Using Heuristics to Speed up Induction on Continuous-Valued Attributes

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Abstract. Induction of decision trees in domains with continuous-valued attributes is computationally expensive due to the evaluation of every possible test on these attributes. As the number of tests to be considered grows linearly with the number of examples, this poses a problem for induction on large databases. Two variants of a heuristic, based on the possible difference of the entropy-minimization selection-criterion between two tests, are proposed and compared to a previously known heuristic. Empirical results with real-world data confirm that the heuristics can reduce the computational effort significantly without any change in the induced decision trees.

1 Introduction

Learning from examples has been recognized as a promising way of acquiring knowledge for classification purposes. Especially top-down induction of decision trees (TDIDT) has been used successfully in a number of applications (see e. g. [2], [3], [5]).

Early TDIDT-methods are restricted to nominal attributes. But in many applications, e. g. in process control, which include sensory data, continuous-valued attributes are predominant and should be treated adequately. A straightforward extension, as briefly mentioned in [4], leads to good results, but has the disadvantage of a rather high computational effort, which increases with the number of examples. On large data bases the runtime of the induction can therefore become unacceptable. This aspect has not found much attention until Fayyad and Irani [1] recently proposed a heuristic to reduce the average complexity while preserving the induced trees.

This paper describes two versions of a supplementary heuristic and argues that a combination with the one developed by Fayyad and Irani can reduce the computational effort significantly. The characteristics of the heuristics are discussed and their payoff is tested on different sets of real-world data.

Sect. 2 briefly describes the *ID3* algorithm and the results of Fayyad and Irani. In Sect. 3 another approach is motivated and two variants of a new heuristic are given. The results of empirical comparisons are presented in Sect. 4. Finally, conclusions can be found in Sect. 5.

2 Induction on Continuous-Valued Attributes

Induction of decision trees typically expects the examples to be represented as a set of attribute-value pairs with respect to a common set of attributes with each example belonging to one of a disjoint set of classes. In general, the algorithms select a test on one or more of the attributes to partition the set of examples into subsets, which are further divided recursively until a termination criterion is met. The generated tree consists of the selected tests at the inner nodes with subtrees for each possible outcome. The leaves are marked with the predominant class of the corresponding subset.

The well-known *ID3* [3] selects the test which minimizes the average entropy $E(T)$ in the resulting subsets. Let $C_1, \dots, C_k$ be the classes occurring in the example set, $P(C_i)$ the corresponding probabilities, T a test on the examples which partitions the set into subsets $T_1, \dots, T_m$, $P(T_j)$ the probability of an example to fall into the subset T_j and let $P(C_i | T_j)$ denote the conditional probability of an example out of T_j belonging to class C_i . The remaining average entropy $E(T)$ after the evaluation of a test T is then given by:

$$E(T) = - \sum_{j=1}^m \sum_{i=1}^k P(T_j) P(C_i | T_j) \log_2(P(C_i | T_j)) \quad (1)$$

For continuous-valued attributes tests of the form $A(x) < V$, with $A(x)$ denoting the attribute value of an example x , are generated by varying the threshold value V . This can be done efficiently by ordering the examples according to their attribute values and representing the tests as a splitting of this list (see Fig. 1). This admits an incremental computation of the probabilities needed in (1).

Fig. 1. Tests splitting the ordered list of examples

The number of possible tests $A(x) < V$ is linear in the number of examples¹, whereas only one test needs to be evaluated for a nominal attribute. Therefore the runtime of TDIDT- algorithms grows significantly in the presence of continuous-valued attributes, so that induction on large data bases becomes a time-consuming effort.

Not every possible test needs to be evaluated, however. In the ordered list of examples (see Fig. 1) two adjacent examples e_r and e_{r+1} often belong to

¹ If different examples have the same attribute value, the number of tests is, of course, sublinear.

the same class. Intuitively, it does not seem to make sense to divide the set of examples between them. Fayyad and Irani prove that in terms of the entropy-minimization selection criterion used in *ID3* either the division before example e_r or after example e_{r+1} is indeed superior to the one between them. They introduce the term *boundary point* for those thresholds whose constituting examples e_r and e_{r+1} belong to different classes.

As tests between boundary points are always inferior to at least one of their neighbouring tests, the following heuristic can be used:

Heuristic 1. *In the test selection process only those tests need to be evaluated which fall on boundary points.*

The achievable reduction in the number of test evaluations depends heavily on the data. Given n examples out of k different classes, a perfectly predicting attribute yields only $k - 1$ boundary points, whereas a totally uncorrelated attribute can be expected to result in $(n-1) \frac{k-1}{k}$ boundary points. This indicates that Heuristic 1 will work particularly well on data sets which contain few classes.

3 Heuristics Based on the Maximum Difference

In order to select a test at a node during the induction, the algorithm has to generate and evaluate the possible tests on all available attributes. The test yielding the minimum remaining entropy (1) is selected. If there are many examples and a continuous-valued attribute is considered, two successive tests T_r and T_{r+1} (see Fig. 1) have a very small difference in their rating by the selection criterion. With an estimation of this difference the evaluation of test T_{r+1} can safely be omitted if test T_r is sufficiently inferior to a previously found one.

Hence an upper bound is needed for the difference between $E(T_{r+1})$ and $E(T_r)$ with the two tests differing only in the example e_{r+1} . Such a bound is given in Theorem 1.

