A Neural Network Algorithm for Solving the Job-Shop Scheduling Problem with Priority

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Abstract

Hopfield-type neural networks have shown part success in solving hard optimization problem. As problem becomes more and more complex, it is increasingly difficult to find a suitable encoding of constraints into an energy function, which will effectively guide the network to a desired solution. This paper describes how the job-shop scheduling problem with priority is encoded into a modified Hopfield neural network, and solved. The neural network algorithm is easy to be virtually realised on digital computers. At the end an example is given.

Keywords

NP-hard, neural network, job-shop scheduling problem, priority

1 INTRODUCTION

The job-shop scheduling problem (JSSP) is a resources to tasks allocation problem. The resources are usually called machines, the tasks jobs. Each job may consist of several subjobs (referred to as operations) which subject to some precedence restrictions. The job-shop scheduling is a classical operations research problem with numerous applications but very few practical solution approaches. Due to the large number of constraints the problem is known to be NP-hard one, so that even a good (which may be not optimal) feasible solution is acceptable in most applications. The scheduling problem with priority is one important class among the scheduling problems, and this paper especially introduces the neural network algorithm for solving it.

2 FORMAL DESCRIPTION OF GENERAL JSSP

Let S_{ik} denote the starting time and t_{ik} the processing time for operation k of job i . The cost function will be the sum of the starting time of each job's last operation: $\sum_{i=1}^n S_{ik_i}$, where n is the number of jobs, k_i is the last operation of job i . The total waiting and processing time for all jobs equals $\sum_{i=1}^n S_{ik_i} + \sum_{i=1}^n t_{ik_i}$ where $\sum_{i=1}^n t_{ik_i}$ is a constant.

The precedence constraints are given by the following inequality:

$$S_{ik} - S_{i,k+1} + t_{ik} \leq 0, \quad i=1, \dots, n; k=1, \dots, k_i-1$$

The starting time should be positive, so

$$S_{ik} \geq 0, \quad i=1, \dots, n.$$

For any two operations (i, k) and (j, p) assigned to the same machine (where the first index denotes the job number and the second index the operation number), the following constraints need to be satisfied in order to avoid the overlap in time between these operations: $S_{ik} - S_{jp} + t_{ik} \leq 0$, if operation (i, k) is performed first; or $S_{jp} - S_{ik} + t_{jp} \leq 0$, if operation (j, p) is performed first.

In conclusion the mathematical formulation for the job-shop scheduling is described as follows (Zhou and etc., 1990):

$$\min \sum_{i=1}^n S_{ik_i} \quad (1)$$

$$\text{s. t. } S_{ik} - S_{i,k+1} + t_{ik} \leq 0, \quad i=1, \dots, n; k=1, \dots, k_i-1 \quad (1)$$

$$S_{ik} \geq 0, \quad i=1, \dots, n \quad (2)$$

$$S_{ik} - S_{jp} + t_{ik} \leq 0 \quad \text{or} \quad S_{jp} - S_{ik} + t_{jp} \leq 0 \quad (3)$$

In addition, there is another constraint,

$$S_{ik_i} \leq \text{expect}_i, \quad i=1, \dots, n \quad (4)$$

If job i has due date expect_i equals the due date, otherwise expect_i may take an enough large number.

3 PROPOSED NEURAL NETWORK FOR GENERAL JSSP

3.1 Energy function

The energy function of the neural network is chosen as follows :

$$E = \sum_i S_{ik_i} + \frac{1}{2} \sum_i \sum_k H_1 F(S_{ik} - S_{i,k+1} + t_{ik}) + \frac{1}{2} \sum_i H_1 F(-S_{ii}) + \frac{1}{2} \sum_i H_1 F(S_{ik_i} - \text{expect}_i) + \frac{1}{2} \sum_{ik,jp} H_2 F(\min(S_{ik} - S_{jp} + t_{ik}, S_{jp} - S_{ik} + t_{jp})) \quad (5)$$

where H_1, H_2 are all big positive parameters, $H_1 \gg H_2$. $F(x) = \begin{cases} x^2, & x > 0 \\ 0, & x \leq 0 \end{cases}$

