Avoiding state explosion by composition of minimal covering graphs¹

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Abstract

In this paper, we study composition and decomposition of Petri nets via a common set of places or transitions. We are interested in automatical verification of Petri nets properties from the point of view of reusability of partial results already obtained. We give two algorithms which allow to compute the minimal covering graph of a Petri net by composing the minimal covering graphs of each of its modules.

1 Introduction

Composition/Decomposition of Petri nets is a key problem, as well as concerns specification as verification of properties. This problem was already tackled in [Bou90], [Hac76], [Sou90], ..., but generally sub-classes of Petri nets were studied (state machine decomposable nets, bounded nets). When a system is specified with general Petri nets, one does not know, a priori, whether the net is bounded or not. Moreover, imposing to the places to be bounded may introduce unwanted deadlocks and change the demanded service.

We studied in [FP91] the properties preserved by composition/decomposition mechanisms. The techniques considered consist in composing two nets by fusion of common transitions or fusion of common places. They are applied to Petri nets in general, i.e. without any restriction. However some properties are not shared by a net and its subnets : two subnets can have the same property P, and their composed net can eventually not satisfy P. The main analysis methods for Petri nets are the calculus of invariants or of the covering graph. The first method does not allow to automatically deduce properties of the Petri net. But we know that the covering graph of a net allows to verify most properties. However, in some cases, the computation of the covering graph is too long. Our approach

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consists in using the minimal covering graph (which is the smallest covering graph) and to compute it not with the usual global and centralised method, but with a local and distributed method using a given decomposition of the Petri net.

So, in this paper, we study composition of minimal covering graphs of Petri nets composed via a common set of places or a common set of transitions. More precisely, we are interested in the construction of the minimal covering graph of a net using the minimal covering graphs of its subnets. Let us notice that a first study from Bourguet ([Bou90]) dealt with composition of reachability graphs for bounded Petri nets. In the case of unbounded Petri nets, covering graphs are needed. As the minimal covering graph of a net is both unique and as small as possible, composition of minimal covering graphs can easily be performed.

Basic notions about Petri nets can be found in [Rei85]. We first recall some notions about the minimal covering graph of a net. Then, we give two new algorithms which compute the minimal covering graph of a net from those of its subnets. The efficiency of these algorithms, as concerns time, can be increased by parallelising the calculus of the minimal covering graphs of each of the subnets.

2 The minimal covering graph of a Petri net

The classical Karp-Miller covering graph ([KM69]) is sometimes too large to be computed. The minimal covering graph ([Fin90]) is the smallest covering graph which allows to verify the same properties as the usual covering graph.

Definition 2.1([Fin90])

A *coverability set* $CS(PN)$ of a Petri net $PN = \langle P, T, V, M_0 \rangle$ is a subset of $(N \cup \{\omega\})^P$ such that the two following conditions hold :

- (i) for every reachable marking $m \in RS(PN)$, there is a marking $m' \in CS(PN)$ such that $m \leq m'$,
- (ii) for every marking $m' \in CS(PN) - RS(PN)$, there is an infinite strictly increasing sequence of reachable markings $\{m_n\}$ converging to m' .

A coverability set $CS(PN)$ is *minimal* iff no proper subset of $CS(PN)$ is a coverability set of PN .

Lemma 2.2([Fin90])

The minimal coverability set is finite and unique.

Definition 2.3([Fin90])

A *coverability graph* of a Petri net $PN = \langle P, T, V, M_0 \rangle$ is a labelled directed graph $\langle N, L, A \rangle$ where the set of nodes N is a coverability set of PN , $L = T$ and there is an arc $(m, t, m') \in A$ iff $m(t) > m'$. The *minimal coverability graph* of a Petri net PN is the coverability graph such that its set of nodes is the minimal coverability set.

The algorithm to construct the minimal covering graph of a net can be found in [Fin90]. However, the algorithms we will present in the next sections are very similar, as they are derived versions.

Theorem 2.4 ([Fin90])

Let PN be a Petri net and MCG(PN) its minimal covering graph.

- (i) The reachability tree $RT(PN)$ is infinite iff there is at least one circuit in $MCG(PN)$.
- (ii) The reachability set $RS(PN)$ is infinite iff there is at least one symbol ω in $MCG(PN)$.
- (iii) A place p is not bounded iff there is at least one marking $m \in MCG(PN)$ such that $m(p) = \omega$.
- (iv) A transition t is quasi-live iff there is at least one marking $m \in MCG(PN)$ such that for every place p , $m(p) \geq \text{Pre}(p, t)$.

