Discussion on response surface approximations for use in structural reliability

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ABSTRACT

Response Surface Methodology (RSM) has been largely discussed by the past, from a biometric viewpoint and also for use in nuclear reactor safety. The main concern comes to perform some approximations as a surrogate for the original model in subsequent uncertainty and sensitivity studies.

We propose in this paper to focus attention on RSM for use in structural reliability analysis. The requalification of offshore platforms through the process of re-assessment confirms that such questions merit to be more investigated.

Obviously there is a large amount of basic statistical techniques as stochastic differential or Monte Carlo analyses which remain a common background. But thinking is that the criteria for constructing response curves are drastically depending of how the models will be managed through the computational procedure and for what physical meanings mainly.

1 - INTRODUCTION

Response Surface Methodology (RSM) is a formal aid-tool for most of the quantitative investigations where the required response y can be considered as the output of a system and which varies in response to the changing levels of several input variables [1][2]. The system is to be modelled by some mathematical function of the random variables $\mathbf{x}$ (or processes) involved in the system, and characterised by statistical informations Θ (moments, free or parametric distribution functions, fourier series, ...),

$$y = f(\mathbf{x}/\Theta) \quad (1)$$

The exact form of f being unknown generally, different targets which may conflict are in considerations :

- it is wished to fit mainly a function in order to describe the data with reduction in storage.
- the function is required to be meaningful in the sense that it is based as large as possible on some understanding of the underlying mechanism.
- the function is to be used for inference purposes.
- the function can be fitted without too complex computational requirements.

Mathematically, assuming that f is a continuous function and the vector $\mathbf{x}$ varies in a finite range, the Weierstrass approximation theorem ensures that one can approximate f by a polynomial function to any desired accuracy.

The most frequently used response curves are linear, quadratic or cubic, as they are very often obtained by least squares estimates of the parameters without too complex calculations. This can be extended by applying the same result to $\Phi(f)$ where Φ is a specified basic scale function (inverse, power, logarithmic, exponential, ...).

It is to notice that these polynomial or derived functions seem to be used as simplest readily-available smoothing curves, without any appeal to their theoretical properties as approximations to the true response function. In particular there is no discussion and even an eclipse of the issue of a polynomial order to be admissible as an asymptotic property.

There is no hope to define a general procedure but when a response curve is chosen, then this choice has to be well argued. Otherwise the results are to be presented as conditional on the model being correct and consequently with loss of generality.

By means of an example (wave actions for structural reliability) we suggest to illustrate application of techniques for validating some response curve following two essential criteria given by physical meanings and statistical distribution effects. Complexity reduction in modelling and computational tractability are introduced at a second stage as increasing use and progress of electronic computer and algorithms offer important potentialities.

2 - WAVE ACTIONS FOR STRUCTURAL RELIABILITY

Since some decades, the Morison equation remains a robust means of predicting wave forces on slender cylinders [3]. The usual form can be already viewed as a response curve by itself,

$$f(t) = \frac{1}{2} \rho D C_d u_t |u_t| + \frac{1}{4} \pi \rho D^2 C_m u_t'' \quad (2)$$

where $f(t)$ is the force per unit span separated into drag and inertial components, ρ is water density, D is cylinder diameter and u_t is the instantaneous flow velocity. The drag and inertia coefficients C_d and C_m closely related to the Reynolds and Keulegan-Carpenter numbers are obtained by applying least squares procedures to measured force and velocity data.

2.1 - Physical meanings

The equation (2) generally predicts the main trends in measured data quite well, once an appropriate joint distribution function for C_d and C_m , depending of sea-state parameters, can be provided [4].

Measurements of Morison coefficients were performed also in many other investigations with particular interest in realistically fouled and roughened cylinders, and in conditions where waves and currents act simultaneously.

Nevertheless some interesting characteristics of the flow are not represented with enough accuracy (e. g. high frequency content, gross vortex shedding effects, ...). So when applying eq.(2) we have to get in mind that some sources of response problems for an offshore platform are missing.

As an example, extensions of eq.(2) are to be evaluated, using the NARMAX modelling technique (Non linear Auto-Regressive Moving Average with eXogenous inputs) [5] [6]. It is obtained from the frequency domain analysis that some systematic structure is present in higher order which is not predicted by Morison's equation. But they don't allow for instance a direct physical interpretation of the additional term. There is probably no hope to propose a general model consistent whatever the application but it is relevant to validate extension of the physics "governed" by eq.(2) and specially for non linear effects.

2.2 - Polynomial approximations

When predicting the stochastic response of offshore platforms under Morison type non linear random wave loading, a lot of works suggest a response curve of $f(t)$ in the form of linear, quadratic or cubic expansion as a function of the instantaneous flow velocity. From this tractable form, the resulting non linear equivalent system can be solved using the Volterra series method or some other integration technique. Then response moments are computed.

We propose to examine this procedure under the designation distribution effect, e. g. the interrelationships between probability distributions functions through the response curves.