3.2 Motion equations

The motion equations of the neural network are stated as follows :

$$\frac{dS_{ik}}{dt} = -\frac{\partial E}{\partial S_{ik}} \quad (6)$$

$$\Delta S_{ik}(t) = \begin{cases} \min(2^q, LS_{ik}(t) - S_{ik}(t)), & \text{if } \frac{dS_{ik}}{dt} \geq 2^{q-1} w_{ik,ik} \\ \max(-2^q, -S_{ik}(t)), & \text{else if } \frac{dS_{ik,ik}}{dt} < -2^{q-1} w_{ik,ik} \\ 0, & \text{else} \end{cases} \quad (7)$$

$$S_{ik}(t+1) = S_{ik}(t) + \Delta S_{ik}(t) \quad (8)$$

$i = 1, \dots, n; k = 1, \dots, k_i; q = 0, \dots, b_m - 1$

The processing times are all integer numbers; even not, they may be converted into integer numbers. S_{ik} can also be thought of as integer numbers, and b_m is the maximum bits of binary digit needed to represent S_{ik} . $LS_{ik_i} = \text{expect}_i - t_{ik_i}$, $LS_{ik} = LS_{i,k+1} - t_{i,k+1}$, $i = 1, \dots, n$; $k = k_i - 1, \dots, 1$.

The initial values of S_{ik} are selected as follows :

$$S_{ii} = 0, S_{ik} = S_{i,k-1} + t_{i,k-1}, i = 1, \dots, n; k = 2, \dots, k_i$$

Determination of $w_{ik,ik}$: Assuming the coefficient matrix of (1) is A , the coefficient matrix of (3) A_1 ,

$$W = H_1 A^T A + \frac{1}{2} H_2 A_1^T A_1$$

and $w_{ik,ik}$ is the ik th diagonal element (corresponding to S_{ik}) of W .

It may be proved that the function E in (5) will monotonously decrease if S_{ik} change according to the motion equations (6), (7) and (8).

3.3 'Hill-climbing' term

Rather than use a stochastic technique to enable the neural network to escape from local energy minima (which may correspond to infeasible solutions), a simple but effective 'hill-climbing' term was added to the motion equations, that is,

$$\frac{dS_{ik}}{dt} = -\frac{\partial E}{\partial S_{ik}} - \sum_{jp} H_3 h(\min(S_{ik} - S_{jp} + t_{ik}, S_{jp} - S_{ik} + t_{jp})) \\ \times \text{select}(S_{ik} - S_{jp} + t_{ik}, S_{jp} - S_{ik} + t_{jp})$$

where H_3 is a positive parameter, $H_3 > H_1$.

$$h(x) = \begin{cases} 1, & x > 0 \\ 0, & x \leq 0 \end{cases}, \quad \text{select}(x, y) = \begin{cases} 1, & x \leq y \\ -1, & x > y \end{cases}$$

3.4 Implemented method

The value of H_3 is selected by trial and error, and is hard to get a suitable amount. If H_3 is too small the 'hill-climbing' term can not make effectiveness, if H_3 is too big the neural network will oscillate. Therefore in the implemented method we avoid the selection of the value of H_3 , taking adjustment instead of 'hill-climbing' term. The rules for adjustment are:

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if  $S_{jp} < S_{ik} < S_{jp} + t_{jp}$ 
  if  $LS_{ik} \geq S_{jp} + t_{jp}$  then  $S_{ik} = S_{jp} + t_{jp}$ 
  else if  $LS_{jp} \geq S_{ik} + t_{ik}$  then  $S_{jp} = S_{ik} + t_{ik}$ 
if  $S_{ik} < S_{jp} < S_{ik} + t_{ik}$ 
  if  $LS_{jp} \geq S_{ik} + t_{ik}$  then  $S_{jp} = S_{ik} + t_{ik}$ 
  else if  $LS_{ik} \geq S_{jp} + t_{jp}$  then  $S_{ik} = S_{jp} + t_{jp}$ 
if  $S_{ik} = S_{jp}$  and  $t_{ik} \leq t_{jp}$ 
  if  $LS_{jp} \geq S_{ik} + t_{ik}$  then  $S_{jp} = S_{ik} + t_{ik}$ 
  else if  $LS_{ik} \geq S_{jp} + t_{jp}$  then  $S_{ik} = S_{jp} + t_{jp}$ 

