RANDOM VIBRATIONS OF STRUCTURES UNDER PROPAGATING EXCITATIONS

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1.Introduction

The problem of vibrations of structures under incoherent or, in particular, propagating excitations is important for large, extended civil engineering structures like bridges, lifelines, dams, offshore structures or for aircraft structures. In seismic engineering spatial ground motion models have been studied for more than a decade but credible, stochastic characteristics are available only since SMART-1 accelerograph array is in operation at Lotung in Taiwan.

Based on the spatial ground motion models the structural response can be analyzed in form of spatial response spectra [1-4] or for various specific types of structures, e.g. [5-11].

In present paper the problem is analyzed again and illustrated by an example of random vibrations of a bridge structure under kinematic wave excitations.

2.Equations of motion and mean square response

Consider equation of motion of discrete systems under kinematic excitations:

$$[M]\{\ddot{q}^t\} + [C]\{\dot{q}^t\} + [K]\{q^t\} = \{0\} \quad (1)$$

where $[M]$, $[C]$, $[K]$ are mass, damping and stiffness matrices, vector $\{q^t\} = \{q_1^t, q_2^t, \dots, q_n^t\}^T$ represents total displacements (fixed reference) and symbol T stands for transposition. These n degrees of freedom can be divided onto n_s structural degrees and n_g degrees associated with ground motion ($n=n_s+n_g$). Then eq.1 takes form

$$\begin{bmatrix} [M_{ss}] & [M_{sg}] \\ [M_{gs}] & [M_{gg}] \end{bmatrix} \begin{Bmatrix} \ddot{q}_s^t \\ \ddot{u} \end{Bmatrix} + \begin{bmatrix} [C_{ss}] & [C_{sg}] \\ [C_{gs}] & [C_{gg}] \end{bmatrix} \begin{Bmatrix} \dot{q}_s^t \\ \dot{u} \end{Bmatrix} + \begin{bmatrix} [K_{ss}] & [K_{sg}] \\ [K_{gs}] & [K_{gg}] \end{bmatrix} \begin{Bmatrix} q_s^t \\ u \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{0\} \end{Bmatrix} \quad (2)$$

where vector $\{u\} = \{u_1, u_2, \dots, u_{n_g}\}^T$ describes the free field motion (excitations). The total response $\{q^t\}$ can be separated onto dynamic $\{q\}$ and pseudo-static $\{q^p\}$.

$$\{q^t\} = \begin{Bmatrix} \{q^p\} \\ \{u\} \end{Bmatrix} + \begin{Bmatrix} \{q\} \\ \{0\} \end{Bmatrix}. \quad (3)$$

Substituting eq.3 into the equation of motion (2) and dropping dynamic terms one obtains the pseudo-static motion

$$\{q^p\} = -[K_{ss}]^{-1} [K_{sg}] \{u\}. \quad (4)$$

Substituting again eqs. 4 and 3 into 2 and assuming either the stiffness proportional damping or that the damping contribution to effective force is negligible leads to:

$$[M_{ss}]\{\ddot{q}\} + [C_{ss}]\{\dot{q}\} + [K_{ss}]\{q\} = \left[[M_{ss}][K_{ss}]^{-1}[K_{sg}] - [M_{sg}] \right] \{\ddot{u}\}. \quad (5)$$

For all the analyzed degrees of freedom vector of the associated generalized elastic forces is equal to

$$\{f\} = [K]\{q^t\}. \quad (6)$$

Separating the forces onto structural $\{f_s\}$ and support $\{f_g\}$

$$\begin{Bmatrix} \{f_s\} \\ \{f_g\} \end{Bmatrix} = \begin{Bmatrix} [K_{ss}] & [K_{sg}] \\ [K_{gs}] & [K_{gg}] \end{Bmatrix} \begin{Bmatrix} \{q^t\} \\ \{u\} \end{Bmatrix} \quad (7)$$

where $\{f_s\} = \{f_1, \dots, f_{n_s}\}^T$ and $\{f_g\} = \{f_1, \dots, f_{n_{fg}}\}^T$. From eq.11

$$\{f_s\} = [K_{ss}]\{q_s^t\} + [K_{sg}]\{u\}, \quad (8)$$

$$\{f_g\} = [K_{gs}]\{q_s^t\} + [K_{gg}]\{u\} \quad (9)$$

Substituting from eq.3 $\{q_s^t\} = \{q^p\} + \{q\}$ and applying eq.4 one obtains

$$\{f_s\} = [K_{ss}]\{q\} \quad (10)$$

$$\{f_g\} = [K_{gs}]\{q\} + \left[[K_{gg}] - [K_{gs}][K_{ss}]^{-1}[K_{sg}] \right] \{u\}. \quad (11)$$

It is interesting to note from eq.10 that the forces associated with structural degrees of freedom depend only on dynamic displacements of the structure which, in turn, depend on "averaged" excitations of the structure. On the other hand the forces associated with support degrees of freedom (eq.11) depend on two terms: dynamic and pseudo-static. The calculated elastic forces may be applied in calculations of any

desired inner forces (shear, axial or moments). In practice however the forces in the structure are calculated using the classic formulae of Finite Element Method (FEM) which are more effective in numerical calculations than formulae (6-11). Regardless of the applied method the calculated forces are linear combinations of structural displacements and the stiffness properties of the structure. The support elastic forces $\{f_g\}$ better represent the propagation effects on structure than dynamic displacements of the structure as they combine both dynamic and pseudo-static response.

