ON SPECIFYING SERVICES AND SYNTHESIZING PROTOCOLS FOR REAL-TIME APPLICATIONS

A. Khoumsi* , G.v. Bochmann and R. Dssouli

Université de Montréal, DIRO, C.P. 6128, Succursale A
Montréal, (Quebec), H3C 3J7

ABSTRACT. This paper deals with synthesizing automatically the specifications of the protocol and of the medium, from the specification of a desired service and the model of the medium containing timing requirements. Firstly, we propose a detailed model for specifying a desired service to be provided by a real-time discrete event system. This model uses a global clock, and several fictitious timers and counters. Next, we propose a method for deriving protocol and medium specifications from service specifications for real-time applications. Such derived specifications use a different model which does not use counters. Compared to [KBD93], the application field is much broader, because two important restrictions are removed. Firstly, temporal requirements are between events which are not necessarily consecutive. Secondly, the systems considered can be concurrent.

KEY WORDS : desired service, protocol synthesis, timing requirements, supremal model of the medium, sequential and concurrent systems, timed automata, untimed automata, temporized automata.

1. Introduction

A discrete event system (DES) is a dynamic system where events are executed instantaneously, causing a discrete change of the state of the system. If sequences of events are a regular language, the system can be specified by a finite automaton. A first example of DES is a telecommunication network; an event can then be the transmission of a packet of data. Another example is a communication protocol, and an event can be execution of a service primitive. For some DESs it is not enough to specify the ordering of events, we must also specify temporal requirements between events. This class of DESs are called Real-time DESs. For specifying a desired service to be provided by real-time DES, we use timed automata (TA) which are defined by using a global clock, and several fictitious timers and counters. And for synthesizing the protocol which provides such desired service, we use the approach which consists of transforming a real-time problem to an untimed problem ([AD90,BW92]). In comparison with [KBD93], two extensions are made: (a) temporal requirements are not only between consecutive events; (b) concurrent systems are considered. Henceforth, DES means Real-time discrete event system.

This paper is organized as follows. In Section 2, we introduce the problem of the protocol derivation. The basic principle used for deriving the protocol is explained. In Section 3, we introduce in detail the model developed for specifying a desired service and for defining a supremal model of the medium. This model is based on *timed automata* which are defined in this section. In Section 4, we present the approach which consists of transforming timed automata into *untimed automata* which are FSMs containing a transition *tick*. In Section 5, we present the approach which consists of transforming untimed automata into *temporized automata*. Let's notice that timed and temporized automata are based on different models. In Section 6, we propose a method for deriving automatically the specifications of the protocol and of timing constraints on the medium (temporized automata), from the specification of a desired service and the supremal model of the medium (timed automata). Both sequential and parallel systems are considered. And at last, we conclude in Section 7. We will notice that the possible concurrency in the parallel systems, and the timing requirements cause a problem of state space explosion and of complexity.

2. Problem of the protocol derivation of real-time systems

In a real-time distributed system (RTDS, Fig.1), n protocol entities (with $n > 1$) communicate : (a) with the user of the system through several service access points (SAP); (b) with each other through a medium assumed reliable. To each SAP corresponds one protocol entity.

In the *user's viewpoint*, the RTDS is a black box where only interactions with the user are visible. These interactions correspond to the executions of service primitives (or simply primitives). Therefore, the specification of the service desired by (or provided to) the user defines the *ordering and timing requirements* between the executed primitives.

But in the *designer's viewpoint*, it is necessary to compute the specifications of the local real-time protocol entities PE_i , for $i=1,2, \dots, n$, which may provide the service desired by the user. The designer must also compute timing requirements which must be respected by the medium. In order to avoid the computation of timing requirements impossible to respect by the medium, the designer may refer to a *supremal model* of the medium, and compute only timing requirements which respect this supremal model. Informally, if for instance we know that the medium needs at least two units of clock time (uct) to

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carry messages between two protocol entities, this information is contained in the supramodel. In this case, the designer will not compute timing requirements such as : some message must be carried in one uct. We will see that the medium not only carries a message, but it also adds an information about the transit delay of the message in the medium.

The problem for designing protocols is then : how can we derive systematically the different local protocol specifications and the timing requirements on the medium, from : (a) a global specification of the service desired by the user ; (b) a supramodel of the medium.

Figure 1. Service and protocol concepts

The approach used for deriving protocols is *synthesis* ([BG86, KBK89, SP90, KHB92, KBD93]). Timing requirements are considered in [KBD93], but they are only between consecutive events, and the systems considered are sequential. In the present study, these two constraints are removed. For the sake of simplicity, we explain the basic principle of protocol derivation only for sequential systems. But parallel systems also are considered, farther in this paper (Section 6.3). The principle is then : if a primitive A is executed by a protocol entity PE_a , and is followed by execution of a primitive B by PE_b , then after execution of A by PE_a , this one sends a message to PE_b to inform it that it may execute B. If after execution of A by PE_a , there is a choice between several primitives executed by different PE_{b_i} , for $i=1,2,\dots,p$, then PE_a selects one PE_{b_i} and sends a message to it to inform it that it may execute one of its primitives. Let's notice that the terms "derivation" and "synthesis" are used as synonyms.

3. Model of the desired service and of the medium

The desired service and the supramodel of the medium are the two starting points of the protocol derivation. For specifying them, we use a global digital clock which generates a fictitious event *tick* at a constant frequency ([Os90, BW92, OW90]). The delay between two *ticks* represents the passing of one unit of clock time (uct). The set $\mathbb{N}$ of natural numbers is our domain of time. The time is then modeled by a global variable, noted τ and called *discrete time* : τ is initially equal to zero and is incremented by one after the passing of each unit of clock time (uct), i.e., after the occurrence of a *tick*.

3.1. Timed traces and Timed languages

A *finite timed trace* trc over an alphabet V is a finite sequence of pairs $\langle \sigma_i, \tau_i \rangle$, where σ_i is an event of V , and τ_i is an integer such that $\tau_{i+1} \geq \tau_i$. Such trace is represented by $trc = \langle \sigma_1, \tau_1 \rangle \dots \langle \sigma_n, \tau_n \rangle$ and contains all events that have occurred before time $\tau_n + 1$. Each $\langle \sigma_i, \tau_i \rangle$ means that the event σ_i has occurred when the discrete time is equal to τ_i . It is clear that there is an inaccuracy of one uct on the exact delay of event occurrences. An *infinite timed trace* Trc over an alphabet V is an infinite sequence of pairs $\langle \sigma_i, \tau_i \rangle$; any finite prefix of Trc is called a finite timed trace over V . Such infinite trace is represented by $Trc = \langle \sigma_1, \tau_1 \rangle \dots \langle \sigma_i, \tau_i \rangle \dots$. Each pair $\langle \sigma_i, \tau_i \rangle$ defined in a trace Trc is called a component of this trace and is noted : $\langle \sigma_i, \tau_i \rangle \in Trc$. Since a τ_i may be equal to τ_{i+1} , several consecutive events may occur at the same discrete time, i.e., during one uct.

Definition 1. (Finiteness property)

An infinite timed trace respects the finiteness property (FP) if the number of events executed during one uct is bounded by an arbitrary constant Mc . Formally, $Trc = \langle \sigma_1, \tau_1 \rangle \dots \langle \sigma_i, \tau_i \rangle \dots$ respects the FP if and only if : $\forall i > 0, \exists j > i$ such that $\tau_j = \tau_i < \tau_{j+1}$ and $j \leq i + Mc$. The FP is differently defined in [TH92], where it only requires that a finite number of events occur in any finite time interval. ■

Example 1. (a) Let Trc be the following infinite trace $Trc = \langle \sigma_1, 2 \rangle \langle \sigma_2, 4 \rangle \dots \langle \sigma_i, 2i \rangle \dots$. Trc respects the finiteness property because one event occurs when τ is even, and no event occurs when τ is odd.

(b) Let Trc be the following infinite trace $Trc = \langle \sigma_1, 1 \rangle^1 \langle \sigma_2, 4 \rangle^2 \dots \langle \sigma_i, 2i \rangle^i \dots$, where $\langle \sigma, \tau \rangle^p$ means that σ occurs p times when the discrete time is equal to τ . Trc does not respect the FP because the number of events during one uct is not bounded. ■

Definition 2. (Timed trace and timed language)

In this paper, we consider only *infinite* timed traces. Such traces, will be simply called *timed traces*.

A *timed language* $\mathcal{L}$ over an alphabet V is a set of infinite timed traces over V .

We say that $\mathcal{L}$ respects the finiteness property (FP) if all its timed traces respect the FP. ■

Infinite timed traces, which will be simply called *timed traces*, are executed by non terminating processes. This is not really a restriction. In fact, a terminating process which may be executed infinitely often, can also be considered as a non terminating process.