Proposition 2.5 ([Fin90])

The minimal covering graph is computable, finite and unique.

We are interested in composing nets either by fusion of common transitions or by fusion of common places. The definitions and properties of these compositions were introduced in [FP91].

We first recall the composition by fusion of transitions.

We consider two nets with distinct places and two kinds of transitions : proper ones, and transitions common to both nets. So, we study two nets N_1 and N_2 such that :

where T_1 (T_2) is the set of transitions proper to N_1 (N_2), T_c is the set of transitions common to both nets, and P_1 (P_2) is the set of places of N_1 (N_2).

We obtain, by fusion of transitions common to N_1 and N_2 , the following net N_0 :

Composition by fusion of places is defined in a similar way. We consider two nets with distinct transitions and two kinds of places : proper ones, and places common to both nets. So, we study two nets N_1 and N_2 such that :

where P_1 (P_2) is the set of places proper to N_1 (N_2), P_c is the set of places common to both nets, and T_1 (T_2) is the set of transitions of N_1 (N_2).

We obtain, by fusion of places common to N_1 and N_2 , the following net N_0 :

Notation : every marking M of N_0 can be decomposed into $M=(M_1, M_c, M_2)$ where M_1 is the marking of places of P_1 , M_2 is the marking of places of P_2 and M_c is the marking of places of P_c .

3 Fusion of transitions

We now build the minimal covering graph of a net N_0 obtained by composing two subnets N_1 and N_2 by fusion of common transitions. A marking M of net N_0 can be divided into two parts : a marking M_1 corresponding to places of P_1 and a marking M_2 corresponding to places of P_2 .

The construction of MCG0, minimal covering graph of net N_0 , from MCG1 and MCG2, minimal covering graphs of nets N_1 and N_2 , is done using the following algorithm :

```

MCG1 := minimal covering tree of  $N_1$ ;
MCG2 := minimal covering tree of  $N_2$ ;
/* construction of MCG0 := minimal covering graph of  $N_0$  */
unprocessed_nodes := {create_node(r, M0)}; /*  $M_0=(M_1, M_2)$  is the marking of root r */
processed_nodes :=  $\emptyset$ ;
while unprocessed_nodes  $\neq \emptyset$ ;
  select some node  $n \in$  unprocessed_nodes;
  unprocessed_nodes := unprocessed_nodes - {n}; /* m is the marking of n and m' the one of n' */
  case n : [1..4] of
    1: there exists a node  $n' \in$  processed_nodes such that  $m = m'$  :
      processed_nodes := processed_nodes + {n};
      exit;
    2: there exists a node  $n' \in$  processed_nodes such that  $m < m'$  :
      remove_node(n; MCT);
      exit;
    3: there exists a node  $n' \in$  processed_nodes such that  $m > m'$  :
       $m_2 := m$ ;
      ancestor := false;
      for all ancestors  $n_1$  of n such that  $m_1 < m_2$  do
        for all places p such that  $m_1(p) < m(p)$  do
           $m_2(p) := \omega$ ;
        endfor
      endfor
      if there exists an ancestor  $n_1$  of n such that  $m_1 < m_2$ 
        then ancestor := true;
           $n_1$  := first processed node, on the path from the root to n such that  $m_1 < m_2$ ;
           $m_1 := m_2$ ;
          remove_tree( $n_1$ ; MCT);
          remove from (processednodes + unprocessednodes) all nodes of tree( $n_1$ ; MCT);
          unprocessednodes := unprocessednodes + { $n_1$ };
        endif
      for every  $n_1 \in$  processednodes such that  $m_1 < m_2$  do

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    remove from (processednodes + unprocessednodes) all nodes of tree(n1; MCT);
    remove_tree(n1; MCT);
    remove_node(n1; MCT);
  endfor
  if ancestor = false
    then unprocessednodes := unprocessednodes + {n};
  endif
  exit;
4: otherwise : /* compute sons of node n with marking m=(m1,m2) */
  nc1 := the node of MCG1 with marking mc1, comparable with m1;
  nc2 := the node of MCG2 with marking mc2, comparable with m2;
  for all transitions t ∈ T1 s.t. there exists an arc mc1 [t> mc1' in MCG1 and m1 [t> m1' do
    create_node+arc((n',t,(m1',m2))); MCT);
    unprocessed_nodes := unprocessed_nodes + {n'};
    processed_nodes := processed_nodes + {n};
  endfor
  for all transitions t ∈ T2 s.t. there exists an arc mc2 [t> mc2' in MCG2 and m2 [t> m2' do
    create_node+arc((n',t,(m1',m2))); MCT);
    unprocessed_nodes := unprocessed_nodes + {n'};
    processed_nodes := processed_nodes + {n};
  endfor
  for all transitions t ∈ Tc s.t. there exists an arc mc1 [t> mc1' in MCG1
    and an arc mc2 [t> mc2' in MCG2 and m1 [t> m1' and m2 [t> m2' do
    create_node+arc((n',t,(m1',m2))); MCT);
    unprocessed_nodes := unprocessed_nodes + {n'};
    processed_nodes := processed_nodes + {n};
  endfor
  exit;
endcase
endwhile
unprocessednodes := maximal(unprocessednodes);
MCS := {label(n) ; n ∈ processednodes };
identify_nodes_having_same_label(MCT; MCG);
for every arc (m,t,m') of MCG do
  if not ( m[t>m' )
    then remove_arc((m,t,m'); MCG);
  endif
endfor

