A Practical Conference Key Distribution System

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Abstract. We consider a variant of the Burmester-Desmedt conference key distribution system proposed at Eurocrypt '94. Our system is also based on public keys and has a low communication and computation complexity. Each party exchanges only three messages, and only a few multiplications are required. The security is based on the intractability of the Diffie-Hellman problem.

1. INTRODUCTION

Secure key distribution is a major issue in an open computer network environment, where data is readily available through distributed processing and distributed data bases. Many two-party key distribution systems based on public keys are described in the literature (e.g. [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]). These employ a key exchange protocol in which a common session key is generated using randomness, to prevent replays, and secret information, for identification. If more than two parties are involved then a conference key distribution system is used. Designing such systems can be quite challenging because of the large number of interactions between the many users. Many conference key distribution systems have been proposed [14, 2, 6, 4, 15, 16]. These however are either impractical, or only heuristic arguments are used to address their security. Our goal in this paper is to present a practical and secure conference key distribution system. Our system is based on the protocol presented in [17] and extends the two-party system proposed in [1, 11]. Its security is based on the intractability of the Diffie-Hellman [18] problem,

The D-H problem: given integers p, α, X, Y , find $X^{\log_{\alpha} Y} \bmod p$, if it exists.

This problem is generally assumed to be hard, even if the factorization of the order of α is known (cf. [19, 20]). It is well known [21, 22, 23] that this problem is easy if the Discrete Logarithm problem is easy then (but the converse may not be true).

This paper is organized as follows. In the following section we present our protocol. Then, in Sections 3,4 we discuss its security: first against attacks by passive adversaries, and then against active adversaries.

2. THE CONFERENCE KEY DISTRIBUTION SYSTEM

We use a discrete logarithm setting with prime modulus p . The security parameter is $|p|$, the binary length of p . The basis is α , whose order q is large [24] (superpolynomial in $|p|$). A Trusted Center chooses both p and α and publishes these, together with q . Each user U_i selects a secret key $s_i \in \mathbb{Z}_q$, and then registers $P_i = \alpha^{s_i} \bmod p$ with the Trusted Centre as its public key.

The protocol

Let $U_1, U_2, \dots, U_n$ be a dynamic set of users who want to generate a common conference key.

Step 1 Each user U_i , $i=1,2,\dots, n$ selects uniformly at random $r_i \in \mathbb{Z}_q$ and computes

$$X_i = \alpha^{r_i} \bmod p.$$

Then U_i sends X_i to the other parties in the conference.

Step 2 Each user U_i , $i=1,2,\dots, n$ computes

$$Y_i = (P_{i+1})^{r_i} / (X_{i-1})^{s_i} \equiv (X_i)^{s_{i+1}} / (P_i)^{r_{i-1}} \pmod{p}$$

where the indices are taken in a cycle. Then U_i sends Y_i to the other parties in the conference.

The Session Key

The session key is,

$$K \equiv \alpha^{r_1 s_1 + r_2 s_2 + \dots + r_n s_n} \pmod{p}$$

which each user U_i in the conference can compute since,

$$K \equiv K_i \equiv (X_{i-1})^{ns_i} \cdot Y_i^{n-1} \cdot Y_{i+1}^{n-2} \dots Y_{i-2} \pmod{p}$$

Figure 1 shows the conference key distribution system for three users.

Figure 1. The Conference Key Distribution System for three users

Remark. This protocol is a modification of a protocol presented in [17]. When $n = 2$ we have

$$Y_1 \equiv Y_2^{-1} \equiv P_2^{r_1} / P_1^{r_2} \pmod{p},$$

which both parties can compute, and therefore there is no need to exchange Y_1, Y_2 . In this case

$$K = \alpha^{r_1 s_2 + r_2 s_1} \pmod{p},$$

and we get the 'non-paradoxical' key distribution system proposed by Yacobi [11].