Definition 3. (Projection of a timed trace)

Let V be a subset of an alphabet W , and let $\text{Trc} = \langle \sigma_1, \tau_1 \rangle \dots \langle \sigma_i, \tau_i \rangle \dots$ be a timed trace over W . The *projection* of Trc on V , noted $\text{Proj}_V(\text{Trc})$, is obtained by removing from Trc all $\langle \sigma_i, \tau_i \rangle$, where $\sigma_i \notin V$. ■

Definition 4. (Projection and Extension of a timed language)

Let V be a subset of an alphabet W . Let $\mathcal{L}_1$ be a timed language over W . The projection of $\mathcal{L}_1$ on V , noted $\text{Proj}_V(\mathcal{L}_1)$, is defined by : $\text{Proj}_V(\mathcal{L}_1) = \{ \text{Trc}, \text{ over } V \mid \exists \text{ Trce} \in \mathcal{L}_1 \text{ with } \text{Trc} = \text{Proj}_V(\text{Trce}) \}$;
Let $\mathcal{L}_2$ be a timed language over V . The extension of $\mathcal{L}_2$ to W , noted $\text{Ext}_W(\mathcal{L}_2)$, is defined by :

$$\text{Ext}_W(\mathcal{L}_2) = \{ \text{Trc}, \text{ over } W \mid \text{Proj}_V(\text{Trc}) \in \mathcal{L}_2 \}. \quad \blacksquare$$

Remark 1. (a) if $W=V$ then $\text{Proj}_V(\mathcal{L}) = \text{Ext}_W(\mathcal{L}) = \mathcal{L}$; (b) $\text{Proj}_V(\text{Ext}_W(\mathcal{L})) = \mathcal{L}$ and $\mathcal{L} \subseteq \text{Ext}_W(\text{Proj}_V(\mathcal{L}))$.

3.2. Timers and counters

A DES may be specified by a timed automaton, or simply a TA, which is an extended FSM accepting a timed language (Def. 2). For defining a TA, we use several fictitious timers and counters.

Definition 5. (Timer)

A fictitious timer T_i is a conceptual entity associated to a variable t_i belonging to the set $\mathbb{N}$ of natural numbers. t_i is automatically incremented after the passing of one uct, and is called the current value of timer T_i . The operations we can do on the timer are :

- **Reset** : a timer T_i , whose value t_i is increasing regularly by one after each uct, can be set to zero. t_i represents therefore the time elapsed from the last reset of timer T_i .
- **Comparison** : the value t_i of timer T_i can be compared to a constant integer. The comparison operators are $=$, $>$ and $\leq$. Other operators $<$ and $\geq$ are not necessary because timer values are integers.

Initially, when the discrete time τ is equal to zero, t_i also is equal to zero. ■

We deduce that if several timers $T_1, T_2, \dots, T_{N_t}$ are used, then their current values $t_1, t_2, \dots, t_{N_t}$ are automatically and *simultaneously* incremented after the passing of one uct, i.e. when the discrete time τ is incremented. Therefore, all the timers are synchronized on the digital global clock.

Definition 6. (Timer state)

Let N_t (or $|T|$) be the number of timers $T_1, T_2, \dots, T_{N_t}$. The N_t -uplet $ts = (t_1, \dots, t_{N_t})$, where t_i is the current value of timer T_i , is called the *current timer state*. ■

Definition 7. (T_Condition, set E_T)

Let $T = \{ T_1, T_2, \dots, T_{N_t} \}$ be a set of timers. A T_Condition $E(ts)$, w.r.t. T , is a boolean function depending on the current timer state $ts = (t_1, \dots, t_{N_t})$. $E(ts)$ is formed from : (a) canonical boolean functions $t_i \sim k$, where t_i is the current value of a timer T_i , $k \in \mathbb{N}^*$, and $\sim$ is $=$, $\leq$ or $>$;

(b) operators AND($\wedge$), OR($\vee$), and NOT($\neg$) on these canonical boolean functions.

The set of all T_Conditions, w.r.t. T , is noted E_T . ■

Definition 8. (Counter) A fictitious counter C_i , w.r.t. an alphabet V_{c_i} , is a conceptual entity associated to a variable c_i belonging to $\mathbb{N}$. c_i is called the current value of C_i , and is automatically : (a) incremented after the occurrence of any event of V_{c_i} ; (b) set to zero after the passing of one uct, i.e., when τ is incremented. ■

Definition 9. (counter state) Let N_c (or $|C|$) be the number of counters $C_1, C_2, \dots, C_{N_c}$. The N_c -uplet $cs = (c_1, \dots, c_{N_c})$, where c_i is the current value of counter C_i , is called the *current counter state*. ■

Definition 10. (F_Condition, set E_C) Let $C = \{ C_1, C_2, \dots, C_{N_c} \}$ be a set of counters. A F_Condition $K(cs)$, w.r.t. C , is a boolean function depending on the current counter state $cs = (c_1, \dots, c_{N_c})$. $K(cs)$ is formed from :

(a) canonical boolean functions $c_i < Mc_i$, where c_i is the current value of a counter C_i , and $Mc_i \in \mathbb{N}^*$;

(b) operator AND($\wedge$) on these canonical functions. The set of all F_Conditions, w.r.t. C , is noted E_C . ■

Example 2. if $K(cs) = (c_1 < Mc_1)$ must be always true, and C_1 is w.r.t. V_{c_1} , then no more than Mc_1 events of V_{c_1} may occur during one uct. ■

3.3 Timed Automata for real-time processes

For defining a TA, we use in general:

- a global digital clock which informs about the passing of one uct,
- a finite set of fictitious digital timers (Def. 5), for specifying the timing requirements,
- a finite set of counters (Def. 8), for respecting the finiteness property (Def. 1).

Definition 11. (Timed transition, and Reset)

Let $A=(Q,V,\delta,q_0)$ be a FSM where Q is a set of states, V is an alphabet, q_0 is the initial state, and $\delta \subseteq Q \times V \times Q$ defines the transitions, i.e., a transition of A can be represented by $[q_1;\sigma;q_2]$.

Let $T=\{T_1, \dots, T_{N_t}\}$ be a set of timers, and let $C=\{C_1, \dots, C_{N_c}\}$ be a set of counters, w.r.t $V_{c_i} \subseteq V$, for $i=1,2, \dots, N_c$. Let E_T (resp. E_C) be the sets of $T_Conditions$ (resp. $F_Conditions$), w.r.t. T (resp. C).

A *timed transition*, w.r.t. T and C , is defined by $Tr=[q_1;\sigma;q_2;E(ts);R;K(cs)]$, with $\sigma \in V$, $q_1, q_2 \in Q$, $E(ts) \in E_T$, $K(cs) \in E_C$, and $R \subseteq T$. R is called *Reset* of the transition Tr . The semantics of Tr is the following. Let q_1 be the current state : (1) σ may occur only if $E(ts)$ (Def.7) and $K(cs)$ (Def.10) are true;

(2) after the occurrence of σ : (a) the state q_2 is reached, timers of R are set to zero, and

(b) c_i is incremented if $\sigma \in V_{c_i}$, for $i=1,2, \dots, N_c$.

Besides, $K(cs)=(c_{i1}<M_{c_{i1}}) \wedge \dots \wedge (c_{ip}<M_{c_{ip}})$, where $c_{i1}, \dots, c_{ip}$ are all counters respectively w.r.t. $V_{c_{i1}}, \dots, V_{c_{ip}}$, such that $\sigma \in V_{c_{i1}} \wedge \dots \wedge V_{c_{ip}}$. ■

Informally, the event σ of V in $Tr=[q_1;\sigma;q_2;E(ts);R;K(cs)]$ may occur only if the $T_Condition$ $E(ts)$ is true. Besides, if $\sigma \in V_{c_i} (\subseteq V)$, then $(c_i < M_{c_i})$ also must be true for occurrence of σ .

Definition 12. (Enabled and Eligible timed transition)

A timed transition $[q_1;\sigma;q_2;E(ts);R;K(cs)]$ is *enabled* if conditions for occurrence of σ are true (Def.11).

A timed transition $Tr=[q_1;\sigma;q_2;E(ts);R;K(cs)]$ is *eligible* if :

Tr is enabled or will become enabled with the passing of time (without occurrence of any event). ■

A timed automaton (TA) A^t can then be constructed from the FSM $A=(Q,V,\delta,q_0)$, the finite sets T (of timers $T_1, \dots, T_{N_t}$) and C (of counters $C_1, \dots, C_{N_c}$). For that, we transform each transition $tr=[q_1;\sigma;q_2]$ of A into a timed transition Tr (Def.11) by associating to it, a $T_condition$ $E(ts)$, a $Reset$, and a $F_Condition$.

In this paper, we consider only TA which accept (Def.14) a timed language, i.e., infinite timed traces.

Definition 13. (Timed automaton)

A timed automaton $A^t=(Q,V,T,\mathcal{V},\delta,q_0)$ is defined as follows. Q is the set of states, q_0 is the initial state, V is the alphabet, T is the set of timers $T_1, T_2, \dots, T_{N_t}$, $\mathcal{V}=\{V_{c_i} \mid \text{for } i=1,2, \dots, N_c\} \subseteq 2^V$, where each V_{c_i} is associated to one counter C_i , and $\delta \subseteq Q \times V \times Q \times E_T \times 2^T \times E_C$ defines the timed transitions (Def.11), where E_T and E_C are the sets of $T_Conditions$ and $F_Conditions$ (Def.7 and 10).

Besides, A^t *accepts only infinite timed traces* (Def. 14), and is called a sequential TA. ■

Examples of timed automata are given in Section 3.6. (Example 3).

Remark 2. Let's notice that the timers and counters are *fictitious*. For example, the desired service of Figure 7 just means that the user wants that there must be at most two uct between primitives A_1 and B_2 . But the timers do not really exist. Therefore, a question such as, *how can we use a same timer in a distributed system*, has no sense. If the timers were real, there would be a problem of using a same timer in different sites.

Definition 14. (Acceptance of a timed trace and of a language, equivalence, partial order relation)

Let A^t be a TA $(Q,V,T,\mathcal{V},\delta,q_0)$ and let $Tr_c = \langle \sigma_1, \tau_1 \rangle \dots \langle \sigma_i, \tau_i \rangle \dots$ be an infinite timed trace.

