Map Generalization as a Graph Drawing Problem

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Abstract. A map may be regarded as a plane graph drawing; and, with the growth of the field of computer cartography, a map is increasingly treated as a straight-line plane graph drawing. Map generalization, the process of redrawing a map at a smaller scale, can, hence, be regarded as a graph drawing problem. The initial version of the map, the input to the process, is a plane graph along with a drawing of that graph. The derived output is also a plane graph that is combinatorially a minor of the input graph. The edges and vertices of the derived graph may be obtained through a sequence of selection and simplification operations. The positioning of those edges and vertices must be obtained through a sequence of placement and displacement operations. Considerable research and development has already been undertaken in the areas of feature selection and simplification. Some generally adequate heuristic solutions to selection and simplification problems are in use today for many types of maps. For other types of maps, such as topographic maps, these solutions are insufficient because they do not simultaneously address the more difficult problems of placement and displacement of features.

1 Introduction

With the growth of a body of knowledge called computer cartography, computers have increasingly assumed the map drawing tasks of cartographers . Many of the mechanical tasks, such as drawing straight lines of a specified thickness, are natural tasks for the computer to assume; and most cartographers have embraced the technology that makes their product more consistent and precise. Many cartographers, however, contend that other tasks, especially those involving design decisions, cannot or should not be yielded to the computer. Map generalization is one such task. The tension between traditional cartographers and specialists in computer cartography will only be broken when the specialists present cartographers with a system that generates a product that is satisfactory in the eyes of the cartographers.

1.1 Background from Cartography

Map Generalization in traditional manual cartography refers to the science and art of re-drawing a map at a smaller scale. The new product cannot accommodate all of the detailed information of the original larger scale map; and, hence, the cartographer must selectively transform or omit features and symbols to accomplish the generalization. There are some hard and fast rules (the science) about what to include and how to represent it in a generalized product, but there are also aesthetic and as-yet-unquantifiable considerations (the art) in the generalization process and in map-making, in general.

Automated map-making has stimulated interest in automated map generalization from the earliest developments. Computer cartographers were quick to recognize that, if they had a working automated map generalization system and a single digital map at any particular scale, they could get maps at all smaller scales "free of charge." Computer cartographers, therefore, encouraged traditional cartographers to help them classify the operations that are used for map generalization. Cartographers generally agreed upon the following primitive operations (among others that appeared on various classification schemes): selection, simplification, exaggeration, and displacement, which are illustrated in Figs. 1 - 4.

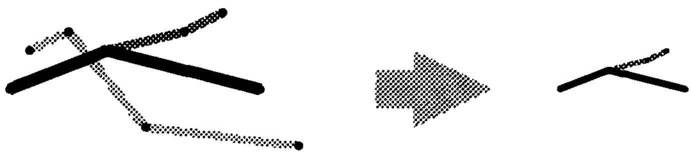

Fig. 1. The linear map generalization process of selection

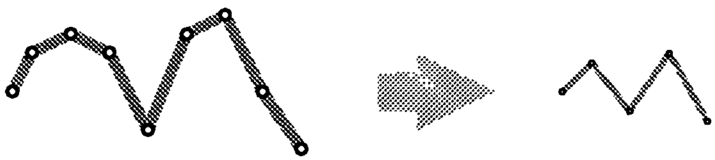

Fig. 2. The linear map generalization process of simplification

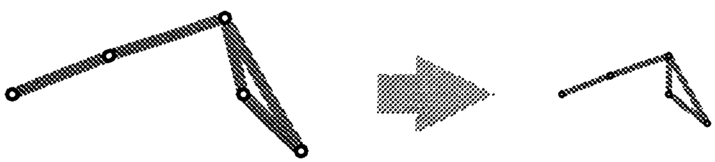

Fig. 3. The linear map generalization process of exaggeration

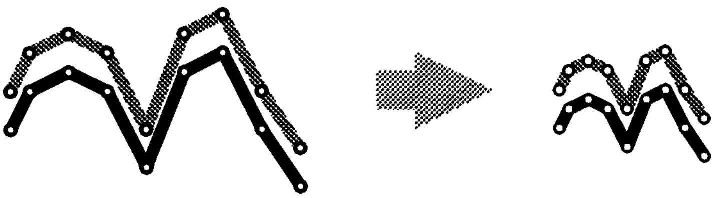

Fig. 4. The linear map generalization process of displacement

1.2 Statement of the Graph Drawing Problem

By an oversimplification that would give many cartographers fits, the set of linear features that constitute a map, especially a map drawn from a computerized cartographic database, may be regarded as a straight-line graph drawing in the plane. In that simplified context, the problem of drawing a derived map of the same area at a smaller scale becomes a problem of modifying both the graph and its drawing. The graph may be modified through a series of reduction operations involving the fusion of graph elements (replacing the graph with a quotient graph or graph minor) or the removal of graph elements (replacing the graph with a subgraph). The drawing may be modified in two ways: (1) assign positions to the new vertices resulting from the structural reduction change in the graph itself, or (2) reposition old vertices to achieve better balance or representation of the mapped area.