Let $Y = \varphi(X)$ be a random variable defined as a function of a random variable X with probability density f_X , where φ is differentiable and bijective over the supports of the distributions. We can obtain the probability density f_Y by :

$$f_Y(y) = f_X(\varphi^{-1}(y)) \cdot \left| \frac{1}{\varphi'}(\varphi^{-1}(y)) \right| \quad (3)$$

Considering φ as a response curve and let P be a polynomial approximation. From eq.(3) and assuming P closed enough of φ , a consistent approximation of φ with respect of the response density f_Y needs also a good approximation of the derivative of φ . Taking P as linear, quadratic or cubic expansions then the derivative P' is constant, linear or quadratic.

The risk of discrepancy between these three last functions is very high and their effects in changing the upper and lower tails of the density f_X are quite different. The choice of a polynomial of low order, so tractable for the computational procedure, although apparently relevant for an approximation of the response curve, can lead to an erroneous probability density output.

In particular this explains results presented in different papers from simulation studies, which show that equivalent systems deduced from quadratization and cubicization methods give inappropriate response skewness and kurtosis coefficients.

This simple example illustrates that the distribution effects can significantly influence response curve construction as giving criteria. It is very often important to diagnose the possible ramifications of these effects. The issue of RSM is an area of sensitivity analysis where interaction among model builders and those conducting the sensibility analysis is essential.

There is no generally applicable technique for determining whether the distribution effect is affecting the computational procedure through the RSM. But there is a clear scope for models of higher order properties than those frequently used for simplification mainly.

This problem increases in complexity, in the case of several variables to be selected. Usual criteria are based on minimization bias and/or variance of the deviation from the true response, integrated over some region. As an extension of eq.(3), there is a need for an additional criterion such as good fitting of the Jacobian matrix,

$$\left(\frac{D(x_1, \dots, x_n)}{D(y_1, \dots, y_m)} \right) \text{ in order to prevent the main distribution effects.}$$

2.3 - Formal geometrical modelling

The eq.(2) is to be quite relevant to compute quasi-static loads under extreme environmental conditions. In the following, the guiding idea is to maintain f as a response curve without any more approximation in order to avoid inherent distribution effects and to focus attention in providing a suitable formal geometrical modelling of the instantaneous flow velocity [7]. At a first stage we consider wave actions for simplicity of presentation only.

Let $Oxyz$ be a canonical repair in the euclidean space R^3 . The origin O is taken on the mean sea level. The axis Oz is vertical and increasing. We note $\overrightarrow{OU}$ the directional vector of waves ($\overrightarrow{OU} = Ox \cos \theta + Oy \sin \theta$ where θ is the wave direction).

The instantaneous kinematics field of waves being a random vector in R^3 , we propose to evaluate this vector field as resulting of geometrical transformations (similarity, rotation, ...) of $\overrightarrow{OU}$.

The Navier Stokes equations are basic for the most mathematical models in fluid mechanics. They are second-order differential equations due to the friction terms, and non linear because of the convective inertia terms.

The small amplitude plane harmonic waves known as Airy waves can be derived from the potential ϕ ,

$$\phi(x, z, t) = \frac{H}{T} \frac{\pi \cosh(kz)}{k \sinh(kd)} \sin(kx - \Psi_t) \quad (4)$$

H : wave height, T : wave period,

k : wave number, Ψ_t : pseudo phase angle,

and d : water depth.

After some algebraic operations the associated gradient field is expressed from geometrical transformations (rotation and similarity product),

$$\bar{V}_{\text{airy}} = H(O, \lambda_1) \circ \mathfrak{R} \left(\overrightarrow{OV}, \alpha \right) \overrightarrow{OU} \quad (5)$$

where the similarity ratio λ_1 , and the rotation angle α are random functions,

$$\alpha = \text{atan} \left[\tanh(kz) \tan(kx - \Psi_1) \right] \text{ and,} \quad (6)$$

$$\lambda_1 = \frac{H}{2} \sqrt{\frac{g}{d}} \sqrt{\frac{\cosh kz}{\cosh kd}} \sqrt{\frac{kd}{\tanh kd}} \sqrt{\cos^2(kx - \Psi_1) + \tanh^2(kz) \sin^2(kx - \Psi_1)} \quad (7)$$

Considering now the Stokes 2nd order expansion, there is an additional term to eq.(5), written in a similar form,

$$\bar{V}_s = H(O, \lambda_s) \circ \mathfrak{R} \left(\overrightarrow{OV}, \alpha_s \right) \overrightarrow{OU} \quad (8)$$

where $\alpha_s \approx 2\alpha$ and λ_s is a random function which varies also with (H, k) .

This gives arguments in discussing the level of complexity to be introduced in kinematics field modelling of waves for structural reliability analyses.

figure 1 Statistical variations of λ_1 with depth (North sea)

figure 2 Statistical variations of λ_s with depth (North sea)

In fact the two similarity ratios λ_1 and λ_s have been compared as functions of the two random variables H and k of which ranges and distributions were performed from North Sea in situ measurements (see figures 1 and 2). The influence of λ_s appears quite of minor importance. It confirms that among Stokes models family, the Airy wave model contains the essential structure of basic random variables to be analysed for structural reliability under quasi-static loads.