- Tr_c is *accepted* by A^t , is formally defined by :

$\forall i \in \mathbb{N}^* : \exists Tr_i = [q_{i-1}; \alpha_i; q_i; E_i(ts); R_i; K_i(cs)] \in \delta$ with : $(\alpha_i = \sigma_i) \wedge ((\tau_i = \tau_i) \Rightarrow (E_i(ts) \wedge K_i(cs) = \text{True}))$.

Informally, a system specified by A^t may execute a trace accepted by A^t .

- A timed language, noted $\mathcal{L}_{A^t}$, is *accepted* by A^t if it contains all and only the traces accepted by A^t .

- A_1^t and A_2^t are *equivalent*, and noted $A_1^t \equiv A_2^t$, if and only if $\mathcal{L}_{A_1^t} = \mathcal{L}_{A_2^t}$.

- A_1^t is *smaller than or equal to* A_2^t , and noted $A_1^t \leq A_2^t$, if and only if $\mathcal{L}_{A_1^t} \subseteq \mathcal{L}_{A_2^t}$. ■

Property 1. Let $A^t=(Q,V,T,\mathcal{V},\delta,q_0)$ be a timed automaton specifying a non terminating system, with $\mathcal{V}=\{V_{c_1}, V_{c_2}, \dots, V_{c_{N_c}}\} \subseteq 2^V$. If $V_{c_1} \cup \dots \cup V_{c_{N_c}} = V$, then the language $\mathcal{L}_{A^t}$ accepted by A^t (Def.14) respects the finiteness property. In this case, we say that A^t respects the finiteness property. ■

Definition 15. (set $\mathcal{T}$ of timer states)

Let $T=\{T_1, \dots, T_{N_t}\}$ be a set of timers used for defining a TA A^t , and let M_{t_i} be the maximum value a timer T_i is compared to, for defining the $T_Conditions$ (Def.7) of all the transitions of A^t . In this case, the value t_i of T_i does not need to be incremented as soon as $t_i = M_{t_i} + 1$. In fact, in this case the

incrementation would have no influence on truths of the $T_Conditions$. Therefore, we can limit the value of t_i by Mt_i+1 , for $i=1, 2, \dots, N_t$, and the set $\mathcal{T}$ of timer states $ts=(t_1, \dots, t_{N_t})$ is equal to or included in $\langle 0; Mt_1+1 \rangle \times \dots \times \langle 0; Mt_{N_t}+1 \rangle$, where $\langle 0; Mt_i+1 \rangle$ is the set of integers belonging to the interval $[0; Mt_i+1]$. ■

For the TA of Fig.2.a (Example 3), $Mt_{11}=5$, and $Mt_{12}=5$, and then $\mathcal{T} \subseteq \langle 0;6 \rangle \times \langle 0;6 \rangle$

Definition 16. (Addition between $\mathcal{T}$ and $\mathbb{N}$)

Let $T=\{T_1, \dots, T_{N_t}\}$ be a set of timers used for defining a TA A^t . The addition between $\mathcal{T}$ and $\mathbb{N}$ is defined as follows : if $ts=(t_1, \dots, t_{N_t}) \in \mathcal{T}$ and $p \in \mathbb{N}$, then $ts+p=(\inf(t_1+p, Mt_1+1), \dots, \inf(t_{N_t}+p, Mt_{N_t}+1))$.

Where $\inf$ is defined by : $\inf(A,B) \in \{A,B\}$ and $(\inf(A,B)=A) \Leftrightarrow (A \leq B)$. ■

Intuitively, if ts is the current timer state, then $ts+p$ is the futur timer state after the passing of p units of clock time (uct). For the TA of Fig.2.a, if $ts=(4,1)$ and $p=3$, then $ts+3=(\inf(4+3;6), \inf(1+3;6))=(6,4) \neq (7,4)$.

Definition 17. (set $\mathcal{C}$ of counter states)

Let $C=\{C_1, \dots, C_{N_c}\}$ be a set of counters used for defining a TA A^t , and let M_{c_i} be the maximum value which bounds the value c_i . Therefore, the set $\mathcal{C}$ of counter states $cs=(c_1, \dots, c_{N_c})$ is equal to or included in $\langle 0; M_{c_1} \rangle \times \dots \times \langle 0; M_{c_{N_c}} \rangle$, where $\langle 0; M_{c_i} \rangle$ is the set of integers belonging to the interval $[0; M_{c_i}]$. ■

3.4. Product of two timed automata over the same alphabet

Let A_1^t and A_2^t be two TA (Def.13) defined over the same alphabet V . An intuitive definition of the synchronized product of A_1^t and A_2^t , noted $A_1^t \times A_2^t$, is the following: $A_1^t \times A_2^t$ is a TA specifying a system which may execute *all and only* the infinite timed traces accepted by both A_1^t and A_2^t .

Definition 18. (Product over a same alphabet)

Let $A_i^t=(Q_i, V, T_i, \mathcal{U}_i, \delta_i, q_{i0})$, for $i=1,2$, be two TA over a same alphabet V , with $T_1 \cap T_2 = \emptyset$, and $\mathcal{U}_i=\{V_{c_{i1}}, \dots, V_{c_{iN_{c_i}}}\}$. Each A_i^t uses then a set $T_i=\{T_{i1}, \dots, T_{iN_{t_i}}\}$ of timers and a set $C_i=\{C_{i1}, \dots, C_{iN_{c_i}}\}$ of counters, where each C_{ij} is w.r.t. $V_{c_{ij}}$. The product, noted $A^t=A_1^t \times A_2^t$, is defined by $A^t=(Q, V, T, \mathcal{U}, \delta, q_0)$, with $\mathcal{U}=\mathcal{U}_1 \cup \mathcal{U}_2$, $T=T_1 \cup T_2$, $Q \subseteq Q_1 \times Q_2$, $q_0=\langle q_{10}, q_{20} \rangle \in Q$, and :

Definition of δ : Let E_{T_1} , E_{T_2} and E_T be the set of $T_Conditions$ (Def. 7), respectively w.r.t. T_1 , T_2 and $T=T_1 \cup T_2$. Let E_{C_1} , E_{C_2} and E_C be the set of $F_Conditions$ (Def. 10), respectively w.r.t. C_1 , C_2 and $C=C_1 \cup C_2$. Then $\forall \langle q_1, q_2 \rangle, \langle r_1, r_2 \rangle \in Q$, $\forall \sigma \in V$, $\forall E \in E_T$, $\forall R \subseteq T$, $\forall K \in E_C$:

$$(\langle [q_1, q_2], \sigma, \langle r_1, r_2 \rangle, E, R, K \rangle \in \delta) \Leftrightarrow (\exists E_1 \in E_{T_1}, \exists E_2 \in E_{T_2}, \exists R_1 \subseteq T_1, \exists R_2 \subseteq T_2, \exists K_1 \in E_{C_1}, \exists K_2 \in E_{C_2}, \\ \text{(with : } R=R_1 \cup R_2, \quad E=E_1 \wedge E_2, \quad K=K_1 \wedge K_2, \text{ and } \\ \langle [q_1, \sigma, r_1, E_1, R_1, K_1] \rangle \in \delta_1, \text{ and } \langle [q_2, \sigma, r_2, E_2, R_2, K_2] \rangle \in \delta_2. \text{)}) \quad \blacksquare$$

Theorem 1. If $\mathcal{L}_{A_1^t}$ and $\mathcal{L}_{A_2^t}$ are respectively the timed languages accepted by A_1^t and A_2^t over the same alphabet, then:

$$\mathcal{L}_{A_1^t \times A_2^t} = \mathcal{L}_{A_1^t} \cap \mathcal{L}_{A_2^t}. \quad \blacksquare$$

Property 2. In Def. 18, if $\forall c_1 \cup \dots \cup \forall c_{1N_{c_1}} = \forall c_2 \cup \dots \cup \forall c_{2N_{c_2}} = V$, then A_1^t , A_2^t , and $A_1^t \times A_2^t$ respect the finiteness property. ■

Remark3. (a) In Def.18, if there exist $i \leq N_{c_1}$ and $j \leq N_{c_2}$ such that $\forall c_1 = \forall c_2$, then counters C_{1i} and C_{2j} may be a same counter for defining A_1^t , A_2^t , and $A_1^t \times A_2^t$. In fact, the values c_{1i} and c_{2j} are incremented and set to zero simultaneously. Therefore, one counter, for example C_{1i} , is sufficient.

(b) From Theorem 1, we deduce that if A_1^t and A_2^t specify two sequential processes over the same alphabet, then their *synchronized product also specifies a sequential process*.

3.5. Product of two timed automata over alphabets V_1 and V_2 with $V_1 \subseteq V_2$

Before defining the product over alphabets V_1 and V_2 , with $V_1 \subseteq V_2$, let's give two definitions.

Definition 19. (Operator $\oplus$ on En)

Let $E_1(ts)$, $E_2(ts)$, ..., $E_k(ts)$ be k $T_Conditions$ (Def. 7), depending on a set of timers $\{T_1, T_2, \dots, T_{N_t}\}$. We define $E(ts)=E_1(ts) \oplus E_2(ts) \oplus \dots \oplus E_k(ts)$ as follows.

$$(E(ts)=\text{false}) \Leftrightarrow \{ \forall i \in \{1, \dots, k\}, \forall p \in \mathbb{N} : E_i(ts+p)=\text{false} \} \quad \blacksquare$$

Informally, $E_1(ts) \oplus \dots \oplus E_k(ts)$ is false if and only if all $E_i(ts)$ are false and remain false with the passing of time. If for instance $T=\{T_1\}$, $E_1(t_1)=(t_1 \leq 5)$, $E_2(t_1)=((t_1 > 2) \wedge (t_1 \leq 6))$, then $E(t_1)=E_1(t_1) \oplus E_2(t_1)=(t_1 \leq 6)$.

