Decomposing Contours into Curves of Different Families

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Abstract. Many contour segmentation methods search for a decomposition into curves of a same family (as, e.g., the polygonal approximation presented in [1]). However, the contours of real objects often are constituted by curves of different families. In this paper an entropic criterion is presented for the decomposition of a given contour using curves of different families. The entropic criterion accounts both for the information cost associated to the use of complex curves and the uncertainty cost associated to the larger curve fitting error deriving from the use of simple curves. Experimental results on real images are reported to show the application of the criterion to two different curve families, namely the linear segments and the circular arcs.

1 Introduction

Many contour segmentation methods decompose a contour into curves of the same family (see, e.g., [2], [4]). Saund [3] presented a scale-invariant criterion given by the ratio between the length of the fitted curvilinear arc and the mean distance between the arc and the points assigned to it. However, this criterion can not be applied to the case of overlapping curve families such as, e.g., the family of the line segments and the family of the circular arcs. In fact, no criterion based on the fitting error would prefer a line segment to the best fitting circular arc, since the latter curve is characterized by a smaller fitting error.

In this paper an entropic criterion is proposed for the decomposition of a set of points into curves belonging to different, but possibly overlapping, families. According to this criterion, the larger fitting error resulting from fitting a set of points to a simpler curve is traded for the lower information cost associated to the representation of the simpler curve. The main issue in this criterion concerns how to combine, in a nonparametric way, the information cost involved in representing a curve and the fitting error related to associating a set of points to the curve.

The determination of a globally optimum decomposition, according to the adopted criterion, would involve a combinatorially exploding complexity. Therefore a method is proposed to determine a suboptimal decomposition. The performance of the suboptimal method are demonstrated on contours extracted from real images.

In Sect. 2, the entropic criterion is introduced and the decomposition problem is formulated. In Sect. 3, the suboptimal method is presented. In Sect. 4, the

performance of the suboptimal method on contours extracted from real images is illustrated. Section 5 contains some concluding remarks.

2 Entropic criterion for contour decomposition

Let us consider a set $\mathcal{P}$ of N points P_i ($i = 1, \dots, N$), which are known to lie within a rectangular planar region of dimensions $R \times R$. A *decomposition* of the point set $\mathcal{P}$ is characterized by a set $\mathcal{C} = \{C_1, \dots, C_n\}$ of curves, and it specifies an assignment of any point $P_i \in \mathcal{P}$ to a curve $C_j \in \mathcal{C}$.

One way to represent the position of the points $P_i \in \mathcal{P}$ in the planar domain $R \times R$ up to a resolution δr_o is to represent their coordinates (x_i, y_i) directly: this involves an information quantity related to both the domain dimension R and the resolution δr_o .

An alternative way to represent the position of the points is through the set $\mathcal{C}$ of curves: any point $P_i \in \mathcal{P}$ is assigned to one of the curves of $\mathcal{C}$. In this case the involved information includes: (i) the information needed to specify the curves C_j with $j = 1, \dots, n$, and, for any point $P_i \in \mathcal{P}$, (ii) the information needed to specify that the point P_i is associated to the curve C_j (and not, e.g., to other curves), and (iii) the information required to represent the coordinates of the point P_i up to a resolution δr_o , once P_i is known to be assigned to the curve C_j .

The proposed criterion is given by the difference between the information associated to representing the set of points through the assigned curves and the information associated to representing the coordinates of the points directly.

To represent the information quantity, the *relative information content* is used. In particular, suppose the a priori and the a posteriori distribution of a random variable U have the same shape: if the standard deviations of the a priori and a posteriori distributions are, respectively, A and δu_o , then the relative information content associated to U is given by $H(U) = \ln(A/\delta u_o)$.

2.1 Direct representation of contour points

In this representation, the coordinates of the contour points are directly specified up to a resolution δr_o .