3. RESILIENCE TO ATTACKS FROM PASSIVE ADVERSARIES

We first consider the resilience of our system to attacks from a passive adversary, i.e. a passive eavesdropper.

Theorem. *If the size of the conference is polynomially bounded in the length of p , and if the Diffie-Hellman problem is intractable, then the conference key distribution system described above is resilient to attacks from passive eavesdroppers.*

Proof. This is similar to proof of Theorem 1 in [17]. Let E be a passive eavesdropper who succeeds with non-negligible probability in computing the key

$$K_1 = (X_n)^{ns_1} \cdot Y_1^{n-1} \dots Y_{n-1} \pmod{p}$$

of U_i . Fix X_n, P_1 . Then select numbers $b_2, b_3, \dots, b_n$ and $d_2, d_3, \dots, d_n$, at random from Z_q , and set:

$$X_1 \equiv X_n \cdot \alpha^{-b_1} \pmod{p}, X_2 \equiv X_1 \cdot \alpha^{b_2} \pmod{p}, \dots, X_{n-1} \equiv X_1 \cdot \alpha^{b_{n-1}} \pmod{p},$$

$$P_2 \equiv P_1 \cdot \alpha^{d_2} \pmod{p}, P_3 \equiv P_1 \cdot \alpha^{d_3} \pmod{p}, \dots, P_n \equiv P_1 \cdot \alpha^{d_n} \pmod{p}.$$

Since

$$Y_i \equiv (P_{i+1})^{s_i} / (X_{i-1})^{s_i} \equiv (P_1 \cdot \alpha^{d_{i+1}})^{s_i} / (X_1 \cdot \alpha^{b_{i-1}})^{s_i} \equiv (X_i)^{d_{i+1}} \cdot (P_i)^{-b_{i-1}} \pmod{p},$$

with $b_1 = d_1 = 0$, it is feasible to compute all the Y_i . So we have the necessary input for E . Then E will succeed with non-negligible probability in computing K_1 , and thus

$$(X_n)^{ns_1} \pmod{p},$$

from which it is feasible to compute

$$(X_n)^{s_1} \equiv X_n^{\log_a P_1} \pmod{p},$$

since it is feasible to extract n -th residues in Z_p when n is polynomially bounded [25,26].

Hence E can compute

$$X_n^{\log_a P_1} \pmod{p}$$

from X_n and P_1 , which violates the intractability of the Diffie-Hellman problem.

4. RESILIENCE TO ATTACKS FROM ACTIVE ADVERSARIES

We consider a very powerful adversary E who can read the calls which the parties exchange, modify the calls, replay old calls, and conduct sessions in parallel with many parties 'playing one against the other' [27,28]. Furthermore the adversary has access to some (chosen) old session keys. However we shall assume that the adversary is polynomially bounded.

4.1. The protocol leaks knowledge

Burmester and Desmedt point out in [29], that the two-party system ($n=2$) leaks knowledge under a general attack. This knowledge can be used by an adversary to compute a session key if the adversary has access to some session keys [30]. We shall now describe such an attack for the case $n=3$ based on the 'triangle attack' in [30].

A known-key attack

Suppose that U_1, U_2, U_3 have computed $K \equiv \alpha^{r_1 s_2 + r_2 s_3 + r_3 s_1} \pmod{p}$, with calls X_1, X_2, X_3 . The adversary $\tilde{U}_4$ is an insider with secret key s_4 , and public key P_4 , who wants to compute the key K . The adversary $\tilde{U}_4$ gets X_1, X_2, X_3 by eavesdropping. To compute K , $\tilde{U}_4$ first gets the partial key

$$k_1 = \alpha^{r_1 s_1} \pmod{p}$$

from the session key

$$K_1 = \alpha^{r_1' s_4 + r_1 s_1} \pmod{p}$$

of the two-party protocol $(U_1, \tilde{U}_4)$ with calls $X_1' = \alpha^{r_1'} \pmod{p}$, X_3 . The adversary $\tilde{U}_4$ can compute k_1 because she knows s_4 . Similarly $\tilde{U}_4$ can get the partial key

