Biases and Their Effects in Inductive Logic Programming

Birgit Tausend

Fakultät Informatik, Universität Stuttgart, Breitwiesenstr. 20-22, D-70565 Stuttgart

Abstract. The shift from attribute-value based hypothesis languages to Horn clause logic as in Inductive Logic Programming (ILP) results in a very complex hypothesis space. In this paper, we study how the basic constituents of biases reduce the size of the hypothesis space in ILP.

1 Introduction

Inductive logic programming (ILP) [Mug93] aims to overcome the limitations of attribute-value based learning algorithms by using Horn logic as hypothesis language. However, a powerful representation language like Horn logic accounts for a very complex hypothesis space. This complexity may prevent systems from finding an appropriate hypothesis in a reasonable time or at all. As a consequence, there is a growing interest in appropriate biases to control the search space, in particular in language biases.

In this paper, we study the influence of basic bias constituents on the complexity of the hypothesis space. As the measure of theory costs, introduced in [PK92], only applies to a restricted subset of hypotheses languages, we first extend it to unrestricted Horn logic. Using the new measure, we show how the upper bound of theory costs varies with respect to the bias constituents.

2 Complexity of the Hypothesis Language in ILP

The inductive logic programming problem is to find a logic program H given the positive examples E^+ , the negative examples E^- and the background knowledge B [Mug93]. H is a set of definite clauses $C : l_0 \leftarrow l_1, \dots, l_n$ in a hypothesis language L_H , and has to be necessary, i.e. $B \not\vdash E^+$, sufficient, i.e. $B \wedge H \vdash E^+$, and consistent, i.e. $B \wedge H \wedge E^- \not\vdash \square$.

The total number of literals to be considered by FOIL [Qui90] for specializing a hypothesis clause with k variables can be measured by the so called theory costs $C_{TH_{FOIL}}$. These costs depend on the number of variabilizations $v(i, k)$, i.e. the choice of variables for the predicate of arity i [PK92], and they are given by:

$$C_{TH_{FOIL}} = 2 * \sum_{i=1}^{MaxA} Pred(i) * v(i, k)$$

where $k \geq 1$, $pred(i)$ is the number of predicates of arity i , $MaxA$ is the maximum arity of any predicate, and the factor 2 indicates that both positive and negative literals may occur.

An upper bound of C_{THFOIL} with respect to the maximum number of old variables *Old* and the total number of predicates *AllPred* is given by [PK92]:

$$C_{THFOIL} \leq 2 * AllPred * (Old + MaxA - 1)^{MaxA}.$$

As C_{THFOIL} applies only to hypothesis languages with function free linked clauses as in FOIL [Qui90], we define a similar measure C_{TH} for languages of unrestricted Horn clauses. We replace the variabilizations $v(i, k)$ by term distributions $t(i, k)$, i.e. the mapping of k old and i new terms to the arguments positions of a predicate. In contrast to a variabilization, a term distribution $t(i, k)$ takes into account structured terms and constants as well. An upper bound of theory costs of unrestricted Horn clauses as hypothesis language is given by

$$C_{TH} \leq 2 * AllPred * (OldTerms + NewTerms)^{MaxA},$$

where *OldTerms* is the number of terms in the hypothesis to be specialized and *NewTerms* is the number of new terms in the new literal.

3 Effects of Biases Constituents

Restricting a hypothesis language L_H means for example reducing C_{TH} by excluding negated literals, or decreasing the parameters of C_{TH} , the total number of predicates *AllPred*, the number of old and new terms, *OldTerms* and *NewTerms*, and the maximum arity *MaxA*. Another way to reduce C_{TH} is to restrict the mapping of terms to argument positions, i.e. the term distributions.

3.1 Reducing *MaxA* and *AllPred*

In ILP, the total number of predicates *AllPred* and the maximum arity *MaxA* are restricted to the predicates occurring in the background knowledge B and the examples E . Only few systems include capabilities to extend these numbers by invented predicates.

Similarly, the parameters *MaxA* and *AllPred* are only restricted in few ILP systems. For example, excluding commutative variants of a literal as in [PK92] reduces *AllPred*. Another bias constituent reducing *AllPred* is to allow only predicates of a certain type, e.g. numeric or list predicates, in the new literal, as in e.g. MOBAL [MWKE93] or the recent version of FOIL [QCJ93].

3.2 Reducing the Number of Terms

Both *OldTerms* and *NewTerms* are subject to several bias constituents in ILP, as the number of terms $Terms = OldTerms + NewTerms$ strongly influences the size of L_H . For example, in [PK92] it is shown that C_{THFOIL} increases exponentially by the number of variables.