Solving the eigenproblem of the analyzed discrete system leads to eigenmatrix $[W]$ and natural frequencies $\omega_1, \omega_2, \dots, \omega_{n_s}$. Introducing normal coordinates $q_1, q_2, \dots, q_{n_s}$ and applying the modal transformation

$$\{q\} = [W]\{q\}, \quad (12)$$

leads, after some algebra [15], to following formulas for mean square displacements

$$\sigma_{q_k}^2 \cong \sum_{i=1}^{n_w} \sum_{p=1}^{n_w} W_{ki} W_{kp} \int_{-\infty}^{\infty} H_i(\omega) H_p^*(\omega) \sum_{j=1}^{n_g} \sum_{r=1}^{n_g} g_{ij} g_{pr} S_{jr}(\omega) d\omega, \quad (13)$$

and forces

$$\begin{aligned} \sigma_{f_k}^2 \cong & \sum_{p=1}^{n_s} \sum_{m=1}^{n_s} K_{kp}^{gs} K_{km}^{gs} \sum_{i=1}^{n_w} \sum_{n=1}^{n_w} W_{pi} W_{mn} \int_{-\infty}^{\infty} H_i(\omega) H_n^*(\omega) \sum_{j=1}^{n_g} \sum_{r=1}^{n_g} g_{ij} g_{nr} S_{jr}^{uu}(\omega) d\omega + \\ & + \sum_{p=1}^{n_s} K_{kp}^{gs} \sum_{i=1}^{n_w} W_{pi} \sum_{j=1}^{n_g} \sum_{r=1}^{n_g} K_{kr}^g \int_{-\infty}^{\infty} \operatorname{Re} \left(H_i(\omega) S_{jr}^{uu}(\omega) \right) d\omega + \\ & + \sum_{j=1}^{n_g} \sum_{r=1}^{n_g} K_{kj}^g K_{kr}^g \int_{-\infty}^{\infty} S_{jr}^{uu}(\omega) d\omega, \end{aligned} \quad (14)$$

where

W_{ij} are elements of eigenmatrix $[W]$,

$H_i(\omega) = (\omega_i^2 - \omega^2 + 2i\xi_i \omega_i \omega)^{-1}$ are modal frequency response functions, $i = \sqrt{-1}$

g_{ij} are elements of matrix $[g] = \operatorname{diag} \left[\frac{1}{m_i} \right] [W]^T \left([M_{ss}] [K_{ss}]^{-1} [K_{sg}] - [M_{sg}] \right)$,

$S_{jr}^{uu}(\omega) = S_{jr}(\omega)$ are cross spectral densities of excitation accelerations,

$S_{jr}^{uu}(\omega) = -S_{jr}(\omega)/\omega^2$ are cross spectral densities displacements-accelerations,

$S_{jr}^{uu}(\omega) = S_{jr}(\omega)/\omega^4$ are cross spectral densities of displacements,

K_{kr}^g are elements of matrix $[K_g] = [K_{gg}] - [K_{gs}] [K_{ss}]^{-1} [K_{sg}]$,

K_{kp}^{gs} are elements of matrix $[K_{gs}]$,

Fig.1 Bridge structure with local $(l-t)$, and principal coordinate system $(x-y)$

3. Numerical example

Consider a bridge structure with coordinate system l, t arbitrarily situated with respect to a system of principal axes x, y in which the axis x is directed towards an epicenter and axis y is perpendicular to x . Axis x is inclined with angle α to axis l (Fig.1). The horizontal ground motions $u_x(t, x, y)$ and $u_y(t, x, y)$. They form a random field with uncorrelated u_x and u_y [13]. Assuming $S_{yy}^{uu}(\omega) = b^2 S_{xx}^{uu}(\omega)$ gives:

$$\begin{bmatrix} S_{xy}^{uu}(\omega) \\ S_{yx}^{uu}(\omega) \end{bmatrix} = \begin{bmatrix} S_{xx}^{uu}(\omega) & 0 \\ 0 & S_{yy}^{uu}(\omega) \end{bmatrix} = \begin{bmatrix} S(\omega) & 0 \\ 0 & b^2 S(\omega) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & b^2 \end{bmatrix} S(\omega) \quad (15)$$

where $S(\omega)$ is the horizontal spectral density of excitation accelerations along propagation direction (axis x) and $b^2=0.7$ here. The transformation of displacements u_x and u_y to u_l and u_t is given through familiar coordinate transformation

$$\begin{Bmatrix} u_l \\ u_t \end{Bmatrix} = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix} \begin{Bmatrix} u_x \\ u_y \end{Bmatrix} \quad (16)$$

The analyzed structure is a r/c highway viaduct modeled using FEM as a 3D frame. The data of the bridge were based on an example from Ref.14. The structure can vibrate

