Model-Based Fatigue Fractographs Texture Analysis

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Abstract. A novel model-based approach for estimation of the velocity of crack growth from microfractographical images is proposed. These images are represented by a Gaussian Markov random field model and the crack growth rate is modelled by a linear regression model in the Gaussian-Markov parameter space. The method is numerically very efficient because both crack growth rate model parameters as well as the underlying random field model parameters are estimated using fast analytical estimators.

1 Introduction

The quantitative microfractography of fatigue failures is concerned mainly with the investigation of the history of a fatigue crack growth process. Specimens of the material are loaded under service conditions and the crack growth process is recorded. Fracture surfaces images produced by a scanning electron microscope (SEM) are studied to relate image morphological information of the crack surface with the macroscopic crack growth rate (CGR). The crack growth process is reconstituted using integration of CGR along the crack growth direction.

Traditional fractographical methods are based on strictly defined fractographic features measurable in the morphology of a fracture surface. In the case of fatigue analysis, these features are striations [12,13], i.e., fine parallel grooves in the fracture surface. However such methods cannot be used when striations are partially occluded, typically due to corrosion. For such cases, a family of methods is being developed called textural fractography [14]–[22]. The proposed method estimates CGR from textural features derived from a Markovian underlying model.

For the application of the textural method, especially suitable is the mezo-scopic dimensional area with SEM magnifications between macro- and microfractography (about 30–500×). These magnifications were traditionally seldom used in the past due to the absence of measurable objects in corresponding images (for example see Fig.1). Setting the magnification is limited by several conditions related to individual images, to the whole set of images and to image

Fig. 1. Cuttings (256×256) from small, medium and great CGR

discretization. Images require preprocessing to eliminate lighting variations to get homogeneous image set for subsequent textural analysis. Fractographic information is extracted in the form of integral parameters of the whole image. Such parameters might be different statistical (e.g., correlations, statistical moments, etc.) or textural features.

In the presented method, the fatigue image is assumed to be described by a Markov random field (MRF) fitted to this image. Although the MRF models generally suffer with time consuming iterative methods for both parameter estimation as well as for the synthesis, the Gaussian Markov random field (GMRF) model used in this paper belongs to few exceptional Markovian models which avoid the time consuming Markov chain Monte Carlo simulations so typical for the rest of Markovian models family.

Modelling monospectral still images require two dimensional models. Among such possible models the Gaussian Markov random fields are appropriate for fractographical texture modelling not only because they do not suffer with some problems (e.g., sacrificing considerable amount of image information, nonlinear parameter estimation, etc.) of alternative options (see [3,4,5,6,10,11] for details) but they are also easy to synthesize and still flexible enough to imitate a large set of fractographical textures. While the random field based models quite successfully represent high frequencies present in natural fatigue textures low frequencies are much more difficult for them. However, the model does not need to generate a realistic fatigue texture. For the crack velocity estimation it is sufficient to produce discriminative features.

2 Fatigue Images Model

Single monospectral SEM images are assumed to be independently modelled by their dedicated Gaussian Markov random field models (GMRF) as follows. The Markov random field (MRF) is a family of random variables with a joint probability density on the set of all possible realisations Y indexed (in our application) on a finite two-dimensional rectangular ($N \times M$) lattice I , subject to following

conditions:

$$p(Y) > 0 \quad , \quad (1)$$

and

$$p(Y_r | Y_s \quad \forall s \in I \setminus \{r\}) = p(Y_r | Y_s \quad \forall s \in I_r) \quad , \quad (2)$$

where $r = \{r_1, r_2\}$ is the multiindex with the row and column indices, respectively. $I_r \subset I$ is a 2D symmetric contextual support set of the monospectral random field.

If the local conditional density of the MRF model (3) is Gaussian, we obtain the continuous Gaussian Markov random field model (GMRF):

$$p(Y_r | Y_s \quad \forall s \in I_r) = (2\pi\sigma^2)^{-\frac{1}{2}} \exp \left\{ -\frac{1}{2\sigma^2} (Y_r - \tilde{\mu}_r)^2 \right\} \quad (3)$$

where the mean value is

$$\begin{aligned} E \{Y_r | Y_s \quad \forall s \in I \setminus \{r\}\} &= \tilde{\mu}_r \\ &= \mu_r + \sum_{s \in I_r} a_s (Y_{r-s} - \mu_{r-s}) \end{aligned} \quad (4)$$

and $\sigma, a_s \quad \forall s \in I_r$ are unknown parameters. The 2D GMRF model can be also expressed [3] as a stationary non-causal correlated noise driven 2D autoregressive process:

$$\tilde{Y}_r = \sum_{s \in I_r} a_s \tilde{Y}_{r-s} + e_r \quad (5)$$

where $\tilde{Y}_r = Y_r - \mu_r$ are centered variables, the noise e_r are random variables with zero mean

$$E\{e_r\} = 0 \quad .$$

The e_r noise variables are mutually correlated

$$\begin{aligned} R_e &= E\{e_r e_{r-s}\} \\ &= \begin{cases} \sigma^2 & \text{if } s = (0, 0), \\ -\sigma^2 a_s & \text{if } s \in I_r, \\ 0 & \text{otherwise.} \end{cases} \end{aligned} \quad (6)$$

Correlation functions have the symmetry property

$$E\{e_r e_{r+s}\} = E\{e_r e_{r-s}\}$$

hence the neighbourhood support set and their associated coefficients have to be symmetric, i.e.,

$$s \in I_r \Rightarrow -s \in I_r$$

and $a_s = a_{-s}$.

