Camera Calibration Based on 3D-Point-Grid ^{*}

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Abstract. In this paper an effective technique of camera calibration based on a 3D-point-grid is presented. The properties of structured space points, defined as 3D-point-grid, and their relations under perspective transformation are analyzed. Upon the correspondence between these 3D-points and their image points a calibration is made not only to compute the parameters of camera model by a linear method simply from the independent points in 3D-point-grid but also to verify, modify and regulate extrinsic parameters of the orientation and position simultaneously from the structure constraint implied in these points. On the basis of the verification, modification and regulation the intrinsic parameters of camera can be computed with more objective criteria. Experimental results show the necessity and advantage of the verification and modification of the orientation and position parameters, and prove that our calibration technique can improve the accuracy of both extrinsic and intrinsic parameters greatly.

1 Introduction

Camera calibration is the process of determining the internal camera geometric and optical characteristics (represented by intrinsic parameters such as focus f , center C_x , C_y and scale factor s_x etc.) and/or the 3D position and orientation of the camera relative to a certain world coordinate system (called extrinsic parameters like rotation matrix R and translation vector T). For a real camera this process includes also a determination of distortion parameters. A lot of papers dealing with the computation of these parameters have been published, e.g. [Tsai86], [LT88], [BM91], [LC93], [WM94], [Faug93], etc.

In most cases the extrinsic and intrinsic parameters are computed by neglecting the distortion. The intrinsic parameters can be verified by some physical methods. But the orientation and position are difficult to verify unless there are exact mechanical settings such as CMM in [BM91]. The methods in above listed

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papers are based on a sequence of correspondent 3D- and 2D-points, but no relationship exists among them, or has been considered and used. The verification of the model parameters can only be made until after all the parameters are computed.

Upon these considerations, a more effective technique, which uses structure-related space points during calibration with the process of verification, modification and regulation, is proposed in this paper. It is proceeded as follows: (1) analysis of the structure of 3D-point-grid, its relation to the transformation parameters, its construction and location of its image counterpart; (2) improved calibration of camera parameters and results; (3) conclusions.

2 3D-Point-Grid

2.1 Definition

Suppose a sequential triple set $\mathcal{L} = \{(i, j, k), i = 1, \dots, M, j = 1, \dots, N, k = 1, \dots, L\}$, $\Delta x, \Delta y$ and Δz are the unit lengths along the three orthogonal directions in space $\mathcal{R}^3$, then the set $\mathcal{Q} = \{(i\Delta x, j\Delta y, k\Delta z), (i, j, k) \in \mathcal{L}\}$ is defined as a 3D-point-grid, $\mathcal{L}$ is its label set. When $(I_0, J_0, K_0) \in \mathcal{L}$ is chosen as the origin of a coordinate system, $\{((i - I_0)\Delta x, (j - J_0)\Delta y, (k - K_0)\Delta z), (i, j, k) \in \mathcal{L}\}$ is the coordinates set of $\mathcal{Q}$.

2.2 Characteristics in structure

Coplanar and parallel relation Let $i = I_1$, a fixed number, then the elements in subset $q_{I_1} = \{(I_1\Delta x, j\Delta y, k\Delta z), j = 1, \dots, N, k = 1, \dots, L\}$ are coplanar. The M planes π_{I_1} , formed by q_{I_1} ($I_1 = 1, \dots, M$), are parallel, and $\mathcal{Q} = \cup_{I_1=1}^M (q_{I_1})$.

Coplanar and perpendicular relation Let $i = I_1$ and $j = J_1$, then we get two subsets $q_{I_1} = \{(I_1\Delta x, j\Delta y, k\Delta z), j = 1, \dots, N, k = 1, \dots, L\}$ and $q_{J_1} = \{(i\Delta x, J_1\Delta y, k\Delta z), i = 1, \dots, M, k = 1, \dots, L\}$ respectively. They form two planes π_{I_1} and π_{J_1} with $\pi_{I_1} \perp \pi_{J_1}$. In the same way we can get q_{K_1} and its π_{K_1} , and $\pi_{J_1} \perp \pi_{K_1}$ or $\pi_{K_1} \perp \pi_{I_1}$.

2.3 The relations among parameters of orientation and position from 3D-point-grid

Between subsets of the coplanar points that form parallel planes Generally, if any coordinate system takes a point in 3D-point-grid, that is $\Delta x_w, \Delta y_w$ and Δz_w away from its previous origin in x, y and z direction respectively, as its current origin and is acquired by translation in three directions, we have

$$R_1 = R_0, \quad T_1 = T_0 + R_0(\Delta x_w, \Delta y_w, \Delta z_w)^T \quad (1)$$

in which $\Delta y_w = \Delta z_w = 0$ corresponds to the situation of parallel plane in x direction, while $\Delta x_w = \Delta z_w = 0, \Delta x_w = \Delta y_w = 0$ to y and z direction respectively.

