac{1-p}{p} P\{\text{buffer full}|A_t = i\} = \frac{1-p}{p} \frac{P\{\text{buffer full} \wedge A_t = i\}}{P\{A_t = i\}} . \quad (53)$$

Let $\underline{p}_{\text{full}}$ be a vector of which the i -th element ($0 \leq i \leq m$) is the probability that the buffer is full, and i connections are active. $\underline{p}_{\text{full}}$ can be defined as follows:

$$(\underline{p}_{\text{full}})_0 = 0 , \text{ and} \quad (54)$$

$$(\underline{p}_{\text{full}})_i = \pi_{in}^T \quad (1 \leq i \leq m) . \quad (55)$$

Substituting $\underline{p}_{\text{full}}$ in Eq. 53 yields

$$E[L|A_t = i] = \frac{1-p}{p} \frac{(\underline{p}_{\text{full}})_i}{P\{A_t = i\}} . \quad (56)$$

Conditioning on the number of active connections yields $E[L]$, the expected number of lost batches for an arbitrary arriving packet. Since L is concerned with arriving packets, we only consider those slots in which packets arrive:

$$E[L] = \sum_{i=0}^m E[L|A_t = i] P\{A_t = i | \text{packet arrival}\} . \quad (57)$$

This can be rewritten as

$$E[L] = \sum_{i=0}^m E[L|A_t = i] \frac{P\{\text{packet arrival}|A_t = i\} P\{A_t = i\}}{P\{\text{packet arrival}\}} . \quad (58)$$

Every cell arrival is with probability p a packet arrival, regardless of the number of active connections, i.e.,

$$E[L] = \sum_{i=0}^m E[L|A_t = i] \frac{p P\{\text{cell arrival}|A_t = i\} P\{A_t = i\}}{p P_{\text{arrival}}} . \quad (59)$$

Substituting Eq. 56, and recognizing that the arrival probability, given i active connections, is by definition given by the sum of all elements on the i -th row of the matrix D^1 , yields

$$E[L] = \sum_{i=0}^m \frac{1-p}{p} (p_{\text{full}})_i \frac{P\{\text{cell arrival}|A_t = i\}}{P_{\text{arrival}}} = \frac{1-p}{p} \frac{p_{\text{full}} D^1 \bar{e}}{P_{\text{arrival}}} . \quad (60)$$

The probability that an arbitrary packet is lost ($p_{\text{packet loss}}$), can be obtained by conditioning:

$$p_{\text{packet loss}} = P\{L \geq 1\} = \frac{E[L]}{E[L|L \geq 1]} . \quad (61)$$

We assume that all arriving cells have an equal probability of causing the loss of a batch of cells. Hence, because of the memoryless property of the packet length distribution, a packet that has already lost a batch remains to have the same probability of losing another batch of cells, i.e.,

$$E[L|L \geq 1] = 1 + E[L] . \quad (62)$$

Substituting Eq. 62, and Eq. 60 in Eq. 61 yields the packet loss probability:

$$p_{\text{packet loss}} = \frac{(1-p) p_{\text{full}} D^1 \bar{e}}{p P_{\text{arrival}} + (1-p) p_{\text{full}} D^1 \bar{e}} . \quad (63)$$

5. MODEL DECOMPOSITION

The model described in Section 3 has a state space of $m(n+1) + 1$ states. A large computational effort is required to solve its balance equations (Eq. 43 and Eq. 44). In the sequel, we will show how the stationary state probability vector of this matrix can be approximated by solving m systems of equations of size $n+1$, and one of size $m+1$. We make use of a property of our model that is called near-complete decomposability, and come up with a new model, which will be referred to as *decomposed model*.

If the values of r and s are small compared to $1/m$, the matrix T is nearly completely decomposable ([10]). This means that it can be expressed as

$$T = T^* + \varepsilon E, \quad \varepsilon \ll 1, \quad (64)$$

where T^* is a stochastic matrix of block diagonal form, i.e.,

$$T^* = \begin{bmatrix} t_0^* & \bar{0} & \dots & \bar{0} & \dots & \bar{0} \\ \bar{0} & T_1^* & \dots & \bar{0} & \dots & \bar{0} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \bar{0} & \bar{0} & \dots & T_i^* & \dots & \bar{0} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \bar{0} & \bar{0} & \dots & \bar{0} & \dots & T_m^* \end{bmatrix}. \quad (65)$$

Furthermore, E is a matrix with zero row sums. The elements of E that fall within the diagonal blocks defined in the structure of T^* are all non-positive, the elements outside these blocks are non-negative. All elements of E have an absolute value less than or equal to one. We have thus approximated T by $m+1$ subsystems with transition matrix T_i^* , of which the states can be interpreted as describing the number of cells in the buffer given i active connections. Because of the definition of T , the probabilities of transition between subsystems can be obtained from the transition matrix of the modulating arrival process (D).