Definition 20. (Extension of a timed automaton)

Let $A^t = (Q, V, T, \mathcal{V}, \delta, q_0)$ be a TA over an alphabet V with $T = \{T_1, \dots, T_{Nt}\}$, $\mathcal{V} = \{V_{C1}, \dots, V_{C_{Nc}}\}$, and then $C = \{C_1, \dots, C_{Nc}\}$. Let E_T (resp. E_C) be the set of T _Conditions w.r.t. T (resp. F _Conditions w.r.t. C). Let W be an alphabet such that $V \subseteq W$. The *extension* of A^t to the alphabet W , noted $\text{Ext}_W(A^t)$, is a TA defined by $(Q, W, T, \mathcal{V}', \delta_{\text{ext}}, q_0)$, where $\delta_{\text{ext}} \subseteq Q \times W \times Q \times E_T \times 2^{T \times E_C}$ is such that :

- (1) $\forall q_1, q_2 \in Q, \forall \sigma \in V, \forall E \in E_T, \forall R \subseteq T, \forall K \in E_C : [(q_1, \sigma, q_2, E, R, K) \in \delta \Leftrightarrow [(q_1, \sigma, q_2, E, R, K) \in \delta_{\text{ext}}]$.
- (2) $\forall q \in Q : \text{Let } E_i \in E_T, \text{ for } i=1, \dots, k, \text{ be all the } T\text{-Conditions of } E_T \text{ such that : } \exists q_i \in Q, \exists \sigma_i \in V, \exists R_i \in 2^T, \exists K_i \in E_C, \text{ with } [q, \sigma_i, q_i, E_i, R_i, K_i] \in \delta, \text{ and let then } E = E_1 \oplus E_2 \oplus \dots \oplus E_k.$
Then $\forall \sigma \in W - V : [(q, \sigma, q', E', R, K) \in \delta_{\text{ext}} \Leftrightarrow (q' = q, E' = E, R = \emptyset, K = \text{True})]$.

If $B^t = \text{Ext}_W(A^t)$, then A^t is called projection of B^t in the alphabet V , and is noted $A^t = \text{Proj}_V(B^t)$. ■

Informally, $\text{Ext}_W(A^t)$ is obtained by adding selfloops of all events of $W - V$ to each state of A^t . The resets of these selfloops are empty, and their T _conditions are defined as follows. The T _Condition of the added selfloops at a state q of A^t is true if at least one of the transitions defined in A^t from q is eligible. The F _Condition for events of $W - V$ is always true, and then $\text{Ext}_W(A^t)$ does not necessarily respect the finiteness property (Property 1).

Lemme 1. If $\mathcal{L}_{A^t}$ is the timed language accepted by a TA A^t over an alphabet V , and if W is an alphabet such that $V \subseteq W$, then : $\mathcal{L}_{\text{Ext}_W(A^t)} = \text{Ext}_W(\mathcal{L}_{A^t})$. (see Def. 4 for $\text{Ext}_W(\mathcal{L}_{A^t})$) ■

Before defining formally the product over V_1 and V_2 with $V_1 \subseteq V_2$, let's give an intuitive definition. Let A_1^t and A_2^t be two TA defined over V_1 and V_2 with $V_1 \subseteq V_2$. The product of these two TA is a TA specifying a system which may execute *all and only* the infinite timed traces which both :

are accepted by A_2^t , and whose projections (Def. 3) on V_1 are accepted by A_1^t .

Definition 21. (Product over V_1 and V_2 with $V_1 \subseteq V_2$)

Let $A_i^t = (Q_i, V_i, T_i, \mathcal{V}_i, \delta_i, q_{i0})$, for $i=1,2$, be two TA (Def.13) over alphabets V_1 and V_2 , with $V_1 \subseteq V_2$, $T_1 \cap T_2 = \emptyset$, and $\mathcal{V} = \{V_{C1}, \dots, V_{C_{Nc1}}\}$, i.e., each A_i^t uses a set $C_i = \{C_{i1}, \dots, C_{i_{Nc1}}\}$ of counters where each C_{ij} is w.r.t. V_{Cij} . Their synchronized product, noted $A_1^t \otimes A_2^t$, is defined by :

$$A_1^t \otimes A_2^t = (Q, V_2, T_1 \cup T_2, \mathcal{V}_1 \cup \mathcal{V}_2, \delta, q_0) = \text{Ext}_{V_2}(A_1^t) \times A_2^t \quad (\text{See Def.18 and 20 for } \times \text{ and } \text{Ext}_{V_2}(A_1^t)). \quad \blacksquare$$

Theorem 2. If $\mathcal{L}_{A_1^t}$ and $\mathcal{L}_{A_2^t}$ are respectively the timed languages accepted by A_1^t and A_2^t respectively over alphabets V_1 and V_2 , with $V_1 \subseteq V_2$, then: $\mathcal{L}_{A_1^t \otimes A_2^t} = \mathcal{L}_{\text{Ext}_{V_2}(A_1^t)} \cap \mathcal{L}_{A_2^t}$. ■

Property 3. Let A_1^t and A_2^t be two TA, respectively over alphabets V_1 and V_2 with $V_1 \subseteq V_2$. If A_2^t respects the finiteness property (FP), then $A_1^t \otimes A_2^t$ respects the FP. ■

Remark4. (a) in Definition 21, if $V_1 = V_2$, then $A_1^t \otimes A_2^t = A_1^t \times A_2^t$ (Def. 18), because $\text{Ext}_{V_2}(A_1^t) = A_1^t$;

(b) From Theorem 2, we deduce that if A_1^t and A_2^t specify two sequential processes respectively over alphabets V_1 and V_2 with $V_1 \subseteq V_2$, then *their synchronized product also specifies a sequential process*.

3.6. General parallel product of two timed automata

Before defining formally the parallel product of two TA A_1^t and A_2^t , respectively over alphabets V_1 and V_2 , let's give an intuitive definition. The product of A_1^t and A_2^t is a TA specifying a parallel system which may execute *all and only* the timed traces over the alphabet $V_1 \cup V_2$: **(a)** whose projections (Def.3) on V_1 are accepted (Def.14) by A_1^t *and* ; **(b)** whose projections on V_2 are accepted by A_2^t .

Definition 22. (Parallel product of two TA)

Let $A_i^t = (Q_i, V_i, T_i, \mathcal{V}_i, \delta_i, q_{i0})$, for $i=1,2$, be two TA over alphabets V_1 and V_2 , with $T_1 \cap T_2 = \emptyset$, and $\mathcal{V} = \{V_{C1}, \dots, V_{C_{Nc1}}\}$. Their parallel product, noted $A_1^t \parallel A_2^t$, is defined by :

$$A_1^t \parallel A_2^t = (Q, V_1 \cup V_2, T_1 \cup T_2, \mathcal{V}_1 \cup \mathcal{V}_2, \delta, q_0) = \text{Ext}_{V_1 \cup V_2}(A_1^t) \times \text{Ext}_{V_2 \cup V_1}(A_2^t). \quad \blacksquare$$

Remark 5. In Definition 22, if $V_1 \subseteq V_2$ then $A_1^t \parallel A_2^t = A_1^t \otimes A_2^t$, and if $V_1 = V_2$ then $A_1^t \parallel A_2^t = A_1^t \times A_2^t$

Theorem 3. If $\mathcal{L}_{A_1^t}$ and $\mathcal{L}_{A_2^t}$ are the timed languages accepted by two TA A_1^t and A_2^t over alphabets V_1 and V_2 , then : $\mathcal{L}_{A_1^t \parallel A_2^t} = \mathcal{L}_{\text{Ext}_{V_1 \cup V_2}(A_1^t) \cap \mathcal{L}_{\text{Ext}_{V_1 \cup V_2}(A_2^t)}}$ ■

Property 4. If two TA A_1^t and A_2^t , respectively over alphabets V_1 and V_2 , respect the finiteness property, then $A_1^t \parallel A_2^t$ respects the finiteness property. ■

Example 3. Let $A_1^t = (Q_1, V_1, T_1, \tau_1, \delta_1, q_{10})$ and $A_2^t = (Q_2, V_2, T_2, \tau_2, \delta_2, q_{20})$ (Figure 2), with $\mathcal{V} = \{V_{c1}\} = \{V_1\}$, $Q_i = \{q_{i0}, q_i\}$, $T_i = \{T_{i1}, T_{i2}\}$, $Mc = Mc_{i1} = 10$, for $i=1,2$. $V_1 = \{a, b\}$, $V_2 = \{a, c\}$. The values of timers T_{11} , T_{12} , T_{21} and T_{22} , are respectively t_{11} , t_{12} , t_{21} and t_{22} , and the values of counters C_1 and C_2 , are respectively c_1 and c_2 .

$\delta_1 = \{[q_{10}, a, q_1, E_{11}, \{T_{11}\}, K_1], [q_1, b, q_{10}, E_{12}, \{T_{12}\}, K_1]\}$, $E_{11} = (t_{11} \leq 5)$, $E_{12} = (t_{11} \leq 2) \wedge (t_{12} \leq 5)$, $K_1 = (c_1 < 10)$.

$\delta_2 = \{[q_{20}, a, q_2, E_{21}, \{T_{21}\}, K_2], [q_2, c, q_{20}, E_{22}, \{T_{22}\}, K_2]\}$, $E_{21} = (t_{21} \leq 3)$, $E_{22} = (t_{21} > 3)$, and $K_2 = (c_2 < 0)$.

$\text{Ext}_{V_1 \cup V_2}(A_1^t)$ and $\text{Ext}_{V_2 \cup V_1}(A_2^t)$ are on Figure 3, and the product of the two parallel TA is on Figure 4.