We chose to represent a point through its distance from a reference curve. In this way, we represent the “lateral” coordinate of the point, while neglecting to specify its “longitudinal” coordinate (i.e., the its coordinate along a direction which is locally parallel to the reference curve). Thus, the relative information content associated to representing a set of contour points is

$$H_d = \sum_{i=1}^N \ln \frac{R}{\delta r_o} = N \ln \frac{R}{\delta r_o}.$$

2.2 Contour points represented through a set of curves

To represent the contour points through a set of curves, first the curves are represented, and then the lateral coordinate of each contour point with respect to its assigned curve is specified up to a resolution δr_o .

Suppose that the decomposition of the contour is through a set of n curves. Each curve is represented by a set of parameters. For instance, straight lines are represented by the polar parameters ρ and θ , which represent the distance of the origin from the line, and the angle formed by the X axis and the normal to the line. A circular arc is represented by its radius and by the cartesian coordinates of the center of the circle (r_j, x_j, y_j) .

If both the a priori and the a posteriori distribution of the parameters of the curve C_j are Gaussians, with covariance matrixes, respectively, Λ_{j_o} and Λ_j , the relative information content of the curve representation is

$$H(C_j) = (1/2) \ln(|\Lambda_{j_o}|/|\Lambda_j|)$$

Here Λ_{j_o} is relative the dimension R of the image screen, while Λ_j is relative to the parameters of the curve C_j fitted to the contour points assigned to it.

Now that the parameters of the curves have been specified, the single contour points have to be represented. For each contour point, first the assigned curve has to be specified, and then the lateral coordinate, relative to the associated curve, must be provided. Suppose that N_j , out of the N contour points, are associated to the curve C_j : then the information quantity needed to specify the curve associated to a generic contour point P_i is

$$H_i = - \sum_{j=1}^n \frac{N_j}{N} \ln \frac{N_j}{N}.$$

Once a contour point P_i is known to be associated to a curve C_j , the lateral dispersion of the point about the curve has to be represented (neglecting the "longitudinal" coordinate of the point) up to a resolution δr_o .

The a priori uncertainty of the position of the point P_i is now related to the variance σ_j^2 of the lateral dispersion about the curve C_j assigned to the point: σ_j^2 is estimated, e.g., by a least squares method. To specify the lateral coordinate of P_i with an accuracy δr_o , the needed information quantity is

$$H_{ij(i)} = \ln(\sigma_{j(i)}/\delta r_o),$$

where $j(i)$ indicates the index of the curve assigned to the point P_i . For a given decomposition of the set of contour points, the relative information content is

$$H_e = \sum_{j=1}^n H(C_j) + N \cdot H_i + \sum_{i=1}^N H_{ij(i)}.$$

2.3 Global criterion

The global criterion proposed in this paper, is the difference between the relative information content of the curve representation of the contour points and the relative information content of their direct representation. Notice that the parameter δr_o vanishes from the global criterion.

$$H = \frac{1}{2} \sum_{j=1}^n \ln \frac{|A_{jo}|}{|A_j|} - \sum_{j=1}^n N_j \ln \frac{N_j}{N} + \sum_{i=1}^N \ln \frac{\sigma_{ij(i)}}{R} \quad (1)$$

2.4 Expression of the determinants

First let us consider a straight line C_j fitted to a set of points $\{P_1, \dots, P_{N_j}\}$. Suppose that the least square estimate of the line parameters are (ρ_j, θ_j) , which represent, respectively, the distance from the origin to the line and the orientation of its normal. Let $\bar{x}_j = (\sum_i x_i)/N_j$, $\bar{y}_j = (\sum_i y_i)/N_j$, and $L_j^2 = \sum_i ((x_i - \bar{x}_j) \sin \theta_j - (y_i - \bar{y}_j) \cos \theta_j)^2 / N_j$. The a posteriori covariance matrix referred to an origin placed at the center $(\bar{x}_j, \bar{y}_j)$ of the line is:

$$A_j = \frac{1}{N_j} \begin{bmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 / L_j^2 \end{bmatrix}$$

Its determinant is $|A_j| = \sigma^4 / (N_j^2 L_j^2)$. It is supposed $|A_{jo}| = R^2$.

