Two Motion-Detection Algorithms for Projection-Reconstruction Magnetic Resonance Imaging: Theory and Experimental Verification

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Abstract. In this paper, projection-reconstruction (PR) magnetic resonance (MR) imaging is considered. We present two new quantitative techniques that allow to check when motion was present during a PR MR experiment. No a priori information about the motion is required since only the measured MR signals are used in the calculations. Moreover, the proposed methods can be implemented on a standard personal computer or workstation. It is experimentally shown that both methods are able to detect motion intervals with an accuracy of one repetition time.

1 Introduction

Magnetic resonance imaging has several advantages over other imaging techniques, including a high spatial resolution combined with a high soft-tissue contrast and its noninvasiveness. An important disadvantage of the technique is that workable images often require long measuring times compared with gross motion and physiological motion (e.g. blood flow, heart pulsation, peristaltic motion and breathing). The presence of motion during MR imaging results in a distortion of the measured MR signals. When images are reconstructed from these distorted signals, severe ghosting and blurring are often seen [1–5]. Several methods have been proposed to minimize these artifacts [6–15], but none of these is able to reduce artifacts caused by general motion completely. Therefore, the appearance of motion artifacts remains one of the major drawbacks in MR imaging.

2 Theoretical Considerations

The aim of two-dimensional (2D) MR imaging is to depict the 2D distribution of the complex transverse magnetization $\overline{M}(x, y)$ in a plane through a patient. In case of PR, a set of one-dimensional (1D) projections $\overline{P}_\theta(r)$ of $\overline{M}(x, y)$ is considered, where $\theta = 0, \frac{1}{N_P}\pi, \frac{2}{N_P}\pi, \dots, \frac{N_P-1}{N_P}\pi$ are the projection angles, N_P is the total number of projections and r is a place variable along the projection axis.

When PR is used in MR imaging, a 1D signal $\bar{S}_\theta(k)$, consisting of N_S samples in the spatial-frequency k -space, is measured for each projection angle θ . The set $\{\bar{P}_\theta(r); \theta = 0, \dots, \frac{N_P-1}{N_P}\pi\}$ is related with the set $\{\bar{S}_\theta(k); \theta = 0, \dots, \frac{N_P-1}{N_P}\pi\}$ by a 1D Fourier transform with respect to k :

$$\bar{S}_\theta(k) = \int \bar{P}_\theta(r) e^{-j2\pi kr} dr.$$

The projections $\bar{P}_\theta(r)$ can thus be calculated from the signals $\bar{S}_\theta(k)$ by performing an inverse Fourier transform on these signals and finally, the global distribution $\bar{M}(x, y)$ can be reconstructed from the set of projections by applying the filtered back-projection algorithm [16].

Generally, $\bar{P}_\theta(r)$ is a complex quantity since the transverse magnetization itself is complex. The signals $\bar{S}_\theta(k)$ are complex as well. However, from now on we consider amplitude images reconstructed by projection reconstruction. In this type of images, the transverse magnetization's magnitude $M(x, y)$ is depicted. Amplitude images can be reconstructed when the set of projection magnitudes $\{P_\theta(r); \theta = 0, \dots, \frac{N_P-1}{N_P}\pi\}$ is known (where $P_\theta(r)$ is the magnitude of the complex projection $\bar{P}_\theta(r)$), by applying the same filtered back-projection algorithm as mentioned before.

The projection magnitude $P_\theta(r)$ is a real number for each value of the place variable r and the projection angle θ . Therefore, the real part $\mathcal{R}_\theta(k)$ and the amplitude $A_\theta(k)$ of the measured signals $\{\bar{S}_\theta(k); \theta = 0, \dots, \frac{N_P-1}{N_P}\pi\}$ are even functions of k , while the imaginary $\mathcal{I}_\theta(k)$ part and the phase $\phi_\theta(k)$ of the signals are odd functions of k .

