NARROWING IN PARTIAL CONDITONAL EQUATIONAL THEORIES

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(Abstract)

We extend the "Conditional Narrowing Algorithm" of H.Hussmann (MIF-8502, Universität Passau 1985) to partial conditional equational theories. Without loss of generality it is assumed that the partial conditional equational theory IMP be canonical, i.e. for every conditional existence equation $(X : G \rightarrow t \stackrel{\circ}{=} t') \in \text{IMP}$ holds $\text{Var}(G)=X$, and that for every term $r \in \text{Term}(t, t') \setminus \text{Term}(G)$ there exists a conditional existence equation $(Y : H \rightarrow p \stackrel{\circ}{=} p') \in \text{IMP}$ and a substitution σ with $H\sigma = G$ and $p\sigma = r$.

Using canonical theories we can drop the explicit declaration of the sets X,Y of variables.

A set K of "ground objects" and a set F of "ground facts", i.e. a set F of existence equations on K, define a model $\mathbb{F}(\text{IMP}, F, K)$ of IMP freely generated by K and F. Given a set H of existence equations, the narrowing algorithm computes solutions of H in $\mathbb{F}(\text{IMP}, F, K)$.

We divide IMP into two parts:

$\text{IMP}_{\text{equ}} = \{(G \rightarrow r \stackrel{\circ}{=} r') \in \text{IMP} / r \neq r'\}$ and $\text{IMP}_{\text{def}} = \{(G \rightarrow t \stackrel{\circ}{=} t') \in \text{IMP}\}$.

In the partial case we have to replace the final unification step by an iterative resolution step using the conditional existence equations of IMP_{def} .

Partial conditional Narrowing Algorithm (PCNA)

begin: $H \rightarrow H$

narrowing: $R \rightarrow H\tau$
 $(G, R[u \leftarrow r'])\sigma \rightarrow H\tau\sigma$

where $(G \rightarrow r \stackrel{\circ}{=} r') \in \text{IMP}_{\text{equ}}$, $u \in O(R)$, R/u is not a variable,
 and σ is a most general ϕ -unifier of R/i and r

partial unification:

$R \rightarrow H\tau$
 $(G, R[(p \stackrel{\circ}{=} p')])\sigma \rightarrow H\tau\sigma$

where $(G \rightarrow t \stackrel{\circ}{=} t') \in \text{IMP}_{\text{def}}$, and σ is a most general ϕ -unifier of t, p ,
 and p' for some $(p \stackrel{\circ}{=} p') \in R$.

test of facts:

$R \rightarrow H\tau$
 $(R[(p \stackrel{\circ}{=} p')])\sigma \rightarrow H\tau\sigma$

where $p \stackrel{\circ}{=} p' \in R$ and $p\sigma = r$, $p'\sigma = r'$ for some $(r \stackrel{\circ}{=} r') \in F$, or
 $p\sigma = p'\sigma = r$ for some $r \in K \cup \text{Term}(F)$.

If the PCNA yields " $\phi \rightarrow H\tau$ " we have found a single special solution τ of H, whereas " $\text{Diag}(\text{Var}(H\tau)) \rightarrow H\tau$ " describes a set of solutions.

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