The F_Conditions (Def. 10) of transitions in $A_1^t \parallel A_2^t$ (Fig. 4) are as follows.

Transitions with event a are enabled only if both $(c_1 < 10)$ and $(c_2 < MA)$ are true ($a \in V_{c1} \cap V_{c2}$).

Transitions with event b are enabled only if $(c_1 < 10)$ is true (because $b \in V_{c1}$).

Transitions with event c are enabled only if $(c_2 < 10)$ is true (because $c \in V_{c2}$).

Figure 2. Two concurrent automata

Figure 3. Extensions of the two concurrent automata of figure 3

Figure 4. Synchronized product $A_1^t \parallel A_2^t$

Definition 23. (Independent and concurrent DES)

Let A_i^t be two TA over alphabets V_i , for $i=1, 2$, specifying two processes.

If $V_1 \cap V_2 = \emptyset$, the two processes are independent with each other.

If $V_1 \cap V_2 \neq \emptyset$, the two processes are concurrent. In fact, they may run in parallel by executing respectively events of $V_1 - V_2$ and $V_2 - V_1$, but they must execute conjointly events of $V_1 \cap V_2$. ■

4. Untiming a timed specification

4.1. Untimed traces and untimed languages

So far, an infinite sequence of events has been represented by a timed trace $\text{Trc} = \langle \sigma_1, \tau_1 \rangle \dots \langle \sigma_i, \tau_i \rangle \dots$

If we use the event *tick* which represents the passing of one uct, the same sequence can be represented by an untimed trace $\text{TRC} = \alpha_1 \alpha_2 \dots \alpha_j \dots$, where each α_j for $j=1, 2, \dots$ is equal to *tick* or to one of $\sigma_1, \sigma_2, \dots$

Example 4. The timed $\text{Trc} = \langle \sigma_1, 2 \rangle \dots \langle \sigma_i, 2i \rangle \dots$ can equivalently be represented by the untimed : ■

$\text{TRC} = \text{tick tick } \sigma_1 \text{ tick tick } \sigma_2 \dots \sigma_{i-1} \text{ tick tick } \sigma_i \text{ tick tick } \sigma_{i+1} \dots$

A formal definition of the untimed trace corresponding to a timed trace is the following.

Definition 24. (Untimed trace and operator *UntimeT*)

Let $\text{Trc} = \langle \sigma_1, \tau_1 \rangle \dots \langle \sigma_i, \tau_i \rangle \dots$ be a timed trace. We define the operator *UntimeT*, for obtaining the untimed trace TRC corresponding to Trc , by : $\text{TRC} = \text{UntimeT}(\text{Trc}) = \alpha_1 \alpha_2 \dots \alpha_j \dots$

with : $(\alpha_{i+\tau_i} = \sigma_i)$ and $(\alpha_j = \text{tick}, \text{ if } \exists k > 0 \text{ such that } j = k + \tau_k)$, for $i, j = 1, 2, \dots$

If Trc respects the finiteness property (Def.1), we also say that TRC respects the finiteness property.

Let's notice that the operator *UntimeT* is a bijection. ■

Property 5. Let $\text{TRC} = \alpha_1 \alpha_2 \dots \alpha_j \dots$ be a infinite untimed trace respecting the finiteness property . $\exists \text{Mc} > 0$ such that : $\forall k > 0, \exists l_1 > k, \exists l_2 > k$ with $\alpha_{l_1} \neq \text{tick}$, $l_2 - k \leq \text{Mc} + 1$, and $\alpha_{l_2} = \text{tick}$. ■

More informally, Property 5 means that the untimed TRC corresponds to an *infinite* timed trace (by $\alpha_{l_1} \neq \text{tick}$) which respects the *finiteness property* (by $l_2 - k \leq \text{Mc} + 1$ and $\alpha_{l_2} = \text{tick}$).

Definition 25. (Untimed language and operator *UntimeL*)

Let $\mathcal{L}$ be a timed language. $\mathcal{L}^u$, which is called untimed language and noted $\mathcal{L}^u = \text{UntimeL}(\mathcal{L})$, is defined by : $\mathcal{L}^u = \text{UntimeL}(\mathcal{L}) = \{\text{TRC} \mid \exists \text{Trc} \in \mathcal{L} \text{ with } \text{TRC} = \text{UntimeT}(\text{Trc})\}$ ■

Theorem 5. Let $\mathcal{L}_1$ and $\mathcal{L}_2$ be two timed languages over a same language. Then :

$$\text{UntimeL}(\mathcal{L}_1 \cap \mathcal{L}_2) = \text{UntimeL}(\mathcal{L}_2) \cap \text{UntimeL}(\mathcal{L}_1). \quad \blacksquare$$

4.2. Untimed automata

Definition 26. (Untimed automaton and operator *UntimeA*)

Let A^t be a TA over an alphabet V which accepts (Def. 14) a timed language $\mathcal{L}$, and let $\mathcal{L}^u = \text{UntimeL}(\mathcal{L})$.

A^{ut} is the minimal FSM over the alphabet $V \cup \{\text{tick}\}$, called untimed automaton (UA) which accepts the untimed language $\mathcal{L}^u$. In other words : $\mathcal{L}_{A^{ut}} = \text{UntimeL}(\mathcal{L}_{A^t})$, (Def. 25, for *UntimeL*)

A sufficient condition of existence of A^{ut} is the finiteness of the set of timers.

We also define the surjective operator *UntimeA* such that : $A^{ut} = \text{UntimeA}(A^t)$. ■

Example 5. Let's consider the timed $A^t = (Q, V, T, \{V\}, \delta, q_0)$ on Figure 5.a, where we use one timer T_1 , and one counter C_1 w.r.t. V , with $\text{Mc}_1 = 5$. In this specification, the value c_1 of C_1 is smaller than or equal to c_1 , and since $\text{Mt}_1 = 5$ (Def. 15), the value t_1 of T_1 is smaller than or equal to 6. The obtained untimed A^{ut} is on Figure 5.b, each state being defined by $\langle q, t_1, c_1 \rangle$, where q is a state in A^t .

Figure 5. Timed and untimed automata

Remark 6. Since untimed traces accepted by A^{ut} correspond to infinite timed traces accepted by A^t , then A^{ut} accepts only infinite untimed traces, and does not contain undesirable states. An undesirable state is either a deadlock state or a state from which only a selfloop *tick* is executable.

- A deadlock in A^{ut} is undesirable, because it has no sense. In fact, a deadlock state means that the event *tick* is not executable. Therefore, the passing of time is stopped!

- A state from which only a selfloop *tick* is executable is undesirable, because it implies that A^{ut} accepts a trace $\text{TRC} = \text{UntimeT}(\text{Trc})$ where Trc is a **finite** timed trace!

Informally, A^{ut} allows to represent a real-time system specified by A^t , as a system without timing requirement, but where a new event *tick* is added. This event, which models the passing of one unit of clock time (uct), is processed like any other event.

Let's give an idea of how A^{ut} is obtained from A^t over an alphabet V , when only one counter C_1 , w.r.t. $V_{C_1} = V$ is used. This implies that A^t respects the finiteness property (Theorem 1). Let $T = \{T_1, \dots, T_{N_t}\}$ be a set of timers used for defining a TA A^t , let Mt_i be the maximum value a timer T_i is compared to, for defining the $T_Conditions$ (Def. 7) of all the transitions of A^t . In this case, the value t_i of T_i does not need to be incremented as soon as $t_i = \text{Mt}_i + 1$ (Def. 15). A state of A^{ut} is defined by $\langle q_1, ts, c_1 \rangle$, where q_1 is a state of A^t , $ts = (t_1, \dots, t_{N_t})$ is a timer state (Def. 6), and c_1 is a value of C_1 . The passing of one uct is represented in A^{ut} by the event *tick*. Execution of *tick* from state $\langle q_1, ts, c_1 \rangle$ leads to state $\langle q_1, ts+1, 0 \rangle$, i.e., timers are incremented and the counter is set to zero. Execution of an event $\sigma \neq \text{tick}$ from state $\langle q_1, ts, c_1 \rangle$ of A^{ut} leads to state $\langle q_2, ts, c_1+1 \rangle$, where q_2 is a state of A^t which is reached by a transition $\text{tr} = [q_1, \sigma, q_2, E, R, K]$ from state

q_1 of A^t (with E true for the current timer state ts , and $c_1 < Mc_1$), and ts' is obtained from ts by setting to zero timers belonging to R . Besides, A^{ut} is minimal.

Remark 7. (a) if $ts = (Mt_1 + 1, \dots, Mt_{N_t} + 1)$ then $ts + 1 = ts$. In this case, an event *tick* is a selfloop in A^{ut} ;

(b) Since two A^{ut} over alphabets $V_i \cup \{tick\}$, for $i=1,2$, are FSM, we can use the classic synchronized product between them, noted $A_1^{ut} \times A_2^{ut}$, where events of $(V_1 \cap V_2) \cup \{tick\}$ are executed conjointly.

(c) The product $UntimeA(A_1^t) \times UntimeA(A_2^t)$ may contain deadlocks, therefore it does not correspond to a real DES. In fact, a deadlock prevents the event *tick*, i.e., the passing of time is stopped.

Lemmes 2. Let $A^t = A_1^t = (Q, V, T, \mathcal{V}, \delta, q_0)$ be a TA and $A^{ut} = UntimeA(A^t)$. Let's remind some notations :

(a) N_t and N_c are the numbers of timers and counters; (b) Mc bounds all the Mc_i , for $i=1, \dots, N_c$; (c) Mt is the maximum constant any timer is compared to; (d) $|Q|$ and $|I|$ are numbers of states and of transitions.