In practical PR, the signals $\bar{S}_\theta(k)$ are sampled symmetrically around the origin $k = 0$ for each $\theta = 0, \dots, \frac{N_P-1}{N_P}\pi$. By calculating the complex mean signal $\langle \bar{S}_\theta(k) \rangle_k$ with respect to k , we find

$$\langle \bar{S}_\theta(k) \rangle_k = \langle \mathcal{R}_\theta(k) \rangle_k.$$

Obviously, $\langle \bar{S}_\theta(k) \rangle_k$ is real, which implies that its phase equals zero:

$$\arg(\langle \bar{S}_\theta(k) \rangle_k) = 0. \quad (1)$$

This conclusion holds as long as no motion is present during the measurement of $\bar{S}_\theta(k)$. Now it will be shown that it does not hold when motion is present during the MR experiment.

Indeed, it is well known that the presence of translational motion during an MR experiment results in a phase distortion of the acquired signals. When the motion contains a rotational component, both the amplitude and phase of the signals are distorted. As a conclusion, when a general type of motion is present during the measurement of the signal $\bar{S}_\theta(k)$, the phase of $\bar{S}_\theta(k)$ will be distorted. Because of this phase distortion, the mean value of the signal's imaginary part will differ from zero when motion is present during the measurement of $\bar{S}_\theta(k)$. As a result, the mean value $\langle \bar{S}_\theta(k) \rangle_k$ will not be a real number in this case and its phase will therefore differ from zero.

From these considerations, a theoretical algorithm that allows to decide whether or not the studied object was moving during the measurement of the signal $\overline{S}_\theta(k)$, can be derived:

1. acquire $\overline{S}_\theta(k)$;
2. calculate $\arg(\langle \overline{S}_\theta(k) \rangle_k)$;
3. decision:
 - if $\arg(\langle \overline{S}_\theta(k) \rangle_k) = 0 \rightarrow$ no motion occurred during the measurement of $\overline{S}_\theta(k)$;
 - if $\arg(\langle \overline{S}_\theta(k) \rangle_k) \neq 0 \rightarrow$ motion did occur during the measurement of $\overline{S}_\theta(k)$.

In practical PR imaging, the measured signals are always noisy in some degree. As a result, the theoretically (i.e. when noiseless data were considered) derived Eq. 1 is never fulfilled exactly, even when the depicted object did not move during the MR experiment. Therefore, in order to be useful in practical situations, the proposed algorithm's decision step should be modified as follows:

3. decision:
 - if $|\arg(\langle \overline{S}_\theta(k) \rangle_k)| \leq \alpha \rightarrow$ no motion occurred during the measurement of $\overline{S}_\theta(k)$;
 - if $|\arg(\langle \overline{S}_\theta(k) \rangle_k)| > \alpha \rightarrow$ motion did occur during the measurement of $\overline{S}_\theta(k)$,

where α is a small critical phase, for which $\alpha = 10^\circ$ is found to be a useful value when the $\arg(\langle \overline{S}_\theta(k) \rangle_k)$ values are calculated on a scale between -180° and $+180^\circ$ (see further).

In the next part of this section, a second method to determine motion intervals in projection-reconstruction magnetic resonance imaging will be explained. In a previous study, it was found that in case of translational motion with a frequency that is smaller than a critical frequency, the projections' amplitudes $P_\theta(r)$ are almost not distorted [3–5]. This critical frequency has a typical value of 50 Hz.

As a result, when translational motion with small frequencies is present during a PR MR experiment, images without relevant motion artifacts can be reconstructed from the set $\{P_\theta(r); \theta = 0, \dots, \frac{N_P-1}{N_P}\pi\}$ by applying the filtered-backprojection algorithm.