2.1 Parameter Estimation

The selection of an appropriate GMRF model support is important to obtain good results in modelling of a given random field. If the contextual neighbourhood is too small it can not capture all details of the random field. Inclusion of the unnecessary neighbours on the other hand add to the computational burden and can potentially degrade the performance of the model as an additional source of noise. We use the hierarchical neighbourhood system I_r , e.g., the first-order neighbourhood is $I_r^1 = \{r - (0, 1), r + (0, 1), r - (1, 0), r + (1, 0)\}$,

$$I_r^1 = \begin{array}{ccc} & \otimes & \\ \otimes & r & \otimes \\ & \otimes & \end{array} \quad I_r^2 = \begin{array}{ccc} & \otimes & \otimes & \otimes \\ \otimes & & r & \otimes \\ & \otimes & \otimes & \otimes \end{array} ,$$

etc. An optimal neighbourhood is detected using the correlation method [7] favouring neighbours locations corresponding to large correlations over those with small correlations.

Parameter estimation of a MRF model is complicated by the difficulty associated with computing the normalization constant. Fortunately the GMRF model is an exception where the normalization constant is easy to obtain. However either Bayesian or ML estimate requires iterative minimization of a nonlinear function. Therefore we use the pseudo-likelihood estimator which is computationally simple although not efficient. The pseudo-likelihood estimate for a_s parameters has the form

$$\begin{aligned} \hat{\gamma}^T &= [\hat{a}_s \quad \forall s \in I_r]^T \\ &= \left[\sum_{r \in I} X_r X_r^T \right]^{-1} \sum_{r \in I} X_r \tilde{Y}_r \end{aligned} \quad (7)$$

where

$$X_r = [\tilde{Y}_{r-s} \quad \forall s \in I_r]^T \quad (8)$$

and

$$\hat{\sigma}^2 = \frac{1}{MN} \sum_{r=1}^{MN} (\tilde{Y}_r - \hat{\gamma} X_r)^2 . \quad (9)$$

Alternatively this estimator can be computed recursively [8,9].

2.2 Crack Growth Rate Model

We assume that the crack growth rate $v(i)$ is linearly dependent on GMRF parameters describing corresponding fatigue images, i.e.,

$$v(i) = \sum_{j=1}^{\nu} b_j a_{j,i} + \epsilon_i = \theta \gamma_i^T + \epsilon_i , \quad i = 1, \dots, n \quad (10)$$

Fig. 2. The fatigue test specimen and the location of images in the fatigue crack surface

where b_s are unknown parameters, $\theta = [b_1, \dots, b_\nu]$, $\nu = \text{card}\{I_r\} + 1$, n is the number of fatigue images and a_s are RF pseudo-likelihood estimates (7),(9). The growth rate is assumed to have independent Gaussian measurement error ϵ_i with standard deviation η_j $j = 1, \dots, \nu$. We can assume an overestimated set of equations, i.e., $n \gg \nu$ hence the b_s parameters can be for example estimated using the least square estimator

$$\hat{\theta}^T = (\Gamma^T \Gamma)^{-1} \Gamma^T V \quad (11)$$

where

$$V = [v(1), \dots, v(n)]^T$$

and

$$\Gamma = [\gamma_1^T, \dots, \gamma_n^T]^T$$

is a $n \times \nu$ design matrix. Finally the velocity estimator is

$$\hat{v}(i) = \hat{\theta} \gamma_i^T. \quad (12)$$

The alternative option is a Bayesian estimator for both - unknown parameters from θ as well as for the optimal model selection (i.e., selection of an optimal subset of the variables $a_{j,i}$).

3 Results

The method was applied on data from four fatigue experiments with specimens from stainless steel AISI 304L used in nuclear power plants. The specimen type was CT (Fig.2) with the initial notch length 12.5 mm. Constant cycle loading with parameters $\Delta F = 3400$ N, $R = 0.3$, $f = 1$ Hz occurred in water at 20° C. The crack length was measured by COD. Fatigue crack surfaces were documented using SEM with magnification $200\times$. The sequence of images was located in the middle of the crack surface (Fig.2) and the images were distanced by 0.4 mm. The direction of the crack growth in images is bottom-up. The real area of one image is about 0.6×0.45 mm (the images overlap by 0.05 mm). The whole experimental set contains 164 images. Fig.1 shows examples of typical textures - cuttings 256×256 pixels from normalized images (size 1200×1600 pixels).

The estimation quality was evaluated using the mean absolute error:

$$\zeta = \frac{1}{n} \sum_{i=1}^n |v(i) - \hat{v}(i)|$$

and the overall velocity estimation error

$$\varsigma = \frac{100\zeta}{\bar{v}}$$

where $\bar{v}$ is the average velocity in the measurements set. These textures were modelled using the fifth order GMRF model. Although all modelled textures are non stationary and thus violate the GMRF model assumption, the crack rate estimates are fairly accurate. The rate estimates can be further improved if we select a subset of variables γ_i for the model for example by eliminating variables with low correlation with velocity.

4 Conclusions

Our test results of the algorithm on stainless steel fatigue images are encouraging. Some estimated crack rates match true velocities within measurement accuracy. Overall velocity estimation error was $\varsigma = 30\%$ but further improvement is possible if we increase the GMRF model order or introduce a multiresolution MRF model. The proposed method allows quantitative estimation of the crack growth rate while it has still moderate computation complexity. The method does not need any time-consuming numerical optimization like for example some Markov chain Monte Carlo method.

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