Between subsets of coplanar points that form perpendicular planes

Suppose that an origin is selected with a label (I_0, J_0, K_0) , then (see Fig. 1(a)) $q_{I_0} = \{(I_0\Delta x, j\Delta y, k\Delta z), j = 1, \dots, N, k = 1, \dots, L\}$ forms π_{I_0} i.e. y_0 - O - z_0 plane $q_{J_0} = \{(i\Delta x, J_0\Delta y, k\Delta z), i = 1, \dots, M, k = 1, \dots, L\}$ forms π_{J_0} i.e. x_0 - O - z_0 plane $q_{K_0} = \{(i\Delta x, j\Delta y, K_0\Delta z), i = 1, \dots, M, j = 1, \dots, N\}$ forms π_{K_0} i.e. x_0 - O - y_0 plane in x_0 - y_0 - z_0 coordinate system. Suppose that different coordinate systems

Fig. 1. (a) Points on perpendicular planes and (b) Selection of coordinate systems

are selected on these three planes by rules: (1) the origins are the same as that of x_0 - y_0 - z_0 ; (2) all are right-handed systems; (3) all x - o - y planes are on these three planes; (4) all z axes are in counter direction of the axes of x_0 - y_0 - z_0 , as shown in Fig. 1(b). The rotation matrices of these coordinate systems, which transform points in coordinate systems O, I, II, III into camera coordinate system, are supposed to be R_0, R_1, R_2, R_3 , then

(i) if R_{ij} stands for a matrix that transforms a point (x_i, y_i, z_i) from i th coordinate system into j th coordinate system, then the matrices for the coordinate transformations among O, I, II and III are listed in Table 1.

(ii) if a point in coordinate system I is transformed into coordinate system O, and then to camera coordinate system, then the rotation matrices have relations $R_1 = R_0 R_{10}$. Similarly we have $R_1 = R_2 R_{12}$, $R_1 = R_3 R_{13}$. Same relations hold for other coordinate systems and can be concluded in Table 2(a).

2.4 Construction of 3D-Point-Grid and extraction of image points

In experiment a 3D-point-grid $Q = \cup_{I_1=1}^M(q_{I_1})$ is constructed by the horizontal movement of a square pattern, or specially made cube with surfaces of square patterns, as shown in Fig.2. By steps: (1) subpixel edge detection; (2) edge tracing; (3) corner detection; (4) computation of corner-coordinates (see [Zhang96]), the image points can be extracted.

Table 1. Transformations of the coordinate systems on perpendicular planes

$i \nearrow j$	O	I	II	III
O	$R_{00}: \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$R_{01}: \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$	$R_{02}: \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$	$R_{03}: \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$
I	$R_{10}: \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$	$R_{11}: \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$R_{12}: \begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$	$R_{13}: \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$
II	$R_{20}: \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$	$R_{21}: \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$	$R_{22}: \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$R_{23}: \begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$
III	$R_{30}: \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$	$R_{31}: \begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$	$R_{32}: \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$	$R_{33}: \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Table 2. (a) Relations of coordinates' transformations on three perpendicular planes and (b) illustration of modification of R_0, R_1, R_2, R_3

R_{ij}	R_0	R_1	R_2	R_3
R_0	$R_0 I$	$R_0 R_{10}$	$R_0 R_{20}$	$R_0 R_{30}$
R_1	$R_1 R_{01}$	$R_1 I$	$R_1 R_{21}$	$R_1 R_{31}$
R_2	$R_2 R_{02}$	$R_2 R_{12}$	$R_2 I$	$R_2 R_{32}$
R_3	$R_3 R_{03}$	$R_3 R_{13}$	$R_3 R_{23}$	$R_3 I$

(a)

$$\begin{array}{lcl}
 R_0 \Rightarrow R_0 I = R_0^0 \searrow \nearrow \bar{R}_0 I \Rightarrow \bar{R}_0 \\
 R_1 \Rightarrow R_1 R_{10} = R_1^1 \searrow \nearrow \bar{R}_0 R_{01} \Rightarrow \bar{R}_1 \\
 R_2 \Rightarrow R_2 R_{20} = R_2^2 \nearrow \bar{R}_0 \searrow \bar{R}_0 R_{02} \Rightarrow \bar{R}_2 \\
 R_3 \Rightarrow R_3 R_{30} = R_3^3 \nearrow \searrow \bar{R}_0 R_{03} \Rightarrow \bar{R}_3
 \end{array}$$

(b)

3 Improved Camera Calibration: Verification, Modification and Regulation , and their Results

3.1 R's Verification and Modification

Case of parallel planes According to (1) there should be $R_0 = R_1 = \dots = R_{M-1}$ for R_j ($j = 0, \dots, M-1$) that are computed initially by Tsai's coplanar methods. In the light of this relation the validation can be verified by checking R_j 's elements or the angles from them. A common rotation matrix can be modified by averaging the orientation angles from them.