Moreover, the root mean square deviation between two successive projections $P_\theta(r)$ and $P_{\theta-\pi/N_P}(r)$ will remain small when the considered object is not moving during the MR experiment. This can be seen as follows. Consider the root mean square deviation between two consecutive projections:

$$RMSE(\theta) = \sqrt{\sum_r \frac{(P_\theta(r) - P_{\theta-\pi/N_P}(r))^2}{N_P}}, \quad (2)$$

where $\theta = \frac{1}{N_P}\pi, \dots, \frac{N_P-1}{N_P}\pi$.

Each projection $P_\theta(r)$ can be considered as a real function $P(\theta, r)$ of two variables, i.e. the projection-angle variable θ and the place variable r . By applying Taylor's expansion theorem [17] with respect to the projection-angle variable, the following expression can be written:

$$\frac{(P_\theta(r) - P_{\theta-\pi/N_P}(r))^2}{N_P} = \left(\frac{\delta P_\theta(r)}{\delta \theta} \right)^2 \frac{\pi^2}{N_P^3} + \mathcal{O} \left[\frac{\pi^3}{N_P^4} \right], \quad (3)$$

where $\mathcal{O} \left[\frac{\pi^3}{N_P^4} \right]$ denotes a term of order $\frac{\pi^3}{N_P^4}$. In practical situations, N_P has a minimum value of 64 and therefore $\frac{\pi^2}{N_P^3}$ and $\frac{\pi^3}{N_P^4}$ will have a maximum order of magnitude of 4.10^{-5} and 2.10^{-6} respectively. From Eq. 2 and 3 it can then be seen that $RMSD(\theta)$ will remain relatively small in this case. This conclusion holds as long as $\left(\frac{\delta P_\theta(r)}{\delta \theta} \right)^2$ remains small, which is only true when no motion or translational motion with a low frequency is present. When the object's motion contains a high-frequency translational or a rotational component during the measurement of the signal $\overline{S}_\theta(k)$, $RMSD(\theta)$ becomes much larger since $\left(\frac{\delta P_\theta(r)}{\delta \theta} \right)^2$ becomes large in this case.

As a result, a second theoretical algorithm to determine whether the studied object moved during the measurement of $\overline{S}_\theta(k)$ can be derived:

1. acquire $\overline{S}_\theta(k)$;
2. calculate $RMSD(\theta)$;
3. decision:
 - if $RMSD(\theta) \leq \beta \rightarrow$ no relevant motion occurred during the measurement of $\overline{S}_\theta(k)$;
 - if $RMSD(\theta) > \beta \rightarrow$ motion did occur during the measurement of $\overline{S}_\theta(k)$,

where β is a relatively small critical $RMSD$ value, for which $\beta = 10$ is found to be a suitable value when the $RMSD(\theta)$ values are calculated on a scale in terms of percentage between 0 and 100 (see further). 'Relevant motion' should be interpreted as 'high-frequency translational motion or rotational motion', since low-frequency translational motion does not give rise to motion artifacts in PR imaging.

3 Materials and methods

In this study, sagittal images of a volunteer's brain were investigated. The corresponding MR signals were measured by using a PR sequence on a 1.5 T Siemens SP63 whole-body scanner. The images' field of view was 300 mm, the total number of projections N_P was 256, the number of k -space samples per signal N_S was 256 and the thickness of the image planes was 10 mm. The PR sequence was characterized by an echo time of 20 ms, a repetition time of 1000 ms and a readout time of 7.68 ms. The signals $\overline{S}_\theta(k)$ were measured in algebraic order: the first signal corresponded with $\theta = 0$, the second with $\theta = \frac{1}{256}\pi, \dots$, the last with

$\theta = \frac{255}{256}\pi$. A reference measurement was performed during which the volunteer was not moving.

In a first experiment, the volunteer's head was moving during two separate and long time intervals. These intervals corresponded with the measurement of the 17th through the 41st signal and the 114th through the 137th signal respectively. The head movement was continuous and contained both translational and rotational components. The maximum displacement in the direction perpendicular to the image plane was 2 mm, while the maximum in-plane displacement was 45 mm.