The angle rotation α is a random function with a specified trajectory (see eq. (6)) of which the stochastic fluctuations are governed by the random variable k . In the case of severe conditions, the range and the statistics of k are well adapted for use of first order Taylor's expansion around the mean wave number expectation $\bar{k}$, when developing the angle rotation function.

Denoting by $\underline{\alpha} = \alpha(\bar{k})$, $\underline{\alpha}' = \alpha'(\bar{k})$, $\lambda_0 = (k - \bar{k}) \cdot \underline{\alpha}'$, $\lambda_2 = \frac{1}{\sqrt{1 + \lambda_0^2}}$. Then after some mathematical operations in the euclidean space $\mathbb{R}^3$, and considering a repair connected with waves ($O, \overrightarrow{OU}, \overrightarrow{OV}, \overrightarrow{OZ}$), we obtain a sharp approximation among geometrical operators,

$$\Re \left(\overrightarrow{OV}, \underline{\alpha} \right) \approx H(O, \lambda_2) \circ \left\{ I_d + H(O, \lambda_0) \circ \Re \left(\overrightarrow{OV}, \frac{\pi}{2} \right) \right\} \circ \Re \left(\overrightarrow{OV}, \underline{\alpha} \right) \quad (9)$$

Introducing this operator approximation inside eq.(5), the kinematics field deduced from the Airy model has the very convenient form,

$$\bar{V}_{\text{airy}} = \mu \bar{A} + \nu \bar{B} \quad (10)$$

where $\bar{A}$ and $\bar{B}$ are deterministic orthonormal vectors defined by,

$$\bar{A} = \Re \left(\overrightarrow{OV}, \underline{\alpha} \right) \overrightarrow{OU} \quad \text{and} \quad \bar{B} = \Re \left(\overrightarrow{OV}, \frac{\pi}{2} \right) \bar{A}.$$

The coefficients μ and ν are random scalar functions with specified trajectories depending of the couple of random variables (H, k) .

The non linear term $u|u|$ (see eq. (2)) is then written easily $\sqrt{\mu^2 + \nu^2} (\mu \bar{A} + \nu \bar{B})$.

Finally $f(t)$ is also approximate in the closed form of a linear combination of deterministic vectors ; the coefficients of which are random functions which depend of a set of basic random variables H, k, C_d, C_m and possibly D_0 the number's effective diameter to take account for estimating the marine growth thickness. For another wave direction θ_1 , then the response curve is obtained by changing $\bar{A}$ and $\bar{B}$ only through a rotation of axis Oz and angle $\theta_1 - \theta$.

This formal geometrical modelling can be extended to study wave/current interactions [8]. The maximum current during a storm rarely occurs at the same time or in the same direction as the maximum waves. A one third decrease in current design value is to be expected into the site under consideration when considering concomitants of order statistics (e.g. distribution of current speed given that significant wave height is at its maximum). Due to the respective ranges of the wave kinematics and current speeds, this implies no important decrease in the near-surface zone ($\approx 5\%$) where loadings are the most severe. Obviously the decreasing effect is very sensitive to depth parameter and when the current profile is very uneven, the current/wave interaction becomes a crucial item.

At a second stage this formal approach allows to identify the external equivalent concentrated nodal forces by a finite linear combination of the form,

$$\sum \lambda_i \bar{F}_N^{(i)} \quad (11)$$

where the λ_i are random multipliers which contain the same basic random variables as mentioned previously, enlarged with λ_w, λ_c wind and current velocities measured at reference levels. The $\bar{F}_N^{(i)}$ are deterministic vectors which depend essentially of the structure topology, and also of the different deterministic profile functions of actions and of wave location with respect to the structure [9].

A computer program was developed and implemented as a module of the general package ARPEJ (Evaluation of Jackets from a Probabilistic Redundant Analysis). It gives the opportunity to perform sensitivity studies and it indicates when necessary, where to concentrate efforts in terms of probabilistic modelling of environmental conditions.

3 - CONCLUSION

The Response Surface Methodology is clearly a relevant scope for Reliability and Optimization of structural system. There is no doubt that response curves for limit state functions, ranking of input variables or sensitivity studies are to be provided.

This paper is briefly discussing the main criteria for response surface construction. It is suggested to start with functions based on some understanding of the underlying mechanism.

It is showned that usual polynomial approximations of low order are very sensitive to distribution effect.

A formal geometrical approach through an illustration to wave actions modelling, comes to response curves in a very convenient form for reliability analysis, and allows to prevent the main distribution effects.

The efficiency of RSM for use in the requalification process depends largely on ability in estimating realistic ranges and distributions for the input variables under consideration.

Finally existing data bases enhanced by satellite measurements of wave and wind, experience feedback provided by in situ measurements at platform or a neighbouring location, are to be worked with a dedicated attention to this extension.

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