Case of perpendicular planes The verification can be made by checking the relations listed in Table 1. And furthermore based upon these relations, R_{ijs} can be modified by the process listed in Table 2(b). The verifying results in cube-surface experiment are given in Table 3.

Fig.2. Original images and the extracted corners for the construction of a 3D-point-grid with parallel planes or surfaces of a cube.

3.2 T's Verification and Regulation

Case of parallel planes Considering the orthogonality of R_j and the real geometrical relation among these coordinate systems, we have $|T_{(j+1)} - T_j| = D_{(j+1)j}$, $j = 0, \dots, M-2$, where $D_{(j+1)j}$ are the distances between the origins of the coordinate system on these planes. These relations can serve as a verification of T_j . They can be changed into one objective function

$$F = \left[\sum_{j=0}^{M-2} (|T_{(j+1)} - T_j| - D_{(j+1)j})^2 \right]^{1/2} = \min \quad (2)$$

and its minimization acts as a goal to the regulation of T_j .

In our experiment the origins of the world coordinate systems are selected at the top-right corner of the top-right square (see Fig.3(a)), $D_{(j+1)j} = Z_{(j+1)j}$. The results show that $|T_2 - T_1|$, $|T_3 - T_2|$ in Table 4 are much nearer to the real distances. Figure 4 gives the shape and contour of the objective function at final iteration, in which the minimum corresponds to optimal values of T_i , f and C_y .

Case of perpendicular planes In this case, when the coordinate systems are chosen on each plane, as shown in Fig. 3(b), if D_{12} , D_{23} and D_{31} stand for the distances among the origins of the selected coordinate systems, the objective function can be constructed as

$$F = \left[(|T_{II} - T_I| - D_{12})^2 + (|T_{III} - T_{II}| - D_{23})^2 + (|T_I - T_{III}| - D_{31})^2 \right]^{1/2} = \min \quad (3)$$

With this function the regulation is similar to that of the parallel plane. In experiment we choose $D_{12} = D_{23} = D_{31} = 0$, the results are shown in Table 5.

Finally improvements are made by optimalization of the total residual error of all correspondent 2D- and 3D-points for the image center (C_x, C_y) . Distortion

parameters k_{1x} , k_{2x} , k_{1y} and k_{2y} are computed based on Tsai's Radial Alignment Constraint model. A set of calibrated parameters are given in Table 6.

(a) Case of parallel planes

(b) Case of perpendicular planes

Fig. 3. Verification and modification of R , T .

4 Conclusions

A method of camera calibration that computes the parameters from 3D-point-grid, a group of structure related 3D points, is proposed in this paper. According to objective criteria deduced from the 3D-point-grid the camera extrinsic parameters are verified, modified and regulated. Based on the accurate estimation, the intrinsic parameters of the camera continue to be optimized. It is proved experimentally that our method can improve the accuracy of both extrinsic and intrinsic parameters greatly. The method can be applied not only in a well-defined lab situation but also in a nature scene with a precisely made structure-pattern.

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Table 3. Verification of R_i 's Relations of the Coordinate System on the Cube