```

Informal explanation :

We start with the initial marking $M_0=(M_{01},M_{02})$ of N_0 . There exists one and only one marking M_1 comparable with M_{01} (related by the $<$ relation) in MCG1 (property of the minimal covering graph). Likewise, there exists a unique marking M_2 comparable with M_{02} in MCG2. If there exists an arc $M_1[t>...$ in MCG1, with $t \in T_1$, then t may be firable in N_0 and, if it is the case, a similar arc is constructed in MCG0. If there exists an arc $M_2[t>...$ in MCG2, with $t \in T_2$, then t may be firable in N_0 and, if it is the case, a similar arc is constructed in MCG0. If there exists an arc $M_1[t>...$ in MCG1 and an arc $M_2[t>...$ in MCG2, with $t \in T_c$, then t may be firable in N_0 , and if it is the case, the arc is constructed.

This is not only valid for the initial marking, but also for any reachable marking of N_0 . To build MCG0, we already know which transitions are firable, and also part of the marking - the one

corresponding to the set of places not concerned by the firing of the transition, i.e. places of P_2 for a transition of T_1 and places of P_1 for a transition of T_2 .

Proposition 3.1

The algorithm terminates.

5 Fusion of places

As for composition by fusion of transitions, we are going to study composition of minimal covering graphs when using composition of nets by fusion of common places.

A marking M of N_0 can be decomposed into three parts : a marking M_1 corresponding to places of P_1 , a marking M_c corresponding to places of P_c and a marking M_2 corresponding to places of P_2 .

We propose the following algorithm to compute the minimal covering graph of the entire net from those of the subnets :

```

finished := false;
∀i, kci := M0(pci); /* initial number of tokens in common place pci */
while not finished do
  MCG1 = minimal covering tree of N1, with M01(pci)=kci;
  MCG2 = minimal covering tree of N2, with M02(pci)=kci;
  ∀i, kci1 := maxMCG1M(pci); /* the higher number of tokens in pci in MCG1 */
  ∀i, kci2 := maxMCG2M(pci); /* the higher number of tokens in pci in MCG2 */
  if ∀i, kci1 = kci2
    then finished := true; /* the bounds of places pci are the same in both nets */
  endif
  ∀i, kci := max(kci1, kci2);
  if there exists a sequence s and i, j, i ≠ j, s.t.:
    M[s > M' M(pci) > M'(pci) and M(pcj) < M'(pcj) in MCG1 or MCG2
    then finished := true; /* if Pc is a singleton, this is not applied */
  endif
endwhile
/* construction of MCG0 := minimal covering graph of N0 */
unprocessed_nodes := { create_node(r, M0) }; /* M0=(M1, M2) is the marking of root r */
processed_nodes := ∅;
while unprocessed_nodes ≠ ∅;
  select some node n ∈ unprocessed_nodes;
  unprocessed_nodes := unprocessed_nodes - { n }; /* m is the marking of n and m' the one of n' */
  case n : [1..4] of
    1: there exists a node n' ∈ processed_nodes such that m = m' :
      processed_nodes := processed_nodes + { n };
      exit;
    2: there exists a node n' ∈ processed_nodes such that m < m' :
      remove_node(n; MCT);
      exit;