Let π_i^* be the stationary state probability vector of a DTMC with transition matrix T_i^* , i.e.,

$$\pi_i^* T_i^* = \pi_i^*, \text{ and} \quad (66)$$

$$\pi_i^* \bar{e} = 1. \quad (67)$$

The stationary probability vector of our lumped model ($\underline{\pi}^T$) can then be approximated by

$$\underline{\pi}_i^T \approx \pi_i^D \underline{\pi}_i^* \quad (1 \leq i \leq m) , \text{ and} \quad (68)$$

$$\pi_0^T \approx \pi_0^D , \quad (69)$$

i.e., the probability of having i active connections and k cells in the buffer can be approximated by the product of the probability of i active connections and the approximation of the probability of k cells in the buffer given i active connections.

The error of this approximation is known to go to zero with ε ([10]).

Let us now define the constituting matrices of T^* . As stated above, T_i^* can be interpreted as the transition matrix of a DTMC of which the states are describing the number of cells in the buffer, given i active connections. In this perspective, it seems to be an appropriate choice to define T_i^* in the same way as T_{ii} (Eq. 29), taking into account the fact that no connections become active or silent. Hence, compared to Eq. 29, the last two terms should be omitted, because they correspond to a transition where a connection becomes silent. Similarly, the coefficients of the B -matrices should be slightly adjusted. We define T_i^* as follows for $1 \leq i \leq m$:

$$T_i^* = \left(1 - \frac{i}{m} \right) B_i^0 + \frac{i}{m} p B_i^{al} + \frac{i}{m} (1-p) B_i^{an} . \quad (70)$$

This can be interpreted as follows. T_i^* is a matrix of which the (k, l) -th element specifies the probability that the buffer contains l cells at the end of a slot, given that it contained k cells at the beginning of the slot, and given that i connections are active, that no silent connections become active, and that no active connections become silent. With probability i/m a cell arrives, which is the last one of a packet with probability p , and which is not with probability $1-p$.

For the border case of $i = 0$, we have to define a 1×1 matrix, of which the only element should be one in order to have a stochastic matrix, i.e.,

$$t_0^* = 1 . \quad (71)$$

So, if no connections are active, and no silent connections become active, the buffer will remain empty with probability 1.

We have decomposed the lumped model according to the time scale at which transitions take place. The modulation of the arrival process is relatively slow,

because of the small values that are expected for r and s . The rates at which the buffer contents change on the other hand are relatively high.

Let us now determine a value of ϵ , for which Eq. 64 holds. By realizing that all elements of the B -matrices are smaller than or equal to 1, and that the coefficients in Eq. 29 sum to d_{ij} , it can be easily seen that

$$M(T_{ij}) \leq d_{ij} , \quad (72)$$

where $M(A)$ denotes the maximum of the absolute values of the matrix elements, i.e., $M(A) = \max_{i,j}(|a_{ij}|)$. Similarly, it can be seen that

$$M(T_i^* - T_{ii}) \leq 1 - d_{ii} , \quad (73)$$

since the coefficient in Eq. 70 sum to 1. Hence, the maximum difference between the elements of T^* and T can be expressed as follows:

$$M(T^* - T) \leq M(D - I) . \quad (74)$$

It can easily be seen from Eq. 2 that the element of $D - I$ with the largest absolute value is either $m \times r$ or $m \times s$. Therefore, if

$$\epsilon = m \times \max(r, s) , \quad (75)$$

a matrix E can be found such that Eq. 64 is valid. So, Eq. 69 is a good approximation if $1/r$ and $1/s$ are some orders of magnitude larger than m , i.e., the probability that a connection becomes active or silent in a time slot is sufficiently low. Indeed, this was a starting point of the analysis.

We adapt the definition of the packet loss probability slightly, so that it matches the decomposed model. Using the decomposition, Eq. 55 changes to

$$(p_{\text{full}}^*)_i = \pi_i^D \pi_{in}^* , \quad (76)$$

where π_{0n}^* is assumed to be zero. This is the probability that the buffer is full, given that the arrival process is in state i , times the probability that the arrival process is in state i . Furthermore, the fundamental cell arrival probability (p_{arrival}) changes slightly from its definition in Eq. 24, to

$$p_{\text{arrival}}^* = \pi^D \bar{p}_a^* , \text{ where} \quad (77)$$

$$\bar{p}_a^* = \begin{bmatrix} 0 \\ 1/m \\ \vdots \\ i/m \\ \vdots \\ 1 \end{bmatrix}, \quad (78)$$

i.e., its i -th element gives the cell arrival probability in case of i active connections (see Eq. 70). Similarly, the factor in Eq. 60 representing the cell arrival probability given i active connections can be replaced by $\bar{p}_a^*$, i.e.,

$$E[L]^* = \frac{1-p}{p} \frac{\mathcal{L}_{\text{full}}^* \bar{p}_a^*}{p_{\text{arrival}}^*}. \quad (79)$$

