Analytical Learning Guided by Empirical Technology: An Approach to Integration (Extended Abstract)

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Abstract. An approach to integration of the analytical and empirical learning strategies for solving the theory-based concept specialization problem is proposed. The emphasis is on learning correct concepts within conjunctive description language isolated from supplied domain theory. In order to overcome expressive limitations of such language, the analytical learning component is guided by a specific empirical version space technology. The approach learns incrementally a correct pure conjunctive or DNF concept definition in dependance on the domain theory.

1 Introduction

The theory-based concept specialization problem (TBCS), defined in [1] and [5], requires a more general formulation for practical applications. There are given: (1) a first order domain theory *DT* from DNF rules with consequents from one predicate; (2) a goal concept *GC* defined by the domain theory; (3) a finite set *TS* of examples described in pure conjunctive first order language, which are positive examples of the concept *GC*, and are positive and negative examples of an unknown concept under study *SC* (*SC* is specialization of *GC*). The task is to find a complete and consistent definition of the concept under study *SC*, described in DNF first order language, that is implied by the positive and not by the negative examples in context of the domain theory.

Most of the previous works deal only with specific cases of the above TBCS problem and solve these cases by integrated empirical-after-analytical learning approaches: IOE [1], MECEA [5]. They produce a complete and consistent pure conjunctive concept definition of *SC* when the domain theory determines a sufficiently expressive concept description language. When the language expressiveness is insufficient (i.e., the definition of *SC* is DNF), these approaches fail. This situation arises when in the process of incremental learning either of the two anomalies occurs [4]; (1) incompleteness: a positive example is encountered, but the current concept definitions cannot be generalized to admit it without admitting negative training; (2) inconsistency: a negative example is encountered, but the current concept definitions cannot be specialized to reject it without also rejecting positive training.

This paper proposes an improved approach, called Multiple Explanation-based learning guided by the Space Fragmenting Algorithm (MESFA), for solving the TBCS problem. The idea is to direct the analytical learning with a specific empirical technology that allows the production of correct concepts regardless of the expressiveness of the concept description language isolated from domain theory. The novelty is that all hypothetical concept definitions within the language are considered as a disjunctive version space. The disjunctive version space is representable by its boundary sets and admits incremental learning with the Space Fragmenting algorithm [4]. Space Fragmenting forms a correct, complete and consistent, pure conjunctive or DNF definition of the concept under study.

2 Representation and Ordering of the Concepts

The concept definitions in MESFA are expressed in a concept description language (CDL_{DT}) isolated from supplied domain theory DT :

Definition 1. *The concept description language is a finite set of concept definitions: $CDL_{DT} = \{ c \mid E \in P(TS), DT \cup E \vdash c \text{ such that } DT \cup c \vdash GC \}$, where $P(TS)$ is powerset of the training set TS and the examples E are subset of TS .*

The finiteness of CDL_{DT} is achieved by a proper theorem prover. Between the elements of CDL_{DT} a relation generalized concept ordering holds. The generalized concept ordering is reflexive, anti-symmetric tautologic and transitive:

Definition 2. *A concept c_2 ($c_2 \in CDL_{DT}$) is more general than or equally general to another concept c_1 ($c_1 \in CDL_{DT}$), written $c_2 \succeq c_1$, in the context of the domain theory DT if and only if the entailment $DT \cup c_1 \vdash c_2$ holds.*

It is proven¹ that all subsets C of CDL_{DT} possess the properties convexity and definiteness [2]. Therefore, the subsets of CDL_{DT} are representable by boundary sets and admit version space learning. MESFA distinguishes by supporting all hypothetical concept definitions in form of a disjunctive version space:

Definition 3. *The set C is a disjunctive version space (DVS_{SC}) of the concept under study SC if and only if it consists of all concept definitions c ($c \in CDL_{DT}$) that are not implied by the negative examples E_N of SC in context of the domain theory: $C = (DVS_{SC})$ iff $C = \{ c \in CDL_{DT} \mid DT \cup E_N \not\vdash c \}$.*

With regard to the TBCS problem, definitions 1 and 2 suggest that the disjunctive version space consists of conjunctive concept definitions. Each conjunctive definition summarizes some positive training (*relaxed completeness*) while remaining consistent with all of the negative training (*strict consistency*). The disjunctive concept definitions are built of conjunctive ones that are: first, not mutually related by generalized concept ordering; and second, the conjunctive definitions together satisfy the completeness requirement.

¹ The complete paper with proofs and examples is available from the authors.

