Approximation of non-normal responses for drag dominated offshore structures

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Abstract

For deep water offshore structures, the modeling of random waves loads by the Morison equation yields an equation of motion which admits no analytical solution except in few limit cases. This also holds for response moments which have a great influence on the results of reliability analyses. If polynomial approximations of the Morison drag loads are introduced, some procedures are available to obtain the stationary moments of the approximate response. These procedures result in large computer codes and time consuming computations if large structures are considered. Therefore, only first order approximations of the drag nonlinearity usually can be handled. In the paper, the ability of two famous linearization schemes to predict response variance and response fourth moment is numerically investigated. Improvements of these schemes are considered. For quasi-static structures, it is shown that except in very simple cases it is practically impossible to optimize linearization schemes to fit response variance or fourth moment.

1. Introduction

Non-linearities are faced in many physical problems. In the engineering field, they arise for example in the modelling of loads and material properties. If one considers the particular case of deep water offshore structures, the main nonlinearity generally is in the wave loading. Indeed, the force exerted by waves on any tubular member is described by the extended Morison equation which consists of a nonlinear drag term and a linear mass term. Other nonlinearities are often neglected. For instance the sea elevation, water particle velocity and acceleration processes generally are assumed stationary Gaussian with zero-mean during sea states of several hours duration and wave-current usually is neglected. For a discussion of these problems see [1].

In the following we consider a single-degree-of-freedom system. Under the aforementioned assumptions, the equation of motion is

$$m\ddot{y} + c\dot{y} + ky = K_D (x - \dot{y}) |x - \dot{y}| + K_M \dot{x} - K_A \ddot{y} \quad (1)$$

where $|\cdot|$ denotes absolute value; $K_D = \frac{1}{2}\rho DC_D$, $K_M = \frac{1}{4}\pi\rho D^2 C_M$ and $K_A = \frac{1}{4}\pi\rho D^2 (C_M - 1)$; C_D and C_M are the drag and the inertia coefficients; ρ is the water density; D is the member diameter; m , c and k are structural parameters; x and y denote the water particle velocity and the structural displacement.

If equation (1) is solved, member forces and stresses can be calculated. Some of these responses may further be used for reliability assessment or in fatigue life estimation. It is

well known that results of these analyses are accurate only if a good fit of the extreme value behaviour of these processes is achieved. If the approximation due to Winterstein [2] is used, only the first four order moments of these processes need to be known. For equation (1), however, the estimation of even low order moments of y is not straightforward. The statistical moments of $y(t)$, of course, can be exactly calculated through time domain simulations but this approach is excessively time consuming. Therefore, approximations must be introduced.

If the nonlinear term $(x - \dot{y}) |x - \dot{y}|$ is approximated by a polynomial, then a Volterra series representation of the response can be obtained [3, 4, 5]. If a spectral description of the response is chosen, then stationary response moments result from the integration of higher order spectra in the frequency domain. This method, however, is very time consuming as it involves laborious multidimensional convolutions. Moreover, it is not clear whether this method could be successfully extended to large problems. So far, only simple linear spectral analysis appears possible in practical cases, i.e. when large structures and numerous load cases (sea states) have to be considered. Then the nonlinear term $(x - \dot{y}) |x - \dot{y}|$ is simply replaced by a linear term $A\sigma_{x-\dot{y}}(x - \dot{y})$, where A is a constant and $\sigma_{x-\dot{y}}$ denotes the standard deviation of $(x - \dot{y})$ as proposed already by Borgman [3].

It is obvious, however, that any first order approximation is a severe simplification because the response process then is globally assumed to be Gaussian. Also, sub- and superharmonic responses are underpredicted. Indeed, some authors found that linearization procedures may be excessively inefficient in predicting the response variance in some cases (see, e.g., [6]). Before investigating the possibility to extend the aforementioned method to larger systems and to implement such method in appropriate computer codes, it therefore appears necessary to study in detail the error involved in the variance linearization schemes. The emphasis here is on two linearization schemes which set out from random seas and are widely used in offshore engineering. Analysis is performed in the time domain for a standardized form of the equation of motion (1) suggested in [7] so that conclusions are valid in general. Further, following an idea by Naess et al [8] possible extensions of the linearization schemes for estimation of higher order moments are investigated. This idea is quite attractive because if response moments up to the fourth order could be determined, then at most four spectral analyses each one with the appropriate linearization constant would enable to fit the response by non-Gaussian models such as proposed by Winterstein [2].