Taking Eq. 76, Eq. 77, and Eq. 79 into account, the packet loss probability (Eq. 63) changes to

$$p_{\text{packet loss}}^* = \frac{(1-p) \mathcal{L}_{\text{full}}^* \bar{p}_a^*}{p p_{\text{arrival}}^* + (1-p) \mathcal{L}_{\text{full}}^* \bar{p}_a^*} = \frac{(1-p) \mathcal{L}_{\text{full}}^* \bar{p}_a^*}{\left(p \pi^D + (1-p) \mathcal{L}_{\text{full}}^* \right) \bar{p}_a^*}. \quad (80)$$

Using Eq. 70, we can solve for π_i^* . However, the expression depends on m , the total number of incoming connections. If we were able to remove m from the expression, we could use the result of Eq. 66 for different experiments, with different values of m . We replace the transition matrix T_i^* by a transition matrix T_i^{**} , which specifies the transition probabilities at cell arrival instants only,

$$T_i^{**} = p B_i^{al} + (1-p) B_i^{an}. \quad (81)$$

The DTMC with transition matrix T_i^{**} has the same stationary state probability vector as the one with T_i^* , using Eq. 30 and Eq. 66:

$$\begin{aligned} \pi_i^* T_i^{**} &= \pi_i^* \left(\frac{m}{i} \left(T_i^* - \left(1 - \frac{i}{m} \right) B_i^0 \right) \right) = \frac{m}{i} \pi_i^* T_i^* - \frac{m-i}{i} \pi_i^* I \\ &= \frac{m}{i} \pi_i^* - \frac{m-i}{i} \pi_i^* = \pi_i^*. \end{aligned} \quad (82)$$

Figure 1. Buffer occupancy distribution ($m=3$, $n=50$).

6. NUMERICAL RESULTS

In order to get insight in the accuracy of the lumped model and the decomposed model, we compare results from these models with results from the overall model. The evaluation of the lumped and the decomposed model is done using two programs in the computing environment MATLAB. The evaluation of the overall model is performed using the software tool UltraSAN ([8]). Since the state space of the overall model grows very fast, we are only able to make comparisons for small m and n . The error bound for the decomposed model, which we have found in Eq. 75, increases linear in m . According to Eq. 75, the error is acceptable if $1/m$ is sufficiently larger than r and s . This is a realistic assumption, since the burst-silence behaviour of the connection is at another time-scale than the cell arrival behaviour.

To illustrate the accuracy of our simplified model, we give the buffer occupancy distribution for $m = 3$ and $n = 50$ (Figure 1). For the other parameters we have taken the values $r = s = 0.0001$ and $p = 0.1$, i.e., a mean packet length of 10 cells. Figure 2 shows the relative error in our approximation. It can be seen from the figure that the relative error is always below 0.1%.

It has been stated already in Section 3 that the overall model has a state space of order $O(m^n)$, and the lumped model of order $O(mn)$. The decomposed model can be considered as consisting of m DTMCs of order $O(n)$. It is known that the steady state probability vector of a DTMC with n states can be obtained with a

Figure 2. Relative Error in Buffer Occupancy Distribution compared to the Overall Model.

computational complexity of order $O(n^2)$. Table 1 gives the order of complexity of the various models. It can easily be seen that with the overall model, it is only possible to obtain results for small m and n . The difference between the computational complexity of the lumped and the decomposed model is of the order of the number of incoming connections (m).

Let us now investigate the required size of the reassembly buffer. Figure 3 shows the relation between the buffer length n , and the packet loss probability $p_{\text{packet loss}}$ for different values for the number of connections terminated in the CLS (m). The values of r , s , and p are as above. The decomposed model is used to obtain the results of this figure.

Table 1
Order of Complexity of the Overall, Lumped, and Decomposed Model

model	order of state space size	order of computational complexity
overall model	$O(m^n)$	$O(m^{2n})$
lumped model	$O(mn)$	$O(m^2n^2)$
decomposed model	$m \times O(n)$	$O(mn^2)$

Figure 3. Packet Loss Probability.

It can be seen that for small m the logarithm of the loss probability is almost inversely proportional to the buffer length. Furthermore, the packet loss probability depends very much on m . The required buffer length for a certain loss probability is less than proportional to the number of incoming connections, e.g., in order to obtain a loss probability of 10^{-5} , the required buffer size is approximately 220 for $m = 10$, and 450 for $m = 40$. This can be explained by the fact that packets share buffer space, i.e., the maximum need for buffer space of the different connections does not occur simultaneously most of the time.

7. CONCLUSIONS

We have modelled and analysed the reassembly buffer, which is used in a CLS operating in message mode to reassemble packets from the incoming cells. We have focused on the loss behaviour of the buffer. The overall model of this buffer has a very high computational complexity. We have presented two models with a much lower complexity, by applying lumping and decomposition techniques. The accuracy of the lumped and decomposed model have been evaluated. For typical values of the parameters, the relative error in the buffer occupancy distribution of both models is below 0.1%.

The developed models have been used to get insight in the loss probability of the buffer, given the number of incoming connections, the packet length, and the characteristics of the arrival process. For the used parameters, the analysis has shown that the required buffer space depends heavily on the number of connections terminated in a CLS.

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