2.a. The number $|Q^{ut}|$ of states of A^{ut} is bounded by : $|Q| * (Mt + 2)^{N_t} * (Mc + 1)^{N_c}$

2.b. The number $|I^{ut}|$ of transitions of A^{ut} is bounded by : $(|Q| + |I|) * (Mt + 2)^{N_t} * (Mc + 1)^{N_c}$

2.c. The complexity for calculating A^{ut} is in :

$$O(|Q^{ut}|^2) = O(|Q|^2 * (Mt + 2)^{2 \times N_t} * (Mc + 1)^{2 \times N_c})$$

$|Q^{ut}|$, $|I^{ut}|$ and the complexity for calculating A^{ut} are then exponential in the numbers of timers and of counters. ■

Remark 8. (a) The number of counters is not really a problem. In fact, in general one counter is sufficient, for ensuring the finiteness property. Therefore, the complexity is essentially due to the number of timers. (b) If the timing requirements are only between consecutive events, one timer is sufficient for specifying temporal constraints. In this case, the complexity is no more exponential.

Properties 6. Let A_1^t and A_2^t be two TA respectively over alphabets V_1 and V_2 .

6.a. If $V_1 = V_2$, then : $UntimeA(A_1^t \times A_2^t) \leq UntimeA(A_1^t) \times UntimeA(A_2^t)$

6.b. If $V_1 \subseteq V_2$, then : $UntimeA(A_1^t \otimes A_2^t) \leq UntimeA(A_1^t) \times UntimeA(A_2^t)$

6.c. If $V_1 - V_2 \neq \emptyset$ and $V_2 - V_1 \neq \emptyset$, then : $UntimeA(A_1^t \parallel A_2^t) \leq UntimeA(A_1^t) \times UntimeA(A_2^t)$

6.d. If $V_1 = V_2$, then : $\mathcal{L}_{A_1^t} \subseteq \mathcal{L}_{A_2^t} \Rightarrow \mathcal{L}_{UntimeA(A_1^t)} \subseteq \mathcal{L}_{UntimeA(A_2^t)}$

6.e. If $V_1 \subseteq V_2$, then : $UntimeL(Proj_{V_1}(\mathcal{L}_{A_2^t})) = Proj_{V_1}(UntimeL(\mathcal{L}_{A_2^t}))$

(where $A \leq B$ means $\mathcal{L}_{A^{ut}} \subseteq \mathcal{L}_{B^{ut}}$). ■

4.3. Why untimed automata are useful.

Problem of using timed automata (TA) :

(a) Respecting the timing requirements does not ensure to avoid deadlock states.

(b) Timing requirements between events are specified by using some fictitious timers. Therefore, if we project a TA into an alphabet, a few events may disappear. In this case, we have to respecify temporal requirements between events, and then we have to redefine new fictitious timers. This is not self-evident.

Interest of using untimed automata (UA) :

A UA is a FSM. Therefore, all known methods used for FSMs can be used. Let's see two examples :

(a) States respecting in general a certain "indesirable" property, in particular deadlocks states, may be removed; (b) a UA defined over an alphabet $W' = W \cup \{tick\}$ can be projected in any alphabet $V \subseteq W'$.

Thus, before making some processings, a TA is untimed for obtaining a UA. But after the processing, it is convenient to transform the processed UA into an automaton without *tick*. This is the object of Section 5.

5. Timing untimed automata

We propose an operator *TimeA* which, from a UA A^{ut} , generates a *temporized* automaton with a new model different than the model of timed automata. A logic question arises : why two different models are used for specifying a DES ?

The first model, previously defined, is used because it is more intuitive. In the case where a TA must specify a desired behaviour, it may be constructed quite easily from an informal specification. In fact, the timers and counters are convenient fictitious entities which may be defined intuitively.

The second model is less intuitive, but it can be automatically and easily constructed from a UA. It uses only timers. Informally, if the alphabet of A^{ut} is $V \cup \{tick\}$, $TimeA(A^{ut})$ is obtained as follows.

(a) A^{ut} is projected into the alphabet V , for obtaining $Proj_V(A^{ut})$; (b) For each state q of $Proj_V(A^{ut})$, several timers T_{q_i} are defined, and their values t_{q_i} are incremented at each *tick*..

- (c) For each transition Tr of $Proj_V(A^{ut})$ which is executable from a state q_1 and leads to a state q_2 :
- Several enabling conditions $E_i(tq_{1i})$ are defined. Each $E_i(tq_{1i})$ depends on a timer Tq_{1i} of state q_1 .
 - Several initializations $tq_{2j} \leftarrow F_j(tq_{1i})$ are defined. Each initialization consists in initializing a timer Tq_{2j} of q_2 in function of the value tq_{1i} of a timer Tq_{1i} of q_1 . The index j also depends on tq_{1i} .

The semantics of the enabling conditions and of the initializations is the following.

- When a timer is initialized by an initialization, it becomes the **active timer**.
- The transition Tr may be executed only if its *enabling condition* depending of the active timer is true.
- When Tr is executed, a timer Tq_{2j} is *initialized* in function of the value of the current active timer. Tq_{2j} becomes the new active timer.

Let's give an idea of how the timers are defined. Each state q of $TimeA(A^{ut})$ (and of $Proj_V(A^{ut})$) corresponds to a group G_q of states of A^{ut} closed under the event tick. The group G_q may be composed by several sequences S_{q_i} of states. Each S_{q_i} contains a state, called first state, without ingoing tick, and all other states of S_{q_i} are reachable from this first state by executing a few ticks. To each S_{q_i} , we associate a timer Tq_i whose value is equal to zero in the first state of S_{q_i} . That is why several timers may be associated to each state of $TimeA(A^{ut})$.

If we consider the untimed A^{ut} of Figure 5.b, $TimeA(A^{ut})$ is represented on Figure 6. One timer T_1 (resp. T_2) is defined for state 1 (resp. state 2). For the transition from state 1 to state 2 : Its enabling condition is $E_1(t_1) = (t_1 \leq 5) \wedge (t_1 > 0)$, and its initialization is $t_2 \leftarrow F_1(t_1) = t_1 - 1$. For the two transitions from state 2 to state 1 : Their enabling condition is True, and their initialization is $t_1 \leftarrow F_2(t_2) = 0$.

Figure 6. Temporized automaton

Figure 7. Supremal model $SupMed_{i,j}^d$ of the medium for a pair (PE_i, PE_j)

We will see in the next section, that the untiming and timing operations may be convenient to resolve a real problem, such as designing real-time protocols.

6. Deriving protocols specifications providing real-time services

The desired service specification and the supremal model of the medium are the two starting points of the protocol derivation, and they are defined by timed automata (Sect. 3).

6.1. Supremal model of the medium

Let's firstly remind that the supremal model of the medium is used in order to avoid the derivation of timing requirements impossible to respect by the medium (Sect. 2). The medium is assumed reliable and its supremal model is the following :

when PE_i sends a message m to PE_j , then the transit delay of m in the medium belongs to an interval $I_{i,j} = [t_{i,j}^{\min}, t_{i,j}^{\max}]$, where $t_{i,j}^{\min}$ and $t_{i,j}^{\max}$ are constant integers such that $1 \leq t_{i,j}^{\min} \leq t_{i,j}^{\max} < \infty$. Therefore, we suppose that there is at least one *tick* (Sect. 3) during the transmission of a message. For each pair (PE_i, PE_j) , this supremal model can be represented by a TA $SupMed_{i,j}^d$ represented on Figure 7.

For each pair (PE_i, PE_j) , the TA $SupMed_{i,j}^d$ (Fig.7) is defined by $(Q_{i,j}, V_{i,j}, \{T_{i,j}\}, \{V_{i,j}\}, \delta_{i,j}, q_{0,i,j})$, where $Q_{i,j} = \{q_{0,i,j}, q_{1,i,j}\}$, $V_{i,j} = \{s_i^j, r_j^i\}$, $\delta_{i,j} = \{(q_{0,i,j}, s_i^j, q_{1,i,j}, True, \{T_{i,j}\}, True), (q_{1,i,j}, r_j^i, q_{0,i,j}, E_{i,j}(t_{i,j}), \emptyset, True)\}$, with $E_{i,j}(t_{i,j}) = (t_{i,j} > (t_{i,j}^{\min} - 1)) \wedge (t_{i,j} \leq t_{i,j}^{\max})$.

$SupMed_{i,j}^d$ uses one timer $T_{i,j}$ (Def.5) for defining timing requirements between s_i^j and r_j^i , where the event s_i^j means "PE_i sends a message to PE_j", and the event r_j^i means "PE_j receives a message coming from PE_i". A counter is not necessary, because timing requirements ensure the finiteness property (Def.1) due to $(t_{i,j} > (t_{i,j}^{\min} - 1))$ in the $T_Condition$ $E_{i,j}(t_{i,j})$. Therefore, the $F_Condition$ is True.

Remark 9. Timing requirements on $SupMed_{i,j}^d$ and $SupMed_{j,i}^d$ may be different.

6.2. Protocol derivation for sequential real-time systems

6.2.1. Service specification

The desired service is, with the supremal model of the medium, one of the two starting points of the

protocol derivation. It is described by a TA, noted SS^t and defined by $(Q, V, T, \{V\}, \delta, q_0)$, where V is the set of interactions with the user, and T is the set of timers (Def.5) used for defining timing requirements between these interactions. Only one counter C , w.r.t. V (Def.8), is used, and the finiteness property (Def.1) is respected (Property 1). Informally, no more than Mc service primitives are executed during one unit of the global clock time (uct). Each event of V is represented by A_i , where A is the name of the primitive executed, and i identifies the protocol which executes A .