2. Standardized equation of motion

Introducing the so called hydrodynamic mass $m_{hyd} = K_A$, we rewrite (1) in the form

$$(m + m_{hyd})\ddot{y} + c\dot{y} + ky = K_D(x - \dot{y})|x - \dot{y}| + K_M\dot{x} \quad (2)$$

The left hand side of equation (2) is further standardized in a classical way

$$\ddot{y} + 2\xi_s\omega_s\dot{y} + \omega_s^2y = \frac{K_D}{m + m_{hyd}}(x - \dot{y})|x - \dot{y}| + \frac{K_M}{m + m_{hyd}}\dot{x} \quad (3)$$

where ω_s and ξ_s denote the system natural frequency and damping. In the following, it is assumed that a Pierson-Moskowitz or a JONSWAP one-sided spectral density $G_{\eta\eta}(\cdot)$

describes the sea elevation process. Let ω_0 denote some dominant, central frequency of the wave excitation. Then dividing equation (3) by ω_0^2 yields a standardized equation of motion [7]

$$\bar{y}'' + 2\xi_s \bar{\omega}_s \bar{y}' + \bar{\omega}_s^2 \bar{y} = (u - \alpha \bar{y}') |u - \alpha \bar{y}'| + \beta a \quad (4)$$

where $\bar{y} = (m + m_{hyd}) \omega_0^2 y / K_D$ is a standardized displacement; $\bar{y}' = d\bar{y}/d(\omega_0 t)$; $\bar{\omega}_s = \omega_s / \omega_0$; $u = x/p$ and $a = \dot{x}/q$ are the standardized water particle velocity and acceleration with the factors p and q obtained from

$$p^2 = \omega_0^2 G_{\eta\eta}(\omega_0) \quad \text{and} \quad q^2 = \omega_0^5 G_{\eta\eta}(\omega_0) \quad (5)$$

Then it is easily proved that the densities of the standardized water particle velocity $u(t)$ and acceleration $a(t)$ written as functions of ω/ω_0 are independent of the sea state. The two forcing function coefficients α and β in equation (4) are respectively a measure of water-structure interaction and a hydrodynamic ratio. They are given by

$$\alpha = \frac{p}{\omega_0} K_D / (m + m_{hyd}) \quad \text{and} \quad \beta = \frac{q}{p^2} K_M / K_D \quad (6)$$

3. Linearization of the drag term

The choice of the linearization procedure is not straightforward because the first derivative of $x|x|$ vanishes in zero. Numerous linearization procedures exist to approximate this nonlinear term by a linear one but none of them can be shown to be optimal to fit a given low order moment of the solution of equation (4). In practice, two linearization procedures are generally preferred to estimate the response variance. The first approach follows from Caughey's work [9] and consists in replacing the nonlinear forcing function in equation (4) by a linear term which minimizes the mean square error

$$E \left[\left((u - \alpha \bar{y}') |u - \alpha \bar{y}'| - A \sigma_{u - \alpha \bar{y}'} (u - \alpha \bar{y}') \right)^2 \right] \quad (7)$$

Here $E[\cdot]$ denotes mathematical expectation. The second linearization procedure follows from an idea by Bolotin [10] and appears to be less common in applications although it is simpler (see [11]). It involves replacing the original nonlinear drag term by a linear term which has same variance

$$E \left[((u - \alpha \bar{y}') |u - \alpha \bar{y}'|)^2 \right] = E \left[\left(A \sigma_{u - \alpha \bar{y}'} (u - \alpha \bar{y}') \right)^2 \right] \quad (8)$$

Both procedures require iterative estimation of the linearization factor A because the forcing function depends on the structural velocity $\bar{y}'$ solution of equation (4). They always yield different results as can be seen when the time-dependent damping is neglected. In this limit case $\alpha = 0$, (7) and (8) respectively lead to $A = \sqrt{8/\pi}$ and $A = \sqrt{3}$.

These procedures can be generalized by simply changing the value of the exponent in (7) or (8). Hence a possible extension of Caughey's procedure may be to minimize the error with respect to another norm than the euclidian one, i.e. the linearization factor A minimizes

$$E \left[\left| (u - \alpha \bar{y}') |u - \alpha \bar{y}'| - A \sigma_{u - \alpha \bar{y}'} (u - \alpha \bar{y}') \right|^n \right] \quad (9)$$

whereas an extension of Bolotin's rule (8) may be to require

$$E \left[|(u - \alpha \bar{y}')| |u - \alpha \bar{y}'|^n \right] = E \left[|A \sigma_{u-\alpha \bar{y}'} (u - \alpha \bar{y}')|^n \right] \quad (10)$$

In both cases n is a positive real. Analytical solutions to both generalized procedures (9) and (10) do not exist in general because of the hydrodynamic interaction. However, if this interaction is negligible ($\alpha = 0$), Bolotin's extension (10), on the one hand, admits a unique analytical solution A for any value of n

$$A = \sqrt{2} \sqrt[n]{\Gamma\left(n + \frac{1}{2}\right) / \Gamma\left(\frac{n}{2} + \frac{1}{2}\right)} \quad (11)$$

where $\Gamma(\cdot)$ denotes the gamma function. On the other hand, an analytical solution to (9) exists for $n = 2$ and 4 only. Extensive computations were performed to estimate A for other values of n . Linearization factors are shown as functions of n on Figure 1.