$i \nearrow j$	O	I
O	$R_{00}: \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$R_{01}: \begin{bmatrix} 1.00 & 0.03 & 0.00 \\ 0.00 & 0.06 & 1.00 \\ 0.03 & -1.00 & 0.06 \end{bmatrix}$
I	$R_{10}: \begin{bmatrix} 1.00 & 0.00 & 0.03 \\ 0.03 & 0.06 & -1.00 \\ 0.00 & 1.00 & 0.06 \end{bmatrix}$	$R_{11}: \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
II	$R_{20}: \begin{bmatrix} 0.02 & 1.00 & -0.06 \\ 1.00 & -0.02 & 0.05 \\ 0.05 & -0.06 & -1.00 \end{bmatrix}$	$R_{21}: \begin{bmatrix} 0.05 & 1.00 & -0.05 \\ 0.11 & -0.06 & -1.00 \\ -1.00 & 0.05 & -0.11 \end{bmatrix}$
III	$R_{30}: \begin{bmatrix} -0.08 & 0.00 & -1.00 \\ -0.05 & 1.00 & 0.01 \\ 1.00 & 0.05 & -0.08 \end{bmatrix}$	$R_{31}: \begin{bmatrix} -0.09 & 0.03 & -1.00 \\ 0.99 & 0.11 & -0.08 \\ 0.10 & -0.99 & -0.04 \end{bmatrix}$
$i \nearrow j$	II	III
O	$R_{02}: \begin{bmatrix} 0.02 & 1.00 & 0.05 \\ 1.00 & -0.02 & -0.06 \\ -0.06 & 0.05 & -1.00 \end{bmatrix}$	$R_{03}: \begin{bmatrix} -0.08 & -0.05 & 1.00 \\ 0.00 & 1.00 & 0.05 \\ -1.00 & 0.01 & -0.08 \end{bmatrix}$
I	$R_{12}: \begin{bmatrix} 0.05 & 0.11 & -0.99 \\ 1.00 & -0.06 & 0.05 \\ -0.05 & -0.99 & -0.11 \end{bmatrix}$	$R_{13}: \begin{bmatrix} -0.09 & 0.99 & 0.10 \\ 0.03 & 0.11 & -0.99 \\ -1.00 & -0.08 & -0.04 \end{bmatrix}$
II	$R_{22}: \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$R_{23}: \begin{bmatrix} 0.00 & -0.14 & -0.99 \\ 1.00 & -0.02 & 0.00 \\ -0.02 & -0.99 & 0.14 \end{bmatrix}$
III	$R_{32}: \begin{bmatrix} 0.00 & 1.00 & -0.02 \\ -0.14 & -0.02 & -0.99 \\ -0.99 & 0.00 & 0.14 \end{bmatrix}$	$R_{33}: \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Table 4. Verification of the Distances among Origins

	Real Distance	before regulation		after regulation	
		Computed Dis.	Differences	Computed Dis.	Differences
$ T_1 - T_0 $	32.35	30.78	-1.57	29.97	-2.38
$ T_2 - T_1 $	49.95	55.23	5.28	50.59	0.64
$ T_3 - T_2 $	49.25	89.85	40.60	49.63	0.38

Note: $(\sum_{i=0}^2 (|T_{(j+1)} - T_j| - Z_{(j+1)j})^2)^{1/2} = 40.97$ (before reg.), 2.49(after reg.)

Fig. 4. Objective Function at the Final Iteration

Table 5. Verification of T_i ' of the Coordinate Systems on the Cube

unit(mm)	T_I	T_{II}	T_{III}	$ T_I - T_{II} $	$ T_{II} - T_{III} $	$ T_{III} - T_I $
T_x	-0.93	-0.11	-0.60	0.82	0.49	0.32
T_y	6.57	6.56	8.61	0.02	2.05	2.04
T_z	272.18	321.87	249.44	49.69	72.43	22.74
distance				49.69	72.46	22.83

Before Regulation, $[(|T_{II} - T_I|)^2 + (|T_{III} - T_{II}|)^2 + (|T_I - T_{III}|)^2]^{1/2} = 90.78$

unit(mm)	T_I	T_{II}	T_{III}	$ T_I - T_{II} $	$ T_{II} - T_{III} $	$ T_{III} - T_I $
T_x	-0.93	-0.11	-0.60	0.82	0.49	0.32
T_y	6.57	6.56	8.61	0.02	2.05	2.04
T_z	226.26	226.7	223.12	0.53	3.67	3.14
distance				0.97	4.24	3.76

After Regulation, $[(|T_{II} - T_I|)^2 + (|T_{III} - T_{II}|)^2 + (|T_I - T_{III}|)^2]^{1/2} = 5.75$

Table 6. Calibrated Parameters from the Square Patterns on four Parallel Planes

Transformation Parameters (the unit for elements of T is mm)						Physical Parameters	Distortional Parameters
R	-0.8488	0.0144	0.5285	θ	-31.90°	$f=8.559$ $C_x=260.0$ $C_y=253.0$ $s_x=1.0214$	$k_{1x}=0.002535$ $k_{2x}=0.002532$ $k_{1y}=-0.001040$ $k_{2y}=0.000791$
	0.3335	0.7903	0.5140	ϕ	142.74°		
	-0.4102	0.6125	-0.6756	ψ	-0.98°		
	$T_0 = (142.61, -102.64, 546.97)^T$						
	$T_1 = (121.62, -116.12, 563.82)^T$						
	$T_2 = (90.19, -137.14, 597.45)^T$						
	$T_3 = (59.12, -158.02, 630.20)^T$						

*NOTE: the selection of Vector T_i depends on the selected world coordinate system