$$k_2 = \alpha^{r_1 s_2} \pmod{p}$$

from the session key

$$K_2 = \alpha^{r_2' s_4 + r_1 s_2} \pmod{p}$$

of the protocol $(U_2, \tilde{U}_4)$ with calls $X_2' = \alpha^{r_2'} \pmod{p}$, X_1 , and

$$k_3 = \alpha^{r_3 s_3} \pmod{p}$$

from the session key

$$K_3 = \alpha^{r_3 s_4 + r_2 s_3} \bmod p$$

of the protocol $(U_3, \tilde{U}_4)$ with calls $X'_3 = \alpha^{r'_3} \bmod p$, X_2 . By multiplying $k_1, k_2, k_3, \tilde{U}_4$ then gets K . Observe that with known-key attacks the adversary has access to some chosen session keys.

Different ways to prevent such attacks are described in [30]. One such way is for the users to take as session key $\text{hash}(K)$, where *hash* is an appropriate hash function, instead of K itself. This will prevent this attack. However the messages Y_i sent in Step 2 of the protocol also leak knowledge which an adversary can use to compute earlier session keys. We now describe such an attack.

Another insider attack

Observe that

$$Y_i = \alpha^{r_i s_{i+1} - r_{i-1} s_i} \bmod p$$

is the session key of the two-party protocol (U_i, U_{i+1}) with calls X_i, X_{i-1}^{-1} . The messages Y_i cannot be hashed as in the previous case because they are needed to compute the session key K . If the adversary $\tilde{U}_{i+1} = U_{i+1}$ uses the inverse of an earlier call X_{i-1} , then $\tilde{U}_{i+1}$ can compute the partial key

$$k_i = \alpha^{r_{i-1} s_i} \bmod p.$$

So by arranging sessions with the other parties, one at a time, the adversary can compute all the factors of K , and hence K itself. To prevent this, the calls X_i must be authenticated. This would make it impossible for the adversary to re-use earlier calls in her attacks. Different ways of doing this are described in [17]. Here we propose that each party U_i in Step 1 of the protocol proves knowledge proof of the discrete logarithm r_i of X_i . This can be done by using a zero-knowledge proof of the discrete logarithm (e.g. [31]). For practicality we propose that a signature scheme based on zero-knowledge proofs is used, as in [32]. This means that each U_i in Step 1 of the protocol needs only send one more message, the signature of the conference key string $P_1, \dots, P_n$ with key X_i .

We now address the general problem of security. With the modified protocol the messages that the adversary may use must be independent of those sent in the earlier sessions. This means that the 'history' of the adversary which consists of messages exchanged earlier, can be simulated. This scenario is analogous to the one encountered when dealing with dishonest provers $\tilde{P}$ in interactive zero-knowledge proofs systems of knowledge [33]. These systems

make it possible for a prover P to prove to a verifier V that it knows some secret information without leaking any knowledge about it to V . If any other prover $\tilde{P}$, possibly dishonest, succeeds in convincing V with non-negligible probability that it knows P 's secret, then $\tilde{P}$ must know the secret, or be able to compute it (the soundness condition [33]). $\tilde{P}$ is allowed to collaborate with a dishonest verifier $\tilde{V}$ who may interact with the honest prover P (in an attempt to learn from P something about the secret). The knowledge of $\tilde{P}$ is checked by an 'extractor' who can obtain the secret from $\tilde{P}$ by 'probing and resetting' $\tilde{P}$. To make the link with our scenario we take the adversary E to be both $\tilde{P}$ and $\tilde{V}$, and the 'broken' secret key to be the secret of P . In the full paper we will show that the techniques of zero-knowledge proofs can be used to prove that our modified protocol (with signatures) is resilient from active adversaries, provided the Diffie-Hellman problem is intractable.

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