1.3 Status of Cartographic Research into Generalization

There are two principal directions that researchers have taken with automated map generalization, which correspond to top-down and bottom-up approaches. The bottom-up approach was used by the earliest researchers, who examined local subproblems, such as line feature simplification (for a single polygonal line feature) [DP73]. These pioneers hoped to build a generalization system by amassing a collection of solutions to all subproblems. This focus on algorithmic analysis of subproblems has continued over three decades because many of the subproblems are accessable and solvable [ZDH84]. Some systems available today consist of a library of subproblem algorithms from which to mix and match [Lee92]. Recently, cartographers expressed concern about the interdependence of subproblems, and a trend has been to try to formalize the overall structure of generalization operations by employing tools and techniques from artificial intelligence.

Miscellaneous Algorithms for Generalization Subproblems The earliest generalization algorithms consisted of a collection of algorithms for reducing the number of points used to represent a linear feature [DP73]. Zoraster, Davis, and Hugus [ZDH84] surveyed over a dozen line generalization algorithms all of which offered methods to simplify the representation of a single polygonal line feature while retaining the cartographic character (i.e. shape) of the feature. Methods included systematic and random selection of vertices, various filtering algorithms, angle selection, smoothing of digital elevation models, various tolerance band algorithms, relaxation techniques, and various mathematical fitting algorithms. All of the surveyed techniques shared a serious deficiency: they work for a single polygonal line, but none could guarantee that two neighboring polygonal lines would be transformed in such a way that they would not conflict with (i.e. intersect) each other.

Feature selection (or elimination) algorithms also proved to be straightforward and easy to implement. Often linear features have a natural hierarchy determined by feature importance or permanence or feature size. A cartographer utilizes the natural hierarchy to make decisions. Interstate highways and major rivers remain on small scale maps. Jeep trails and intermittent streams are among the first features to disappear as maps are scaled down through generalization.

Displacement algorithms were developed to detect and correct feature intersection and overlap (or to anticipate and prevent such overlap). Displacement algorithms were developed in part because existing line simplification algorithms were known to introduce geometric conflicts. Any particular displacement algorithm must strike a balance between the opposing goals of (1) separating features sufficiently to distinguish them and (2) placing features close enough to be in their correct position. A notable rule-based system developed at Rensselaer Polytechnical Institute focuses on a few line simplification techniques followed by corrective line displacement techniques [Nic88].

Algorithms to exaggerate features are a consequence of imposing constraints that guarantee that the cartographic character of a feature be preserved. In order to depict a bay, for example, with a 0.05 mm line weight, the opening to the bay must occupy at least 0.2 mm and the bay itself should have an extent of at least 0.3 mm, no matter the scale.

Formalizing the Overall Generalization Process Many cartographers have argued that an effective generalization system must look at *all* interactions of *everything* being generalized or modified [Mor74], [Bea87] and not at individual generalization processes such as the process of line feature generalization or even more specialized processes such as linear feature simplification. Nearly all of the recent research on automated map generalization has concentrated on determining a comprehensive set of *rules* for a knowledge-based, rule-based, or expert system [BW88] for automated map generalization [BM91b]. A great deal of discussion has gone into deciding what the rules should be [Rat67], [BM91a]. A lack of consensus on the rules has resulted in very little effort expended toward implementing those rule sets efficiently and effectively. Another difficulty is that many of the rules themselves are non-constructive—they merely describe desired or required characteristics of the output (e.g. legibility, spatial balance, correct topology, no feature overlap or contact).

2 A Mathematical Framework for Generalization

Formalizing the overall generalization process may be too difficult a task to undertake. Looking at all interactions among all map components may be too daunting. Looking at interactions among linear map features is not only more manageable, but it fits nicely into some recognizable mathematical frameworks, namely, graph theory and plane geometry. Although some cartographers might argue that seeking a partial solution involving only linear features is unacceptable to them (because it does not solve the "big" problem), nevertheless, focusing on representing and drawing linear features does permit the formulation of some interesting graph drawing problems.

2.1 Maps as Plane Graphs

Although map-making encompasses far more than simply drawing graphs in the plane, many of the interesting and challenging problems of computer cartography and map generalization can be formulated either as problems in drawing plane graphs, or as problems in storing, retrieving, and transforming those graphs by means of efficient data structures. The underlying mathematical model used for many contemporary computer mapping systems [F*92] consists of a cellular decomposition of a (usually rectangular) 2-D surface into a finite collection of 0-cells, 1-cells, and 2-cells. The 0-cells are points; the 1-cells may be taken to be straight-line segments joining two 0-cells; and together, the 0-cells and 1-cells make up the 1-skeleton, a straight-line plane graph. The 2-cells are the connected components of the complement of the 1-skeleton. The 1-cells in the mathematical model have zero width or thickness. This particular model necessitates the incorporation of additional theory to deal with the reality of lines on actual maps having thickness or non-zero width.