Theorem 1. *Given n examples $e_1, \dots, e_r, e_{r+1}, \dots, e_n$ and two successive tests T_r and T_{r+1} which split the examples after e_r and e_{r+1} (resp.) the following holds:*

$$E(T_r) - E(T_{r+1}) \leq \frac{2 \log_2(n+1) - \log_2 r}{n} = \text{max-diff}(T_r) \quad (2)$$

Let now $E_{\min}$ be the minimum remaining entropy reached so far by a test during the test selection. If a test T_r results in an entropy $E(T_r)$ such that $E(T_r) - \text{max-diff}(T_r) > E_{\min}$, the following test T_{r+1} does not have to be considered. Furthermore, as the function $\text{max-diff}(T_r)$ decreases strictly monotonously in r , even more tests can eventually be eliminated. This leads to:

Heuristic 2. *If the minimal remaining entropy reached so far is given by $E_{\min}$, the next test to be evaluated after test T_r is $T_{r+\Delta}$ with:*

$$\Delta = \max \left(1, \left\lceil \frac{E(T_r) - E_{\min}}{\text{max-diff}(T_r)} \right\rceil \right) \quad (3)$$

On close examination the heuristic shows the following properties:

- As Δ depends on E_{min} , the number of tests to be omitted is influenced by the order in which the attributes are processed. To achieve high savings the most relevant attribute should be chosen first.
- Since $\max\text{-diff}(T_r)$ is monotonously decreasing in r , the Δ -values are higher in the second half of the list.
- In general, Heuristic 2 will be more effective on irrelevant attributes than on relevant ones, provided a relevant attribute has been processed previously.

The following extension takes advantage of some of these properties. As higher benefits are obtained in the second half of the lists, it can be useful to start with a test in the middle of the list of examples and to proceed successively to the borders, using Heuristic 2.

Now the first test to be evaluated for an attribute is the one which splits the set of examples in the middle. These tests will have different entropy measures depending on the class distribution in the two subsets. Their entropy measures can be used to estimate the relevance of the attributes. By processing those attributes first that yield low entropy measures, an optimal order of the attributes can be approximated. This method is stated in the following heuristic:

Heuristic 3. *For each attribute A_i the examples are partitioned at the median into two roughly equally large sets by a test T_{m_i} . The examples in the set with attribute values below or above the median are ordered by decreasing or increasing attribute values resp. These lists are ordered by increasing values of $E(T_{m_i})$ and processed according to Heuristic 2.*

4 An Empirical Comparison

As the efficiency of the heuristics discussed depends heavily on the characteristics of the data, an evaluation using real-world data is needed. In these experiments the heuristics are used on their own as well as in combinations.

The data sets used are listed in Table 1 together with a brief description. Except for the solar data all of them are publicly available; as far as training and test sets are supplied separately, they were joined for the following experiments. Examples with values missing were removed.

As the heuristics aim for a reduction in the number of tests to be evaluated by the entropy-minimization selection-criterion, the number of tests actually considered during the induction gives a first metric for comparison. It is a machine- and implementation-independent efficiency measure of the heuristics, which ignores the overhead associated with them. The results for different data sets are given in Fig. 2.

Some observations should be discussed:

- The heuristics based on the max-diff bound perform poorly on the medical domains *cleveland*, *cleveland2* and *pima*. These domains contain many examples with identical attribute values, thus reducing the number of possible tests that are to be omitted despite high Δ -values.

Table 1. Brief description of the data sets

Domain	Name	Examples	Attributes	Classes
Heart-diseases	cleveland	297	13	2
Same as above, but 5 classes	cleveland2	297	13	5
Glass types	glass	214	9	6
Diabetes	pima	768	8	2
Semiconductor films	solar	540	10	4
Sonar signals	sonar	208	60	2
Vowel recognition	vowel	990	10	11

- Heuristic 1 is more efficient on data sets with few classes. It yields exceptionally high reductions on the sonar data and performs better on the cleveland data with 2 classes than on the corresponding set cleveland2 with 5 classes.
- Only on the vowel data the more sophisticated Heuristic 3 performs considerably better than Heuristic 2, where obviously a more suitable order of attributes is found.
- The combinations of two heuristics consistently achieve good results. They reduce the number of test evaluations by at least 43 % on the cleveland2 data up to 83% on the vowel data.

Reductions in the number of test evaluations are achieved at the expense of the computational overhead associated with each heuristic. From a practical point of view the only interesting metric is reduction in total runtime. As expected, no substantial reductions are attained in the medical domains cleveland², cleveland2 and pima. However, in most cases the overhead of the heuristics is compensated by reductions in the number of test evaluations; only the large overhead of Heuristic 3 did not pay off.

Especially the combinations work reliably on the other data sets. The combination of Heuristic 1 with Heuristic 2 results in a speed-up between 20% (sonar data) and 30% (vowel data); the combination with Heuristic 3 saves between 10% (glass data) and 39% (vowel data).

5 Conclusion

When decision trees are induced by means of the entropy-minimization selection-criterion, the number of tests to be evaluated can be reduced by using simple heuristics. This result has been confirmed by empirical experiments. As the heuristics exploit different characteristics of the data, a combination of them will work reliably for a wide variety of applications.

The heuristics apply first of all to the entropy-minimization selection-criterion. In [1] Fayyad and Irani mention that their heuristic (Heuristic 1 in this paper) can be used with other selection measures as well. It should be possible to derive

² In [1] Fayyad and Irani report a runtime reduction of more than 15% for their heuristic on this data set.

Fig. 2. Number of tests evaluated

bounds – similar to the one found for the entropy measure – for other selection criteria, so that Heuristics 2 and 3 could be applied to them as well.

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