Example 6. Here is a very simple service specified by $SS^t = (Q, V, T, \{V\}, \delta, q_0)$ (Fig.8.a), with $Q = \{q_0, q_1\}$, $V = \{A_1, B_2\}$, $T = \{T_1\}$, $\delta = \{[q_0, A_1, q_1, E(t_1), \{T_1\}, K(c_1)], [q_1, B_2, q_0, E(t_1), \{T_1\}, K(c_1)]\}$, with $E(t_1) = (t_1 \leq 2)$, and $K(c_1) = (c_1 < 1)$. Informally, SS^t uses one timer T_1 and one counter C_1 , and specifies that : (a) events A_1 and B_2 are executed alternatively; (b) at most one event occurs between two ticks (F_Condition $K(c_1)$); (c) at most two ticks occur between two consecutive events (T_Condition $E(t_1)$). $SS^u = \text{UntimeA}(SS^t)$ (Def.26) is represented on Figure 8.b, and its states are $\langle q, t_1, c_1 \rangle$ where q is a state of SS^t .

Figure 8. Timed and untimed sequential service specification

Definition 27. (outgoing, ingoing, out(q), in(q), out_i(q), nbout(q))

Let SS^t be a TA specifying a timed sequential service, and let q be one of its states.

Outgoing (resp. *ingoing*) transitions of q are transitions which are executable from (resp. lead to) q .

out(q) (resp. in(q)) contains identifiers of SAP where outgoing (resp. ingoing) transitions of q occur.

out_i(q) is the set of states of SS^t reachable from q by transitions executed by PE_i .

nbout(q) is the number of transitions executable from q . ■

Example 7. for SS^t of Example 6 (Figure 8.a), $\text{in}(q_0) = \{2\}$, $\text{in}(q_1) = \{1\}$, $\text{out}(q_0) = \{1\}$, $\text{out}(q_1) = \{2\}$, $\text{out}_1(q_0) = \{q_1\}$, $\text{out}_2(q_0) = \emptyset$, $\text{out}_1(q_1) = \emptyset$, $\text{out}_2(q_1) = \{q_0\}$, $\text{nbout}(q_0) = \text{nbout}(q_1) = 1$. ■

Since the starting points of the protocol derivation, i.e., the supremal model of the medium and the specification of the desired service, have been defined, we can propose a systematic method for deriving the specifications of : (a) the local protocol entities; (b) the necessary timing requirements on the medium, (Sect. 2). These derived specifications are first untimed automata (Def.26), and then are timed by using operator *TimeA* (Sect. 5).

6.2.2. Transformation of the service specification

The first thing to do is to transform SS^t into another timed automaton TSS^t (Transformed SS^t) with the following rules.

First step : each timed transition of SS^t :

is replaced by :

A new state q_1' is then inserted between each pair of states q_1 and q_2 connected by a transition.

q_1 and q_1' are connected by an internal transition $i(q_2)$ parameterized by q_2 .

After this first step, we obtain a TA noted TS^t . Let's notice that if a state of TS^t is reachable by an internal transition $i(q)$, then its outgoing transitions are not internal.

Second step : The specification TS^t is transformed into an equivalent (Def.14) TSS^t , such that every state q_1 of TSS^t respects either condition C1 or condition C2, defined below.

C1 = only an internal transition $i(q)$ is executable from q_1 (Fig. 9.a),

C2 = no internal transition is executable from q_1 , and all outgoing transitions (Def.27) of q_1 are executable by a same protocol entity, i.e., cardinal of $\text{out}(q_1)$ is equal to one ($|\text{out}(q_1)| = 1$) (Fig.9.b). On Figure 9.b., $\text{out}(q_1) = \{k\}$ and $\text{out}_k(q_1) = \{r_1, \dots, r_p\}$.

9.a. internal outgoing transition

9.b. non internal outgoing transitions

Figure 9. Outgoing transitions in a state of the transformed specification TSS^t .

received by a PE_j with a different second parameter $q'2$ (event $r_j^i(q, q'2)$). This means that the medium not only carries messages, but it also modifies their second parameters. This modification informs the receiving protocol entity about the transit delay of the message in the medium.

Step 6 : For each PE_i , the untimed automaton PS_i^{ut} is derived by projecting GPS^{ut} in the alphabet $V_i \cup \{tick\}$, where V_i contains all events in GPS^{ut} executed by PE_i . An event of V_i may correspond to :

(a) execution of a primitive by PE_i ; (b) an event $s_i^j(q, q2)$; (c) an event $r_j^i(q, q2)$, with $j, k \neq i$.

Step 7 : For each pair (PE_i, PE_j) and each q , where PE_i sends to PE_j a message whose first parameter is q (i.e., events $s_i^j(q, *)$ and $r_j^i(q, *)$ exist in GPS^{ut}), the untimed automaton $ReqMed_{i,j}^{ut}(q)$ is generated by projecting GPS^{ut} in the alphabet $V_{i,j}(q) \cup \{tick\}$. An element of $V_{i,j}(q)$ may be any event $s_i^j(q, *)$ and $r_j^i(q, *)$ of GPS^{ut} . The obtained $ReqMed_{i,j}^{ut}(q)$ specifies the behaviour of the medium when it carries, from PE_i to PE_j , a message whose first parameter is q .

The informal semantics of the different PS_i^{ut} (Step 6) and $ReqMed_{i,j}^{ut}(q)$ (Step 7) is the following. Let n be the number of protocol entities PE_i , for $i=1, \dots, n$. If PE_i are specified by PS_i^{ut} , and if the medium respects the specifications $ReqMed_{i,j}^{ut}(q)$, then the service SS^t is totally or partially provided (Def.29 and 30).

Step 8 : The untimed specifications PS_i^{ut} and $ReqMed_{i,j}^{ut}(q)$ obtained at Steps 6 and 7 are timed, by using the operator *TimeA* (Sect.5). End of *Der_Seq_Prot* ■

Remark 11. (a) In Steps 1, 2, 3, SS^t , TSS^t , $MedSS_b^t$, and $MedSS^t$ use the same counter $C1$, w.r.t. the alphabet V of SS^t ; (b) $MedSS_b^t$ and $MedSS^t$ use timers of SS^t and a timer $T_{i,j}$ for each pair (PE_i, PE_j) where PE_i sends a message to PE_j .

Definition 29. (Provided service $PrSS^{ut}$) For obtaining an untimed automaton (with event *tick*) specifying the service provided to the user, and noted $PrSS^{ut}$, one only has to project $MedSS^{ut}$ (Step 4) in $V \cup \{tick\}$, where V is the alphabet of SS^t . Informally, this projection consists in keeping visible, in sequences accepted by $MedSS^{ut}$, only events of SS^{ut} . ■

Definition 30. (Service totally or partially provided)

Let SS^{ut} and $PrSS^{ut}$ be untimed automata specifying respectively the desired and the provided service.

The service is said *totally provided* if and only if : $PrSS^{ut} \equiv SS^{ut}$, i.e., $\mathcal{L}_{PrSS^{ut}} = \mathcal{L}_{SS^{ut}}$,

The service is said *partially provided* if and only if : $PrSS^{ut} < SS^{ut}$, i.e., $\mathcal{L}_{PrSS^{ut}} \subset \mathcal{L}_{SS^{ut}}$. ■

Theorem 6. If SS^t specifies a desired service, let $SS^{ut} = \text{UntimeA}(SS^t)$ (Def.26), and let $PrSS^{ut}$ be the specification of the provided service (def.30). Then : $PrSS^{ut} \leq SS^{ut}$ (i.e., $\mathcal{L}_{PrSS^{ut}} \subseteq \mathcal{L}_{SS^{ut}}$). ■

The safety is then ensured.

6.2.4. Example

We consider the SS^t of Example 6 (Fig.8.a), with $q0=1$ and $q1=2$. The supremal model of the medium is defined by $SupMed_{1,2}^t$ and $SupMed_{2,1}^t$ (Sect.6.1, Fig.7), with $t_{1,2}^{\min} = t_{2,1}^{\min} = 1$ and $t_{1,2}^{\max} = t_{2,1}^{\max} = 2$. Informally, during the transmission of a message, one or two ticks of the global clock may occur.

By using the procedure *Der_Seq_Prot* (Sect.6.2.3), we obtain, after the seventh step, the specifications

PS_1^{ut} and PS_2^{ut} of Figure 12, and $ReqMed_{1,2}^{ut}(2)$ and $ReqMed_{2,1}^{ut}(1)$ of Figure 13.

By using the operator *TimeA* (Sect. 5), we obtain the temporized automata represented on Figure 14.

As an example, *TimeA*(PS_1^{ut}) (Fig. 14.a) is detailed below.

TimeA(PS_1^{ut}) : Timers $T11$ and $T12$ are respectively associated to states 1 and 2.

Timers $T13_1$ and $T13_2$ are associated to state3.

Transition $Tr11$: Event a ; EC : $t11 \leq 2$; ----- In : $t12 \leftarrow 0$.

Transition $Tr12$: Event $s_1^2(2,3)$; EC : $(t12=0)$; ----- In : $t13_1 \leftarrow 0$.

Transition $Tr13$: Event $s_1^2(2,7)$; EC : $(t12=1)$; ----- In : $t13_2 \leftarrow 0$.

Transition Tr14 : Event $r_1^2(1,2)$; ECs : $(t13_1 > 1) \wedge (t13_1 \leq 3)$, $(t13_2 = 1)$; ----- In : $t11 \leftarrow 1$.