Now let us consider a circular arc C_j fitted to a set of points P_i ($i = 1, \dots, N_j$) characterized by their polar coordinates (ρ_i, θ_i) . (For the fitting of a circular arc to a set of points see [5].) Suppose that these polar coordinates are referred to an origin which coincides with the estimated center of the circle. Let σ^2 be the variance of the lateral dispersion of the points about the circular arc (σ^2 can be approximated by the mean squared distances of the points from the best fitting circle). The inverse of the a posteriori covariance matrix of the circle parameters is:

$$A_j^{-1} = \frac{1}{\sigma^2} \begin{bmatrix} N_j & \sum_i \cos \theta_i & \sum_i \sin \theta_i \\ \sum_i \cos \theta_i & \sum_i \cos^2 \theta_i & \sum_i \cos \theta_i \sin \theta_i \\ \sum_i \sin \theta_i & \sum_i \cos \theta_i \sin \theta_i & \sum_i \sin^2 \theta_i \end{bmatrix}.$$

The form of $|A_j|$ is particularly simple if the reference frame is rotated such that $\sum_i \sin(2\theta_i) = 0$. In this case is

$$|A_j| = \frac{(\sigma^6 / N_j)}{(\sum_i \cos^2 \theta_i)(\sum_i \sin^2 \theta_i) - (\sum_i \cos^2 \theta_i)(\sum_i \sin \theta_i)^2 - (\sum_i \sin^2 \theta_i)(\sum_i \cos \theta_i)^2}$$

For the a priori covariance matrix, it is assumed that $|A_{jo}| = R^6$.

3 Criterion minimization

Finding the decomposition which minimizes the criterion (1) would require a time exponential in the number N of points. Therefore in this work a suboptimal method is adopted. A set of threshold values are considered. For each threshold value, the following procedure is executed.

The contour is decomposed into curves of the broadest family (in our case, into the circular arcs) by means of an edge-following technique [2]. The considered threshold value is adopted to compare the distance between the currently analyzed point and the currently followed edge. After the decomposition, the entropic criterion is calculated and a series of local refinements are attempted.

A first local refinement, called *merging*, consists in substituting two neighboring curves by a single curve, by fitting one curve to the set of points associated to the two above curves. The curve may belong to any of the considered families. A second local refinement, called *splitting* consists in substituting a single curve with two curves belonging to any of the considered families.

First merging steps are attempted. After each refinement, the new criterion value is calculated and compared with the previous one. If the criterion value decreases, then a further refinement step is attempted on the curve obtained at the previous step. If the criterion value does not decrease, the situation previous to the currently attempted step is recovered, and splitting steps are iterated on the last modified contour segments, until the criterion value does not decrease anymore. Once the process ends, the same process is carried out starting from a different set of neighboring segments.

The above operations are carried out for each of the considered threshold values. Once all the threshold values of the set have been considered, the best decomposition is retained (among those obtained in the previous steps).

4 Experimental results

The suboptimal method described in Sect. 3 has been applied to the contours, extracted from real images, shown in Figs. 1a-2a. In our case the dimension of the image is $R = 256$. The threshold values used in the process range from 1 to 6 pixel. The obtained contour decompositions are shown in Figs. 1b-2b. The obtained contour decomposition shown in Figs. 1b and 2b capture the correct decomposition of the model of the reproduced objects.

5 Conclusions

An information-theoretic criterion has been presented for driving the decomposition of contours into curves of different families. The criterion allows to evaluate both the fitting error, deriving from fitting a curve to a set of points, and the information cost involved in representing the fitted curve. The reported experimental results seem promising, in that they capture the correct contour

decomposition of the objects present in the original images. Future work will address the introduction of further curve families, the search for better optimization strategies, and the investigation of the applicability of the criterion to the fitting of 3D data.

References

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Fig. 1: A contour (a) and its decomposition (b)

Fig. 2: A contour (a) and its decomposition (b)