Figure 1. Linearization factor A as function of the exponent n ($\alpha = 0$)

4. Numerical work

In the following we investigate the ability of the classical linearization procedures (7) and (8) to yield approximate solutions with accurate second and fourth order moments. Time histories of the stationary Gaussian standardized water particle velocity u and acceleration a are simulated by using the spectral representation method. A Pierson-Moskowitz spectrum is assumed for the sea-elevation process. Harmonics with random phase and deterministic amplitude are used. Time histories with duration 4096 sec and time step $\Delta t = 0.125$ sec are simulated with $N=16384$ harmonics so that they are almost perfectly Gaussian according to the central limit theorem. The constant average acceleration step-by-step method proposed by Newmark is used to solve equation (4) numerically. For $\alpha \neq 0$, the forcing function depends on the response process and the "exact" response time history is computed iteratively. Also time histories resulting from the linearized forcing function are computed. Finally, statistics of the response time histories are calculated.

Figure 2. Error in response variance with Bolotin's method ($n=2$) Figure 3. Error in response variance with Caughey's method ($n=2$) Figure 4. Error in response fourth moment with Bolotin's method ($n=2$) Figure 5. Error in response fourth moment with Caughey's method ($n=2$)

However, it is well known that higher moments of simulated time histories of water velocity and acceleration exhibit some scatter [12]. Therefore, the aforementioned procedure is repeated several times until convergence is reached in the fourth order response moments.

Moments are computed for several values of the frequency ratio ω_0/ω_s and different structural data (α, β, ξ_s) . The frequency ratio ω_0/ω_s is taken in $(0;1)$ ranging from static to dynamic excitation. Ratios above 1.0 are not considered because they may only be observed for very flexible structures for which non linearities cannot be handled by simple linearization. The error in response variance, on the one hand, and in response fourth order moment, on the other hand, is reported as function of the frequency ratio ω_0/ω_s in Figure 2-3 and 4-5, for $\alpha = 0$ and 1, $\beta = 0.0, 0.5$ and 1.0, and $\xi_s = 2\%$. Further, the exponent n in (9) and (10) which yields exact second or fourth order response moment was estimated. Results for $\alpha = 0$ are reported on Figures 6 and 7.

Figure 6. Optimal values of the linearization exponent n in Caughey's extension (9) to fit response second and fourth order moments ($\alpha = 0$)

Figure 7. Optimal values of the linearization exponent n in Bolotin's extension (10) to fit response second and fourth order moments ($\alpha = 0$)

5. Discussion and conclusions

It is first observed that classical linearization procedures (7) and (8) provide satisfactory fits of both second and fourth order response moments for high values of the frequency ratio ω_0/ω_s (see Figures 2-5). In fact, a limit theorem by Rosenblatt proves that the response process asymptotically becomes Gaussian as ω_0/ω_s increases to infinity (see [13, 14]). Thus accurate extreme response or fatigue analyses of a dynamic structure can be performed with second order statistics estimated from linear analysis only.

Most structures, however, are designed to behave essentially in a static manner. Then responses can no longer be assumed Gaussian and fourth order moments of approximate responses are excessively inaccurate. Our results further show that linearization procedures may be excessively inefficient to fit the response variance in this case. For example, the classical Caughey procedure (7) strongly underestimates (by 40% for $\beta = 0$, i.e. when drag-dominated) the response variance in the case of quasi-static excitation ($0 < \omega_0/\omega_s < \frac{1}{3}$). In this case Bolotin's method ($n = 2$) also underestimates the variance but it yields exact variance estimates in the static case. This last feature is due to the fact that condition (8) is satisfied by the whole forcing function for $\omega_0/\omega_s = 0$ because the sea elevation process $\eta(t)$ is taken stationary. More generally, as already noticed by Hu et al [7], it is found that linearization procedures strongly underestimate the response variance at the subharmonic $\frac{1}{5}$ and not at the subharmonic $\frac{1}{3}$ as usually expected [6]. As the water-structure interaction α increases, the fit of the response variance improves but predictions of the response fourth moment remain extremely inaccurate (see Figures 2-5). It follows that an increase of hydrodynamic interaction yields more non-Gaussian behaviour of the response. This is in agreement with results reported in [15]. As β increases, the nonlinear term however becomes negligible and fairly good results are obtained. This also agrees with our expectations.

If α can be assumed close to zero, optimal linearization schemes can be derived from the reported results. Again, optimal values of the exponent n strongly depend on the forcing coefficient β for quasi-static excitations. It must be recognized that the value of β cannot be estimated accurately because large model uncertainties are attached to the hydrodynamic coefficients. Moreover, it may be difficult to define the frequency ratio ω_0/ω_s for structures having several natural frequencies and experiencing the whole range of possible sea states. If modal decomposition is applied to analyse a multi-degree-of-freedom system then our results can be used under the restricting assumption that each modal equation of motion is of the kind (4), i.e. that modal loads reduce to a single Morison equation. In fact, modal forces are a combination of Morison forces correlated in space. Therefore, it is practically rather difficult to use optimum linearization schemes to fit response statistics of quasi-static structures except in very simple cases. It can be proposed to choose Bolotin's rule ($n = 2$) instead of Caughey's procedure ($n = 2$) to estimate the response variance of such structures as this former proves to be simpler and more conservative. It further yields the exact variance of the quasi-static response.

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