In the second experiment, the volunteer's head was moving in a similar way, but only during one short time interval, i.e. during the measurement of the 47th through 56th signal.

For each experiment, the intervals during which the volunteer was moving were determined from the measured sets of MR signals. This was done by calculating $RMSD(\theta)$ in terms of percentage on a scale between 0 and 100 and $\arg(\langle \bar{S}_\theta(k) \rangle_k)$ on a scale between -180° and $+180^\circ$, for all projection angles $\theta = \frac{1}{256}\pi, \dots, \frac{255}{256}\pi$. When $RMSD(\theta)$ was higher than $\beta = 10$ or when the absolute value of $\arg(\langle \bar{S}_\theta(k) \rangle_k)$ was higher than 10° , it was decided that motion occurred during the measurement of $\bar{S}_\theta(k)$, according to the algorithms that were described previously.

4 Results and discussion

Figures 1 and 2 show the $RMSD(\theta)$ and $\arg(\langle \bar{S}_\theta(k) \rangle_k)$ values for the reference measurement. All $RMSM(\theta)$ and $\arg(\langle \bar{S}_\theta(k) \rangle_k)$ are lower than the critical values $\beta = 10$ and $\alpha = 10^\circ$ respectively, as was expected from the theoretical study.

Fig. 1. Root mean square deviation as function of the signal number, corresponding with the reference image.

Fig. 2. Phase of the signals' complex means as function of the signal number, corresponding with the reference image.

The $RMSD(\theta)$ values that were found when motion occurred are shown in figures 3 and 4, while the corresponding $\arg(\langle \bar{S}_\theta(k) \rangle_k)$ values are shown in figures 5 and 6. Using the mentioned threshold $\beta = 10$ for $RMSD(\theta)$, the motion intervals can be derived from figures 3 and 4. For the first experiment, it is decided that motion occurred during the measurement of signals 17 through 40 and during the measurement of signals 113 through 137. In the second experiment, it is decided that motion was present during the measurement of signals 47 through 57.

The motion intervals can also be determined from figures 5 and 6 by using the threshold of $\alpha = 10^\circ$. In this case, motion intervals are found by the motion-detection algorithm that correspond with the measurement of signals 17 through 40 and 115 through 138 for the first experiment, and 47 through 56 for the second experiment.

Fig. 3. Root mean square deviation as function of the signal number, in case of 2 long motion intervals.

Fig. 4. Root mean square deviation as function of the signal number, in case of 1 short motion interval.

Fig. 5. Phase of the signals' complex means as function of the signal number, in case of 2 long motion intervals.

A summary of the results is given in table 1. Both motion-detection methods prove to be able to detect motion intervals in PR signals with high accuracy: the maximum difference between the estimated intervals and the actual motion intervals is 1 signal number or 1 repetition time, both in case of long and repetitive motion (first experiment) and in case of short motion (second experiment).

5 Conclusion

We presented two new methods to detect motion intervals from a set of PR signals, based on a theoretical study. They do not require any a priori information on the motion and can be implemented easily. It was experimentally shown that

Fig. 6. Phase of the signals' complex means as function of the signal number, in case of 1 short motion interval.

	real motion intervals	estimated motion intervals $RMSD(\theta)$	estimated motion interval $\arg(\langle \bar{S}_\theta(k) \rangle_k)$
1st experiment	17-41 114-137	17-40 113-137	17-40 115-138
2nd experiment	47-56	47-57	47-56

Table 1. Summary of the experimental results.

both methods are able to detect these motion intervals with a high accuracy. This information can be used to remove the limited number of motion-corrupted signals from the set $\{\bar{S}_\theta(k); \theta = 0, \dots, \frac{N_P-1}{N_P}\pi\}$. Afterwards only this limited number of signals must be measured again to obtain a complete non-distorted set.

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