Transition Tr15 : Event $r_1^2(1,5)$; EC : $(t13_1 > 2) \wedge (t13_1 \leq 4)$, $(t13_2 = 3)$; ----- In : $t11 \leftarrow 2$.

Figure 12. Obtained untimed protocol

Figure 13. Obtained untimed medium

Figure 14. Obtained timed specifications

6.3. Protocol derivation for parallel and concurrent real-time systems

6.3.1. Introduction

For the sake of simplicity, we only consider a parallel system composed by only two sequential systems.

A desired parallel service is then specified by two TA (Def.13) $SS^i[i]$ over alphabets $V[i]$, for $i=1,2$. Each $SS^i[i]$ specifies a sequential desired service. Let's consider three cases :

a) $V[1] \subseteq V[2]$: $SS^1 = SS^1[1] \otimes SS^1[2]$ (Def.21) is a sequential service (Remark 3.b), and we may use the procedure *Der_Seq_Prot* (sect.6.2.3) for deriving the protocol providing the service specified by SS^1 .

b) $V[1] \neq \emptyset$ and $V[1] \cap V[2] = \emptyset$, for $i,j=1, 2$, and $i \neq j$: $SS^1[1]$ and $SS^1[2]$ are independent and compose a parallel system (Def.23). We may process each sequential service separately, i.e., for each $SS^i[i]$, we use *Der_Seq_Prot* for deriving the sequential protocol which provide $SS^i[i]$.

c) $V[1] - V[2] \neq \emptyset$ and $V[1] \cap V[2] \neq \emptyset$, for $i,j=1, 2$, and $i \neq j$: $SS^1[1]$ and $SS^1[2]$ are dependent and compose a concurrent system (Def.23). This case is studied in detail in the present Section 6.3.

6.3.2. Solution for the problem of the choice

In a concurrent system, we think that one of the main problems consists in avoiding possible deadlocks.

For that, we use the following approach, already proposed in [KHB92] :

During the running of a concurrent distributed system, when a choice is possible between execution of different transitions, this choice is made locally by a same protocol entity.

Let PE_i , for $i=1, \dots, n$, be the n protocol entities which execute the transitions of $SS^1[1]$ and $SS^1[2]$. For the sake of simplicity, we suppose that all choices are made by a same protocol entity PE_c , with $c > n$. In other words, if there is a choice to make, the protocol entities PE_i , for $i=1, \dots, n$, "pass the buck" to PE_c . Such constraint seems too restrictive, and we intend to weaken it in a next version. To enforce explicitly this

choice, we must add to $SS^i[1]$ and $SS^i[2]$, some timed events (Def.11) noted $(i_c, \text{True}, \emptyset, \text{True})$, where i_c is executed by PE_c . These timed events are added as follows : for each state q of $SS^i[i]$, for $i=1, 2$, where $\text{nbout}(q) > 1$ (Def.27), its outgoing transitions $Tr_1, \dots, Tr_m$ represented in Figure 15.a. are replaced by the structure of Figure 15.b. The obtained specifications are noted $SS^i_1[1]$ and $SS^i_1[2]$. Let's now propose a procedure of protocol derivation for concurrent systems.

Figure 15. Adding events i_c

6.3.3. Procedure of protocol derivation for a concurrent system

Let two TA $SS^i[i]$ over alphabets $V[i]$, for $i=1,2$, and a TA $\text{SupMed}_{u,v}^{\#}$ for each pair (PE_u, PE_v) , the procedure of protocol derivation for concurrent systems, called *Der_Conc_Prot*, consists of nine steps.

Step one : $SS^i[i]$ are modified into $SS^i_1[i]$, for $i=1,2$, (sect.6.3.2.). Besides, any two states of respectively $SS^i_1[1]$ and $SS^i_1[2]$ must be identified differently. This is necessary for not confusing exchanged messages, which are parameterized by identifiers of states (see *Der_Seq_Prot* in Sect.6.2.3.).

Step two : Steps 1 to 5 of *Der_Seq_Prot* are applied to each $SS^i_1[i]$ for obtaining $\text{GPS}_e^u[i]$, for $i=1,2$, but with the following difference. At the third step of *Der_Seq_Prot*, not only transitions ϵ , but also transitions $(i_c, \text{True}, \emptyset)$, are removed. Let $V_g[i] \cup \{\text{tick}\}$ be the alphabet of $\text{GPS}_e^u[i]$, then $V[i] \subseteq V_g[i]$.

Step three : The synchronized product $\text{GPS}_e^u = \text{GPS}_e^u[1] \times \text{GPS}_e^u[2]$ is computed (Remark 7.b).

Step four : Indesirable states are removed from GPS_e^u for obtaining GPS^u . A state is undesirable if it is either a deadlock or only a selfloop *tick* is executable from it (Remark 6). For removing undesirable states, we may use a fixpoint method similar to the one used in the control theory for computing controllable languages (WR87,KBD94).

Step five : The protocol specification PS^u of PE_c (Sect.6.3.2) is obtained by projecting GPS^u in alphabet $V_c \cup \{\text{tick}\}$. V_c contains all events of GPS^u executed by PE_c , and these events are of the forms $s_c^*(*,*)$ and $r_c^*(*,*)$ (see Sect.6.1, and Step two of *Der_Seq_Prot*), where $*$ may be any parameter.

Step six : the sequential $\text{GPS}^u[i]$ are obtained by projecting GPS^u in alphabets $V_g[i] \cup \{\text{tick}\}$ of $\text{GPS}_e^u[i]$, (Step 2), for $i=1, 2$. The sequential processes specified by $\text{GPS}^u[i]$, for $i=1,2$, interact with PE_c specified by PS^u and do not lead to an undesirable state.

Step seven : For each $\text{GPS}^u[i]$ (for $i=1,2$), we apply Step 6 of *Der_Seq_Prot* for obtaining the untimed automata (UA) $\text{PS}_j^u[i]$ corresponding to PE_j ($j=1, \dots, n$).

Step eight : For each $\text{GPS}^u[i]$ (for $i=1,2$), we apply Step 7 of *Der_Seq_Prot* for obtaining the UA $\text{ReqMed}_{j,k}^u(q)$. Each $\text{ReqMed}_{j,k}^u(q)$ depends implicitly on i , because q identifies a state of $SS^i_1[i]$, and states of $SS^i_1[1]$ and $SS^i_1[2]$ are identified differently (see Step one).

The informal semantics of PS_j^u (Step 5), of $\text{PS}_j^u[i]$ (Step 7), and of $\text{ReqMed}_{j,k}^u(q)$ (Step 8) is the following.

If each PE_j , for $j=1, \dots, n$, is a parallel system specified by two $\text{PS}_j^u[i]$, for $i=1,2$, and if the medium respects the specifications $\text{ReqMed}_{j,k}^u(q)$, then the desired concurrent service specified by $SS^i_1[1]$ and $SS^i_1[2]$ (step one), is totally or partially provided by the help of PE_c specified by PS_e^u (Step 5).

Step nine : The untimed specifications obtained at Steps 5, 7 and 8 are timed, by using the operator *TimeA* (Sect.5).

End of *Der_Conc_Prot* ■

7. Conclusion

In this paper, we present a model we have developed for specifying real-time discrete event systems. An application of the model for designing real-time protocols is also proposed. The synthesis approach used for deriving a real-time protocol providing a desired service is inspired by other works, but our main contribution has been to consider *real-time* systems, i.e., systems containing timing requirements. We conclude this study by making an informal and succinct comparison between our model for specifying

DESSs, and the two models which have mainly inspired us. A few extensions are also proposed.

First model ([Os90, BW92, OW90]) : For defining a TA A^t , a global clock and a set of timers are used. For each transition Tr_i of A^t corresponds one timer T_i . This timer is reset only when a state q_1 , from which Tr_i is executable, is reached. This same timer is not used in another state $q_2 \neq q_1$. The enabling condition of Tr_i is that the value of T_i must belong to an interval. We think that our model is more general because (in our model): (a) the enabling condition of a transition may depend on several timers; (b) a same timer may be used in the enabling conditions of several transitions; (c) a timer may be reset at the occurrence of any transition; (d) the finiteness property may be ensured by using counters; (e) the operator *TimeA* is defined.

Second model ([AD90, TH92]) : For defining a timed automata, a dense time and a set of clocks are used. These clocks are used as we use the timers in our model, but the semantics is quite different, because the clocks are not synchronized. Besides : (a) Our operators *UntimeA* and *TimeA* are different than operators *Untime* and *Time* proposed in [TH92]. In fact in [TH92], from a TA A^t , the operator *Untime* is used for obtaining a UA A^{ut} . Some processing is then made on A^{ut} for obtaining a UA B^{ut} . And then the operator *Time* can be applied on B^{ut} only if the processing for obtaining B^{ut} from A^{ut} makes no projection, i.e., A^{ut} and B^{ut} have a same alphabet; (b) The finiteness property is supposed respected in [TH92], but it is not ensured. Another advantage with our model is then that we can ensure the finiteness property. (c) In [TH92] the composition is defined only when the two TA have a same alphabet, and in [AD90], authors only specify how events executed conjointly by the two composed systems are processed. Our limitation is that the time is discrete.

Extensions : The exponential complexity in the number of timers imposes to investigate how to choose classes of systems which avoid this computational blow-up. The simplest but also the most restrictive class contains systems respecting the following condition : timing requirements are only between consecutive events. With such systems, only one timer is necessary, therefore the complexity becomes polynomial. In the presented model, the enabling conditions use only operators $\leq$, $>$ and $=$ for defining canonical boolean functions. We are also investigating how we can modify systematically several existing protocol entities, which provide an old service, for providing a new desired service. For that, we intend to use control theory of the discrete event systems .

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