Fig.2 Excitations (u), main response coordinates (q), and shear forces (f)

in horizontal and vertical directions in vertical plane (longitudinal and vertical directions) and out of vertical plane (transversal direction). The supports of the structure have eight horizontal directions of possible excitations (u_1+u_8 in Fig.2). In addition to these eight excitation directions Fig.2 presents also selected FEM displacement coordinates and shear forces of vector $\{f_g\}$. Applying transformation formula (16) to all excitation directions and using eq.15 gives matrix of acceleration cross spectra which is required in eqs. 13 and 14:

$$[S_{j_r}^{uu}(\omega)] = \gamma_{j_r} [D] \begin{bmatrix} 1 & 0 \\ 0 & b^2 \end{bmatrix} [D]^T S(\omega) \quad (17)$$

where $[D]$ is an extended transformation matrix, γ_{j_r} is a complex coherency function of signals at directions "j" and "r" and $S(\omega)$ is a real, point spectral density [15-17]:

$$S(\omega) = \frac{\omega_g^4 + 4\xi_g^2 \omega_g^2 \omega^2}{(\omega_g^2 - \omega^2)^2 + 4\xi_g^2 \omega_g^2 \omega^2} \frac{\omega^4}{(\omega_1^2 - \omega^2)^2 + 4\xi_1^2 \omega_1^2 \omega^2} S_0, \quad (18)$$

where S_0 is an intensity factor and the remaining constants are as follows: $\omega_g = 4\pi$, $\xi_g = 0.6$, $\omega_1 = 1.636$, $\xi_1 = 0.619$. The coherency function may be written in exponential form

$$\gamma_{j_r}(\omega, d_{j_r}^x, d_{j_r}^y) = |\gamma_{j_r}(\omega, d_{j_r}^x, d_{j_r}^y)| \exp(i\omega d_{j_r}^x / v_g). \quad (19)$$

The exponent in formula 19 describes wave propagation effects along axis x . It depends on apparent wave velocity v_g and projection of distance between points "j" and "r" on axis x d_{jr}^x (d_{jr}^y is respective projection on axis y). The modulus of coherency in formula 19 can be found in the Literature following field measurements of seismic ground motion. In this study a formula given by Hao [8] will be applied:

$$|\gamma_{jr}(\omega, d_{jr}^x, d_{jr}^y)| = \exp(-\beta_1 d_{jr}^x - \beta_2 d_{jr}^y) \exp \left[- \left(\alpha_1(\omega) \sqrt{d_{jr}^x} + \alpha_2(\omega) \sqrt{d_{jr}^y} \right) \left(\frac{\omega}{2\pi} \right)^2 \right], \quad (20)$$

where

$$\begin{aligned} \alpha_1(\omega) &= \frac{2\pi a_1}{\omega} + \frac{a_2 \omega}{2\pi} + a_3 \\ \alpha_2(\omega) &= \frac{2\pi a_4}{\omega} + \frac{a_5 \omega}{2\pi} + a_6 \end{aligned} \quad \text{for } 0.314 \leq \omega \leq 62.83. \quad (21)$$

For $\omega \geq 62.83$ $\alpha_1(\omega) = \alpha_1(62.83) = \text{const}$ and $\alpha_2(\omega) = \alpha_2(62.83) = \text{const}$. The constants β_1, β_2 and $a_1 + a_6$ have been taken from event 45 of SMART 1, [8].

Three main parameters affect the response of structure in this case: apparent wave velocity v_g , the angle of propagation α , and the loss of coherency $|\gamma_{jr}|$. The effect of the first two of them on structural response is analyzed in Fig.3 where the root mean square displacements $\sigma_{y_{35}}$ (Fig.3a) and σ_{f_6} (Fig.3b) are presented as the functions of apparent wave velocity and angle α . The presented rms displacements and forces are normalized with respect to uniform excitations. It can be seen from this figure that the incoherent motion of supports reduces the dynamic displacements of the structure, but the force response can be either decreased or increased depending mostly on the propagation velocity. A substantial decrease can be observed for $v_g \approx 100 \div 200 \text{ m/s}$. For very low propagation velocity (less than about 50 m/s) the incoherent excitations make the increase of force response mostly due to increasing contribution of pseudo-static motion. The effect of angle α on the response is also substantial. For high velocity v_g it reflects simply the projections of excitations on the structural coordinate system ($l-t$). For lower propagation velocity the response is very sensitive to changes of angle α .

Final remarks and conclusions

3D random vibrations of a bridge structure under kinematic wave excitations are considered. Both dynamic displacement response and force response are analyzed. The displacement response is reduced compare to coherent excitations while the force response can be either reduced or amplified depending on propagation velocity and angle. Pseudo-static motion contributes substantially in structural response for low

Fig.3 Rms displacements q_{35} (a) and shear force f_6 (b) vs. propagation angle and velocity, normalized with respect to uniform excitations

propagation velocity ($v_g < 50\text{m/s}$).

Acknowledgements

A partial support of Committee of Scientific Research (Grant number PB 0373/P4/94/06) is gratefully acknowledged.

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