```

```

3: there exists a node  $n' \in \text{processed\_nodes}$  such that  $m > m'$  :
   $m_2 := m$ ;
  ancestor := false;
  for all ancestors  $n_1$  of  $n$  such that  $m_1 < m$  do
    for all places  $p$  such that  $m_1(p) < m(p)$  do
       $m_2(p) := \omega$ ;
    endfor
  endfor
  if there exists an ancestor  $n_1$  of  $n$  such that  $m_1 < m_2$ 
  then ancestor := true;
     $n_1 :=$  first processed node, on the path from the root to  $n$  such that  $m_1 < m_2$ ;
     $m_1 := m_2$ ;
    remove_tree( $n_1$ ; MCT);
    remove from (processednodes + unprocessednodes) all nodes of tree( $n_1$ ; MCT);
    unprocessednodes := unprocessednodes + { $n_1$ };
  endif
  for every  $n_1 \in \text{processednodes}$  such that  $m_1 < m_2$  do
    remove from (processednodes + unprocessednodes) all nodes of tree( $n_1$ ; MCT);
    remove_tree( $n_1$ ; MCT);
    remove_node( $n_1$ ; MCT);
  endfor
  if ancestor = false
  then unprocessednodes := unprocessednodes + { $n$ };
  endif
  exit;

4: otherwise : /* compute sons of node  $n$  with marking  $m=(m_1, m_c, m_2)$  */
 $nc_1 :=$  node in one of the MCG1 computed, of marking comparable with  $(m_1, m_c)$ ;
 $nc_2 :=$  node in one of the MCG2 computed, of marking comparable with  $(m_2, m_c)$ ;
/* in both cases, value 0 is returned if the node could not be found */
if  $nc_1 = 0$ 
then for all  $t \in T_1$  such that  $(m_1, m_c) [t > (m_1', m_c')] do
  create_node+arc(( $n', t, (m_1', m_c', m_2)$ ); MCT);
  unprocessed_nodes := unprocessed_nodes + { $n'$ };
  processed_nodes := processed_nodes + { $n$ };
endfor
else for all  $t \in T_1$  s.t. there exists an arc  $(mc_1, mc_c) [t > (mc_1', mc_c')]$ 
  in one of the MCG1 computed and  $(m_1, m_c) [t > (m_1', m_c')] do
  create_node+arc(( $n', t, (m_1', m_c', m_2)$ ); MCT);
  unprocessed_nodes := unprocessed_nodes + { $n'$ };
  processed_nodes := processed_nodes + { $n$ };
endfor
endif
if  $nc_2 = 0$  /* both "if" are similar, but they are duplicated for easier reading */
then for all  $t \in T_2$  such that  $(m_2, m_c) [t > (m_2', m_c')] do
  create_node+arc(( $n', t, (m_1, m_c', m_2)$ ); MCT);
  unprocessed_nodes := unprocessed_nodes + { $n'$ };
  processed_nodes := processed_nodes + { $n$ };
endfor
else for all  $t \in T_2$  such that there exists an arc  $(mc_2, mc_c) [t > (mc_2', mc_c')]$ 
  in one of the MCG2 computed and  $(m_2, m_c) [t > (m_2', m_c')] do
  create_node+arc(( $n', t, (m_1, m_c', m_2)$ ); MCT);
  unprocessed_nodes := unprocessed_nodes + { $n'$ };
  processed_nodes := processed_nodes + { $n$ };
endfor
endif
exit;$$$$ 
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| endcase
endwhile
unprocessednodes := maximal(unprocessednodes);
MCS := { label(n) ; n ∈ processednodes };
identify_nodes_having_same_label(MCT; MCG);
for every arc (m,t,m') of MCG do
| if not ( m[t>m' )
| | then remove_arc((m,t,m'); MCG);
| endif
endfor

```

Informal explanation :

In the first loop, we compute the minimal covering graphs of each module. One of the subnets can add tokens to a common place. In this case the number of tokens in this place will be greater than the initial marking. So, these tokens can be used by the other subnet. Thus, we have to take this information into account.

The union of minimal covering graphs is defined as follows. We start with the initial state M_0 of net N_0 . It can be decomposed into three parts : $M_0=(M_1,M_c,M_2)$, where M_1 (resp. M_2) is the marking of places of P_1 (resp. P_2) and M_c is the marking of places of P_c , common to both nets. Let us suppose that in one of the MCG1 (resp. MCG2) computed, there exists a marking (M_1',M_c') (resp. (M_2',M_c')) comparable with (M_1,M_c) (resp. with (M_2,M_c)). Let t be a transition of T_1 such that $(M_1',M_c')[t>$ in MCG1 (for MCG2, the operation is similar). If t is firable in the global net, then, we will obtain in MCG0 an arc $(M_1,M_c,M_2)[t>(M_1'',M_c'',M_2)$. We apply the same operation for the markings obtained.

When processing a marking M , if a marking M' comparable with marking M , was found in one of the MCG1 computed, then only transitions of T_1 firable from M are transitions firable from M' . It is the same for transitions of MCG2.