The disjunctive version space $\mathcal{DVS}_{SC}$ of the concept under study SC is represented by its maximal and minimal boundary sets G and S :

$$\mathcal{DVS}_{SC} \langle G, S \rangle = \{ c \in CDL_{DT} \mid \exists s_i \in S, \exists g_i \in G \text{ such that } s_i \preceq c \preceq g_i \}$$

The set G is the set of maximally general concept definitions in CDL_{DT} , which are not implied by the negative examples E_N of the concept SC :

$$G = \{ g_i \in CDL_{DT} \mid DT \cup E_N \not\models g_i \text{ such that } \neg \exists g \in CDL_{DT}: g_i \prec g, DT \cup E_N \not\models g \}$$

The set S consists of elements s_i that are more specific than some corresponding g_i . Every s_i is the minimally specific concept definition implied by all elements of its set S_i . Every S_i is a set of minimally specific concept definitions s_{ik} implied by the positive examples E_{P_i} of SC that also imply g_i :

$$\begin{aligned} S_i &= \{ s_{ik} \in CDL_{DT} \mid DT \cup E_{P_i} \vdash s_{ik}, g_i \succeq s_{ik} \text{ such that } \neg \exists s: s \prec s_{ik}, DT \cup E_{P_i} \vdash s \} \\ S &= \{ s_i \in CDL_{DT} \mid DT \cup S_i \vdash s_i, g_i \succeq s_i \text{ such that } \neg \exists s: s \prec s_i, DT \cup S_i \vdash s \} \end{aligned}$$

For some g_i ($g_i \in G$) the sets of positive examples E_{P_i} implying them may be empty. The corresponding s_i to these g_i are specified with the symbol $\perp$ (a bottom element implying all elements in the ordered CDL_{DT}), which is an empty definition that classifies every example as negative.

3 Scenario for Learning with MESFA

MESFA is developed employing the Space Fragmenting algorithm. The Space Fragmenting algorithm supports a disjunctive version space of all hypothetical concept definitions of SC . It carries out efficient incremental learning by manipulating only the boundary sets G and S .

$$\text{Initialize: } G = \{ (\text{antecedent of } GC)_1 \} \quad S = \{ (\perp)_1 \}$$

For each next example update $\mathcal{DVS}_{SC} \langle G, S \rangle$ into a new one $\mathcal{DVS}'_{SC} \langle G', S' \rangle$:

In case of a positive example e_p :

$$G' = G$$

$$S' = \{ s'_i \in CDL_{DT} \mid DT \cup S'_i \vdash s'_i, g'_i \succeq s'_i \text{ such that } \neg \exists s: s \prec s'_i, DT \cup S'_i \vdash s \}$$

where for each i :

$$\text{if } g_i \succeq e_p \text{ then } S'_i = \{ s'_{ik} \in CDL_{DT} \mid DT \cup \{s_i\} \cup \{e_p\} \vdash s'_{ik}, g'_i \succeq s'_{ik} \text{ such that } \neg \exists s: s \prec s'_{ik}, DT \cup \{s_i\} \cup \{e_p\} \vdash s \}$$

$$\text{if } g_i \not\succeq e_p \text{ then } S'_i = \{s_i\}$$

In case of a negative example e_n :

$$G' = \{ g'_i \in CDL_{DT} \mid DT \cup \{e_n\} \not\models g'_i, DT \cup \{g'_i\} \vdash g_i$$

$$\text{such that } \neg \exists g \in CDL_{DT}: g'_i \prec g, DT \cup \{e_n\} \not\models g, DT \cup \{g\} \vdash g_i \}$$

$$S' = \{ s'_i \in CDL_{DT} \mid DT \cup \{s'_i\} \vdash s_i, s'_i \preceq g'_i$$

$$\text{such that } \neg \exists s: s \succ s'_i, DT \cup \{s\} \vdash s_i, s \preceq g'_i \}$$

MESFA always terminates when the explanations of the positive examples are different from the explanations of the negative ones. This is due to the avoidance of the incompleteness and inconsistency anomalies. Two terminations of the learning process are possible. First, when the elements g_i of G are identical with their corresponding s_i of S , a *completely learned definition* of the concept SC is generated. Second, when the boundary sets are not focused, an *incompletely learned definition* of the concept SC is generated. Depending on the power of the set S , the resulting concept definition is pure conjunctive or DNF.

4 Comparisons and Relevance to Other Works

MESFA and IOE [1] are two approaches to solving the TBCS problem. IOE uses domain theory to build an explanation only from the positive examples and forms definitions of SC by an empirical technique over the explanation. A shortcoming of IOE is that it is not capable of learning from negative examples, and therefore it cannot be applied in real domains where the concepts are DNF rather than pure conjunctive.

MESFA is close to the Hirsh's combination of empirical and analytical learning with IVSM [3]. His integrated approach syntactically processes analytically generalized definitions of the examples. This and the kinds of learned concept definitions suggest that MESFA proposes a more general form of learning.

5 Conclusion

A novel approach, called Multiple Explanation-based learning guided by the Space Fragmenting Algorithm (MESFA), was presented. The main advantage of MESFA is that it generates a pure conjunctive or DNF concept definition in an incremental manner in dependence on the provided first order domain theory.

The MESFA approach allows careless design of the domain theory, requires a small number of examples and few computation resources, and may construct a correct definition for several unknown concepts SC .

References

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