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if $S_{ik} = S_{jp}$ and $t_{jp} < t_{ik}$
 if $LS_{ik} \geq S_{jp} + t_{jp}$ then $S_{ik} = S_{jp} + t_{jp}$
 else if $LS_{jp} \geq S_{ik} + t_{ik}$ then $S_{jp} = S_{ik} + t_{ik}$

In conclusion S_{ik} ($i=1, \dots, n$; $k=1, \dots, k_i$) evolve in light of the motion equations (6), (7) and (8), and are sequentially adjusted by the above-mentioned rules. The evolution and adjustment are repeated until S_{ik} freeze not only in evolution but also in adjustment.

It may be proved that the implemented method will certainly converge and the solution obtained by it must be feasible.

4 ALGORITHM FOR THE JSSP WITH PRIORITY

In the JSSP with priority, the jobs have different priorities. The jobs may be scheduled in order according to their priorities (from the highest to the lowest). Moreover, when scheduling the jobs with low priority, the time table of the jobs with higher priorities is not admitted to be delayed. For the consideration, the algorithm for the JSSP with priority may consists of the following steps:

1. Assuming LS_{ik} is the latest start time of operation (i, k) , let $LS_{ik} = \infty$.
2. Schedule the job(s) i with the highest priority, and then let $LS_{ik} = S_{ik}$, $k = 1, \dots, k_i$.
3. According to priority, schedule the job(s) i with lower priority, and then let $LS_{ik} = S_{ik}$, $k = 1, \dots, k_i$.
4. If there is(are) job(s) unscheduled, return to step 3.
5. End.

5 NEUMERIAL RESULTS

A program to simulate the neural network algorithm was written in C language on VAX 785. We tested many problems, and found that the neural network algorithm had been quite successful in finding valid schedules, and the solutions had been superior to those found by heuristic algorithms.

Table 1 gives a JSSP with priority. Figure 1, 2, 3 are the results found by the neural network algorithm, and the results have been known to be very good through comparison.

Table 1 A JSSP with priority

Jobs	Machine Process time	Priority
1	$\frac{M_1}{5} \quad \frac{M_2}{8} \quad \frac{M_3}{2}$	3
2	$\frac{M_3}{7} \quad \frac{M_1}{3} \quad \frac{M_2}{9}$	3
3	$\frac{M_1}{6} \quad \frac{M_2}{3} \quad \frac{M_3}{2}$	2
4	$\frac{M_2}{4} \quad \frac{M_3}{3} \quad \frac{M_1}{4}$	2
5	$\frac{M_3}{6} \quad \frac{M_2}{4}$	2
6	$\frac{M_1}{2} \quad \frac{M_2}{3}$	1
7	$\frac{M_3}{5} \quad \frac{M_1}{6} \quad \frac{M_3}{2}$	1

Figure 1 Schedule of jobs with priority of 3.

Figure 2 Schedule of jobs with priorities of 3 and 2.

Figure 3 Schedule of all jobs.

6 CONCLUSION

The job-shop scheduling with priority is a kind of NP-hard problem. Although it has many application situations, there is few efficient method to solve it. After many tests we think that the neural network algorithm is a good candidate and merits further studies.

7 REFERENCE

- Zhou, D. N. , Cherkassky, V. , Baldwin, T. R. and Hong, D. W. , (1990) " Scaling Neural Network for Job-Shop Scheduling", Proc. IJCNN' 90, Vol. 3, 889-92, Washington, D. C.

8 BIOGRAPHY

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