The number of terms may be infinite in Horn logic as terms are constructed by applying functors in the signature to other terms. However, the construction of terms for a hypothesis clause of ILP is often restricted either to use only functors and constants occurring in the examples, or to avoid term construction by excluding structured terms not occurring as terms and subterms in the examples. This leads to a finite number of terms *Terms*.

A widely used restriction on terms is to use function-free Horn logic, e.g. in FOIL [Qui90], CLINT [DR91] or MOBAL [MWKE93]. This restriction strongly reduces CTH as it excludes literals including n -place functors where $n \geq 1$. The number of terms is $Terms = (OldVC + NewVC)$ in a function-free language, and $OldVC$ is the number of old variables and constants and $NewVC$ the number of new variables and constants. Similar results can be achieved by using constant-free Horn logic.

Additional restrictions of function free languages mainly reduce the number of new variables in a literal not occurring in the preceding literals in the clause.

The number of new variables can be limited to a fixed number i as in some languages series of CLINT [DR91], i.e. $Terms = OldVC + i$. Other language series vary this number with respect to the depth of the new literal. Then, $Terms = OldVC + i(d)$, and $i(d)$ is the number of new variables at depth i . The depth is defined by the maximum length of a connection path established by shared variables.

Definition 1. Let $C : l_0 \leftarrow l_1, \dots, l_n$ be a clause. The depth $depth(X)$ of a variable X is defined by $depth(X) = \min\{length(cp(X)) | cp(X) \text{ is a connection path of } X\}$, and $cp(X)$ is a connection path as in definition 3. The depth of a clause C (a literal l_i) is the maximum depth of its variables.

The restriction to constrained clauses sets the number of new variables to 0, i.e. $Terms = OldVC$, and the body literals only include variables of the head.

Another basic constituent reducing $Terms$ is to exclude unlinked clauses, as in ITOU [Rou92], CLINT [DR91], GOLEM [MF90], SIERES [WO91].

Definition 2. A clause $C : l_0 \leftarrow l_1, \dots, l_n$ is *linked* if all of its literals are linked. A literal l_i is linked if it $\begin{cases} \text{is the head literal of } C, \text{ i.e. } i = 0 \\ \text{contains a variable } X, X \in vars(l_j) \text{ and } l_j \text{ is linked.} \end{cases}$

As excluding such literals does not change the success set of a clause¹, they can be omitted. The linkedness restriction is very weak as it reduces the number of terms only by one, i.e. $Terms = (OldVC + NewVC - 1)$.

3.3 Restricting the Mappings of Terms to Positions

The maximum number $MaxTD$ of term distributions $t(i, k)$ given by $MaxTD = Terms^{MaxA}$ can be reduced by limiting the mapping of terms to argument positions without restricting the maximum number of terms $Terms$.

A basic bias constituent applied in ILP for this purpose are argument types, as in INDICO [STW93]. As a term may not occur at an argument position with conflicting type, $MaxTD = (Terms(Type_1) * \dots * Terms(Type_{MaxA}))$ where $Terms(Type_i)$ is the total number of old and new terms of argument type $Type_i$. A bias constituent of a similar effect is given by mode declarations defining the arguments of a literal in terms of input and output arguments.

Another way to restrict $t(i, k)$ is to require unique variables, as in FOCL [PK92] or some languages of CLINT [DR91]. The number of terms $Terms(i)$ at an argument position i is reduced by the number of all terms including variables that already occur at other argument positions.

¹ except in the failure case

The range restriction, e.g. in CLINT [DR91] or GOLEM [MF90], also reduces *MaxTD* as all head variables must occur in the body literals.

A boundary case of restricting the mapping is given by the rule models [MWKE93] as the term distribution is fixed. Consequently, the number of term distributions is limited to the set of instantiable literals in all partially instantiated rule models.

Graph-based approaches, as described in e.g. [WO91], weaken this restriction by specifying particular connection paths by their edges.

Definition 3. A connection path of a variable X in a clause $C : l_0 \leftarrow l_1, \dots, l_n$ is $cp(X) = \begin{cases} \emptyset & \text{if } i = 0 \\ cp(Y) \circ l_i & \text{if } \exists Y \in vars(l_i), Y \neq X, \text{ and } cp(Y) \text{ does not include } l_i. \end{cases}$

This restriction limits the mapping of terms to argument positions to mappings that do not conflict with specified the connection paths.

4 Conclusions

The basic constituents of biases in ILP mainly reduce the number of terms and term distributions. Using the measure of theory costs, upper bounds for these constituents can be defined. The restriction to function free Horn clauses is particularly useful as it limits the terms to a finite set. Controlling the complexity of languages that are not function free by extending constituents applicable to function free languages or by defining new ones should be subject to further research.

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