2.2 Maps as Graph Drawings

The depiction of 1-cells in any printed map, (i.e. the drawing of linear features), will have non-zero width or thickness. Non-zero line width and human visual resolution limitations are two reasons that a smaller scale map cannot be simply a shrunken scaled exact copy of the original map [CZH84]. One cannot draw and differentiate 50 parallel lines that are less than 10^{-10} inches apart! On the other hand, one can easily draw and distinguish them if they are 10 inches apart. Line thickness plays a key role in the drawability and legibility of certain plane graphs. Scale changes affect the ratio between line thickness and size of free space between lines. Nickerson and Freeman [Nic88] ingeniously simulated scale reduction by increasing line width, thereby transforming the problem of detecting feature interference due to scale change to a problem of expanding line widths until the thick representations make contact with each other.

2.3 Graph/Drawing Operations of Map Generalization

A cartographic database produces a satisfactory map at its nominal scale. An image produced at a scale that is significantly smaller than the nominal scale will appear cluttered and perhaps even unreadable. A satisfactory generalization process should generate a map at any scale that is smaller than the nominal scale. To do that, the process must unclutter the graph. The following map/graph/drawing operations are standard generalization techniques that are regularly used to accomplish some uncluttering:

1. Polygonal line simplification to reduce sinuosity of curves (e.g. the exact location of a stream is less important than merely showing that it is there on the smaller scale maps) [DP73]: Replace graph by graph minor, position new vertices, reposition old vertices along polygonal line.
2. Polygonal line simplification to reduce the drawing burden (e.g. a polygonal line that is one inch in length should not be made up of 10,000 segments): Replace graph by graph minor, position new vertices, reposition old vertices along polygonal line.
3. Feature removal for features of lesser importance (e.g. transportation network hierarchy shows fewer road at smaller map scales: only interstate highways appear on maps at very small scales): Replace graph by subgraph.
4. Feature enlargement to keep shape character visible (e.g. distinctively shaped features: San Francisco bay, Italy): Reposition old vertices along distinctive feature.
5. Feature consolidation to accommodate attribute merging (e.g. ten soil types reduce to five soil types at smaller scale): Replace graph by subgraph, replace graph by graph minor.
6. Dimension reduction at much smaller scales (e.g. a city or lake becomes a point feature at very small scale) [BW88]: Replace graph by graph minor.
7. Feature displacement to maintain visual separation (e.g. a road adjacent to a river must be drawn far enough away so that it does not touch or coincide with the river) [Nic88]: Reposition old vertices to prevent features from getting too close to each other.

2.4 Missing Theory

Existing methods and algorithms provide local modifications to the graph and its drawing. These modifications are made independently of one another, and, therefore, they may produce conflicting and incorrect representations of relative feature positions. This problem is especially conspicuous for topographic maps of contour lines. Contour lines are closed curves that generally do not intersect, tend to be nearly parallel, reflect the steepness of slope by means of separation distance to the neighboring contour, and highlight local geomorphology through their degree of non-parallelness [Ram92].

Global Considerations Managing feature selection, simplification, and displacement in a global context is indeed a formidable challenge, but such a theory is needed to guarantee that processes terminate and do not precipitate endless chains of operations. Also needed is a displacement theory/algorithm that always results in a stable configuration or drawing (or reports that such a drawing is impossible). Nickerson and Freeman [Nic88] have offered a substantial insight into one possible approach through their use of line thickening to simulate scale reduction. Critical points in a continuous line thickening procedure occur when segments touch and begin to merge.

Dynamic Generalization An automated generalization system should be able to make decisions about when and how it will modify features in order to draw the map at each and every possible smaller scale. One way to make those decisions is iteratively. One may envision the map's scale as a continuously decreasing parameter of the drawn map product. As scale decreases, the generalization system itself should prevent clutter or inadequate feature separation by adjusting features accordingly. Feature interaction and interference can be eliminated by watching for feature pairs to fall below predefined nearness thresholds. Scale reduction (which corresponds to bringing all features uniformly proportionally closer to each other) could trigger a graph modification if that reduction results in feature distances within the nearness threshold.

One elegant way to demonstrate an automated generalization system would be to animate a shrinking map, with scale and size decreasing continuously over time. The combinatorial graph structural changes and the drawing displacement changes would take place at appropriate scale moments.

3 Issues and Challenges

Some of the very key issues in automating map generalization are:

1. Understanding the underlying principles well enough to formalize them in a mathematical model for linear feature generalization that is comprehensive, correct, and produces products which are acceptable to cartographers.
2. Addressing the "big picture" by taking feature interactions into account. Algorithms that modify features one by one will not satisfy this requirement.
3. Finding local operations that satisfy global constraints. Although it is not sufficient to modify features one at a time, it would be prohibitively inefficient for each feature modification to precipitate potential alterations in every other feature.

Here are my challenges to graph drawing experts:

1. Outline a comprehensive mathematical theory of line generalization.
2. Design and implement line simplification routines that work in unison on the collection of all line features. Prove that these routines always work.

3. Develop a global feature displacement strategy that accomplishes a balance between threshold separation and correct positioning of features.
4. After accomplishing all of the above, try to:
   (a) Design and implement an efficient and effective automated map generalization system for the line network of a digital map.
   (b) Make your system dynamic (animate it!)
   (c) Conduct experiments with *real cartographers* to evaluate your system and to upgrade your system's performance.

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