Proposition 5.1

The algorithm terminates.

Proof

The problem of the algorithm termination is not as simple as for the algorithm concerning composition by fusion of transitions. So we are going to detail the proof. For that purpose, proving that the first loop of the algorithm terminates is enough.

Let s be a generic term used to denote a firing sequence such that $M[s>M'$. Two cases may be encountered :

- $(\forall s \in T_1^*) ((\forall p \in P_c M'(p) \leq M(p)) \text{ or } (\forall p \in P_c M(p) \geq M'(p)))$.

Let us consider the first subcase : $\forall p \in P_c M'(p) \leq M(p)$. The bound of places is preserved. So, the algorithm terminates.

Let us now consider the second subcase : $\forall p \in P_c M'(p) \geq M(p)$. The nodes are uncomparable, by definition of the minimal covering graph. So, there exists a place p' , not in P_c , such that $M(p') > M'(p')$, i.e. the marking of p' decreases when firing s . So, the algorithm stops.

• $\exists s \in T_i^* \text{ tq } ((\exists p \in P_c \text{ tq } M'(p) > M(p)) \text{ and } (\exists p' \in P_c \text{ tq } M'(p') < M(p)))$: in this case, we force the algorithm termination by setting variable finished to true.

6 Example

Let us consider a railway with a two-way section. Trains circulate from East to West and from West to East. The driving rules on the two-way part of the railway are the following :

- there cannot be two trains going in opposite ways (otherwise, there would be collisions);
- there can be one or several trains going in the same direction;
- there are at most two trains on the section (for security reasons).

We model this problem with the following Petri net.

Trains going from East to West (from West to East) are generated by transition t1 (t'1) and are then in their departure place, DE (DW). The first train to go on the critical section fires transition t2 (t'2), while the following one would fire transition t3 (t'3). Place PEW (PWE) contains the trains going from East to West (from West to East) on the two-way part of the railway. Place Max contains the maximal number of trains which can still be allowed on this part. When a train wants to exit this system, it fires transition t4 (t'4).

We are going to calculate the minimal covering graph of this net. We use, in order to compare the different methods, two time criteria : the tests of transitions (is a transition firable?) and the calculus of the marking of a place (what is the new marking of a place p?).

First, we do the direct construction of the graph. The cost of this operation is 56 tt (tests of transitions) and 76 pm (calculus of places markings).

Secondly, we decompose the net into two nets communicating via place Max : the first net NP1 is composed of places DE, PEW, Max and transitions t1, t2, t3, t4; the second net NP2 is composed of places DW, PWE, Max and transitions t'1, t'2, t'3, t'4. The cost of the whole operation is 38 tt and 37 mp, when the covering graphs of the components are computed sequentially. When computed in parallel, the cost becomes 22 tt and 20 mp.

Thirdly, we decompose the net into three nets communicating via transitions : the first net NT1 is composed of place DE and transitions t1, t2, t3; the second net NT2 is composed of places PEW, PWE, Max and transitions t2, t3, t'2, t'3; the third net NT3 is composed of place DW and transitions t'1, t'2, t'3. The cost of the whole operation is 50 tt and 33 mp, when the covering graphs of the components are computed sequentially. When computed in parallel, the cost is 38 tt and 31 mp.

7 Conclusion

In this paper, we studied the composition of Petri nets by fusion of a set of common transitions or a set of common places. The goal of this work was to make verification of properties easier by reusing already computed results.

We presented two new algorithms to compute the minimal covering graph of a Petri net, knowing the minimal covering graphs of its modules.

The complexity (in terms of tests of firability for transitions) mainly depends on the chosen decomposition. If the decomposition of the net is badly chosen, the calculus by composition can be worse than a direct computation. The problem is thus to find out an optimal decomposition for a more efficient covering graph computation. In general, it would certainly be possible to obtain a decomposition such that the parallel computation of the n modules covering graphs would have almost the complexity of the direct computation divided by n . This problem was exemplified in the previous section.

From a practical point of view, these algorithms will be implemented within the Petri nets environment PAPETRI ([BJP90]).

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Minimal covering graph of net N:

Minimal covering graph of net NP1:

(NP2 with t' instead of t and markings (DW, PWE, Max))

(DE, PEW, Max)

cost:
transitions: 16
places: 17

Composition of minimal covering graphs of NP1 and NP2:

Minimal covering graph of net NT1: (NT3 with t' instead of t and markings (DW))

Minimal covering graph of net NT2:

Composition of minimal covering graphs of NT1 and NT2:

Composition with the minimal covering graph of NT3: