Automatic Verification of Pointer Data-Structure Systems for All Numbers of Processes*

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Abstract. Real-world concurrent software may be implemented with any number of processes which are linked together to form complex and dynamic network configurations. We formally model such concurrent software as processes running algorithms on data-structures with pointers. We show that the verification problem of such algorithms is undecidable. A new automatic approximation method is then proposed to safely verify such algorithms. The central idea is to construct a finite *collective image set (CIS)* which collapses reachable state representations for all implementations of all numbers of processes. Our collapsing scheme filters out unimportant information of system behaviors and results in CIS's with manageable space requirements. Analysis shows our method can automatically generate a CIS of size 1619 to verify that a version of Mellor-Crummey & Scott's algorithm preserves mutual exclusion for all numbers of processes.

1 Introduction

With the success of automatic verification technology for hardware systems in recent years[6, 8], people are now naturally looking forward to automating the tasks of software verification. However, with vast variety of abstract devices like unlimited concurrencies, pointers, dynamic data-structures, range-unbounded variables, unbounded buffers,, etc., software systems are far more sophisticated than hardware systems. Straightforward extension of the existing state-based technology[4, 9, 10, 13, 19, 21, 24] is at an inappropriate abstractness level and has generally bumped into steep complexities of the verification problems for software systems[1, 2, 14, 22, 23]. In this paper, we propose a formal system to model such concurrent softwares and show that in general the corresponding verification problem is undecidable. Then we present a new automatic approximation verification method which reasons at an abstractness level similar to that

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Fig. 1. modified MCS locking algorithm

of humans by filtering out unimportant details of system behaviors for efficient verification.

We are dealing with concurrent systems of unspecified number of processes running different copies of a same algorithm with data-structures of enumerate variables and pointers which are either *global* or *local*. With such pointers, the processes can dynamically compose sophisticate network configurations.

Example 1. : Mellor-Crummy & Scott's algorithm MCS locking algorithm[18] is an example protocol in which a global waiting queue of processes is explicitly used to insure mutual exclusion to the critical section in a concurrent system. In figure 1, a modified version of MCS locking algorithm for a process is drawn, as a finite-state automaton, while the original version is given in figure 5 in the appendix. The critical section is mode 4. The queue is constructed with one global pointer *L* to the tail of the queue, and each process's local pointers: *next* and *prev* which respectively point to the successor and predecessor processes of the local process in the queue. *P* is the special symbol for the address of the running process. Each process also has one Boolean variable *locked* which is set to true when the process can enter the critical section. A process can access and manipulate other processes' information through global and local pointers.

MCS algorithm maintains mutual exclusion by insuring that at any moment at most one process has its local *locked* set to *true* and the process sets its waiting queue successor's *locked* to *true* after it changes its own *locked* to *false*. ||

In section 3, we shall prove that the verification problem of such algorithms is undecidable, i.e. no computers with finite memory can answer such a question. Since the verification problem is extremely difficult, we instead develop an automatic approximation method which can answer the safety of a large class of such algorithms regardless of the number of processes. The idea is to construct a finite *collective image set (CIS)* whose elements are reachable state images describing the behaviors of all implementations with any number of processes. Engineers' intelligence and experiences in design and verification is encoded in the mapping from states to images in CIS's and seems to result in small CIS's even for complicate data-structures. For safety analysis, if we can construct a finite CIS which contains no images of states violating the safety specification, then it is good enough to conclude that the algorithm is safe for any number of processes. However if there is a state image violating the safety specification in the CIS, then no conclusion can be made because the image may be included due to insufficient approximation precision.

With the known high complexities of most verification problem models[1, 2, 14, 22, 23], it is clear that current technology cannot identify a large class of concurrent algorithms subject to efficient verification. On the other hand, we argue that our technology can serve to identify such a large class of "well-designed" concurrent algorithms which can be efficiently verified. In section 5, we shall establish the mighty lemma 3 which allows us to eliminate much combinatorial complexity in CIS without sacrificing approximation precision. In section 6, we shall analyze our method on the modified MCS algorithm (in example 1) in which local pointers are set to null whenever the current values of the pointers will not be used in the future. The modification is consistent with good programming practice of elimination of "stray" pointers. The interesting thing here is that our method can generate a small CIS of size 1619 for the modified MCS algorithm while fails to do so for the original one. This shows that our verification method is indeed more efficient for "good" designs.

We shall adopt the following notations. Given a set or sequence K , $|K|$ is the number of elements in K . For each element e in K , we also write $e \in K$. We let $\mathcal{N}$ be the set of nonnegative integers.

2 Related Work

Apt and Kozen already showed that in general verification of systems with unknown number of concurrent processes is undecidable[3]. This means that such verification problems are extremely hard and we can only rely on semi-decision procedures or, as in this work, approximation algorithms to answer them. Otherwise, we can also investigate to find out decidable subclasses of the problem. In the following, we briefly describe some of the related work.

Browne, Clarke, and Grumberg [5] use bisimulation equivalence relation between global state graphs of systems of different sizes. The equivalence relation must be strong enough for the method to work. Thus the construction of the equivalence relation is difficult to mechanize.

Clarke, Grumberg, and Jha[11] propose to use regular languages to specify properties in a linear network with unknown number of processes. Then state-equivalence relation is defined based on the regular languages and a mechanical method is defined to synthesize a network invariant $\mathcal{I}$ in the hope that $\mathcal{I}$ can be contained by the specification. But there is no guarantee that $\mathcal{I}$ is a model of the specification even if the system indeed satisfies the specification. Moreover, it is not known whether using the specification regular languages to derive equivalence class properly preserves the reasonings behind the system design. Lesens, Halbwachs, and Raymond[17] furthered the approach by designing a language for the specification in systems with complex structures and by using fixed-point resolution with different heuristics to calculate many network invariants. Compared to our approach, we argue that the technique of CIS better captures the design reasoning that the relations between processes in different states are far more important than the actual numbers of processes in different states. We believe in verifying complex systems, without utilizing the reasoning behind the system designs, state-explosion problem cannot be properly dealt with.

Kurshan and McMillan[16] proposes to use network structural induction which is not guaranteed to terminate. Also inductive hypothesis is difficult to construct, although once it is ready, the whole approach is usually very efficient. Compared to our approach, we are using an approximation algorithm which captures the engineers' view of linear list. Users only have to guess the value of bound B , used in CIS construction, which for many real-world concurrent algorithms, small value like 1 will do.

Emerson and Naamjoshi[12] specialized on static token ring networks. They prove that for certain properties, verification on small size networks can be used to guarantee the verification of large size networks. In contrast, our method is applicable to all different configurations of “dynamic” networks of processes.

Boigelot and Godefroid[7] choose to use state-space exploration to handle the verification problems of systems with unbounded FIFO queues. Their state-space representation is constructed by collapsing FIFO queues. Their approach does not guarantee termination.

Recently, the author also has researched on the technique of collective quotient structures on dynamic linear networks [20]. The idea is similar to that of CIS in that they both collapse state-spaces of all implementations into single structures. However my work here is more general for pointer data-structures which allows the development of lemma 3, in section 5, and can lead to significant reduction in time and space complexity.

3 Concurrent Algorithms and Safety Bound Problem

We are dealing with concurrent algorithms with a local data structure for each process. The address of a data structure can be viewed as the identity of the corresponding process. We shall have the convention that if a process is named p , then p is also the address of process p 's data-structure.

Two types of variables can be declared. The first is the type of *enumerate variables* with predefined finite integer value ranges. For convenience, we can also give symbolic names to those integer values. Traditionally, *false* is interpreted as 0 while *true* as 1. The second is the type of *pointers* (*address variables*) to processes (data-structures). As in example 1, L is a pointer to the tail of a queue. Variables can be declared as *global* variables which all processes can access, or *local* variables of a process which only the declaring process can directly access. The same name can be used to represent the respective local variables of different processes. For example, in example 1, different processes access different variables which are all locally called **locked**.

Test can be made to determine if an enumerate type variable's content equals to a constant, if a pointer is **null**, or if two pointers point to the same process. We can also assign a constant to an enumerate type variable or to assign a process address to a pointer. In the following, we shall first formally define the syntax and semantics of our systems, and then define the safety bound problem.

3.1 Syntax of Algorithm Descriptions

Conceptually, a concurrent algorithm S is a tuple $(\text{Enu}_g, \text{Ptr}_g, \text{Enu}_l, \text{Ptr}_l, A(P))$ where Enu_g and Enu_l are respectively the sets of *global* and *local enumerate variables* (as in Pascal programming language), Ptr_g and Ptr_l are respectively the sets of *global* and *local pointers*, and $A(P)$ is the *process program template*, with *process identifier* symbol P .

Given algorithm $S = (\text{Enu}_g, \text{Ptr}_g, \text{Enu}_l, \text{Ptr}_l, A(P))$, a *process predicate* pred of S can be used to describe the triggering condition of state transitions and has the following syntax.

$$\begin{aligned} \text{pred} &::= \text{exp}_1 = \text{exp}_2 \mid \text{ref}_1 = \text{ref}_2 \mid \neg \text{pred} \mid \text{pred}_1 \vee \text{pred}_2 \\ \text{exp} &::= c \mid x \mid z \mid y \rightarrow x \mid w \rightarrow x \\ \text{ref} &::= \text{null} \mid P \mid w \mid y \mid w \rightarrow y \mid y_1 \rightarrow y_2 \end{aligned}$$

where $c \in \mathcal{N}$, $x \in \text{Enu}_l$, $z \in \text{Enu}_g$, $w \in \text{Ptr}_g$, and $y \in \text{Ptr}_l$. Traditional shorthands are $\text{exp}_1 \neq \text{exp}_2 \equiv \neg(\text{exp}_1 = \text{exp}_2)$, $\text{ref}_1 \neq \text{ref}_2 \equiv \neg(\text{ref}_1 = \text{ref}_2)$, $\text{pred}_1 \wedge \text{pred}_2 \equiv \neg((\neg \text{pred}_1) \vee (\neg \text{pred}_2))$, and $\text{pred}_1 \Rightarrow \text{pred}_2 \equiv (\neg \text{pred}_1) \vee \text{pred}_2$. Thus a process may operate on conditions of the global and local variables, and also on the local variables of the processes pointed to by global pointers. We let Predicates_S be the set of all process predicates of S .

Once the triggering condition is satisfied by a running process, the process may execute a finite sequence, say *actseq*, of actions to update the state information. The finite sequence *actseq* of actions has the following syntax.

$$\begin{aligned} \text{actseq} &::= \mid \text{act actseq} \\ \text{act} &::= \text{lhsexp} := \text{exp}; \mid \text{lhsref} := \text{ref}; \\ \text{lhsexp} &::= x \mid z \mid w \rightarrow x \mid y \rightarrow x \\ \text{lhsref} &::= y \mid w \mid w \rightarrow y \mid y_1 \rightarrow y_2 \end{aligned}$$

Here exp and ref are defined as in the syntax of process predicates. act defines what an *action* looks like. The set of all finite sequences of actions of S is named Aseqset_S .

Given a concurrent algorithm $S = (\text{Enum}_g, \text{Ptr}_g, \text{Enum}_l, \text{Ptr}_l, A(P))$, $A(P)$ is the program template for each process with identifier symbol P . Program template $A(P)$ has the syntax similar to that of finite-state automata. $A(P)$ is conceptually a tuple (Q, q_0, E, τ, π) with the following restrictions.

- Q is a finite set of operation modes.
- $q_0 \in Q$ is the initial operation mode.
- $E \subseteq Q \times Q$ is the set of transitions among operation modes.
- $\tau : E \mapsto \text{Predicates}_S$ is a mapping which defines the triggering condition of each transition.
- $\pi : E \mapsto \text{Aseqset}_S$ is a mapping which defines the action sequence performed at the happening of each transition. *Atomicity of the transition is assumed.*

We do require that there is a variable **mode** $\in \text{Enum}_l$ which records the current operation mode of the corresponding process. However, while drawing $A(P)$ as an automaton, we omit the description of **mode** values in the triggering conditions and action sequences for simplicity and clarity.

3.2 Computation of Systems

Let Π be the set of all processes (conceptually represented by either their identifiers, or their data-structure addresses) in an implementation. For each enumerate-type variable x , we let D_x be $\{0\}$ unioned with the set of all constants assigned to x in process program $A(P)$ labeled in all transition's assignment sequence. Especially, $D_{\text{mode}} = Q$ and **mode** = 0 means the process is in its initial operation mode.

A *process state* π is a mapping from $\text{Enum}_l \cup \text{Ptr}_l$ to $\mathcal{N} \cup \Pi \cup \{\text{null}\}$ such that $\pi(x) \in D_x$ if $x \in \text{Enum}_l$; and $\pi(x) \in \Pi \cup \{\text{null}\}$ if $x \in \text{Ptr}_l$.

A *global state* of $S\Pi$ is a pair (ψ, ϕ) with the following restrictions.

- ψ is a mapping from $\text{Enum}_g \cup \text{Ptr}_g$ to $\mathcal{N} \cup \Pi \cup \{\text{null}\}$ such that $\psi(x) \in D_x$ if $x \in \text{Enum}_g$; and $\psi(x) \in \Pi \cup \{\text{null}\}$ if $x \in \text{Ptr}_g$.
- ϕ is a mapping from Π to the set of all process states running algorithm A .

Given a global state $\nu = (\psi, \phi)$, a process $p \in \Pi$, and a process predicate $\text{pred} \in \text{Predicates}_S$, we define the relation of p *satisfies* pred at ν , written $p, \nu \models \text{pred}$, in the following inductive way. Assume that $x \in \text{Enum}_l$, $z \in \text{Enum}_g$, $y, y_1, y_2 \in \text{Ptr}_l$, and $w \in \text{Ptr}_g$.

- $p, \nu \models \text{exp}_1 = \text{exp}_2$ iff $\text{value}(p, \nu, \text{exp}_1) = \text{value}(p, \nu, \text{exp}_2)$
- $\text{value}(p, \nu, c) = c$
- $\text{value}(p, \nu, x) = \phi(p)(x)$
- $\text{value}(p, \nu, z) = \psi(z)$
- $\text{value}(p, \nu, y \rightarrow x) = \phi(\phi(p)(y))(x)$
- $\text{value}(p, \nu, w \rightarrow x) = \phi(\psi(w))(x)$
- $p, \nu \models \text{ref}_1 = \text{ref}_2$ iff $\text{value}(p, \nu, \text{ref}_1) = \text{value}(p, \nu, \text{ref}_2)$
- $\text{value}(p, \nu, \text{null}) = \text{null}$
- $\text{value}(p, \nu, P) = p$

- $\text{value}(p, \nu, w) = \psi(w)$
- $\text{value}(p, \nu, y) = \phi(p)(y)$
- $\text{value}(p, \nu, w \rightarrow y) = \phi(\psi(w))(y)$
- $\text{value}(p, \nu, y_1 \rightarrow y_2) = \phi(\phi(p)(y_1))(y_2)$
- $p, \nu \models \neg \text{pred}$ iff it is not the case that $p, \nu \models \text{pred}$
- $p, \nu \models \text{pred}_1 \vee \text{pred}_2$ iff $p, \nu \models \text{pred}_1$ or $p, \nu \models \text{pred}_2$

Given an action act of S , the new global state obtained by applying act to p at ν , written $\text{next_state}(p, \nu, \text{act})$, is defined in the following way.

- $(\psi', \phi') = \text{next_state}(p, \nu, x := \text{exp};)$ is identical to ν except that $\phi'(p)(x) = \text{value}(p, \nu, \text{exp})$.
- $(\psi', \phi') = \text{next_state}(p, \nu, z := \text{exp};)$ is identical to ν except that $\psi'(z) = \text{value}(p, \nu, \text{exp})$.
- $(\psi', \phi') = \text{next_state}(p, \nu, w \rightarrow x := \text{exp};)$ is identical to ν except that $\phi'(\psi(w))(x) = \text{value}(p, \nu, \text{exp})$.
- $(\psi', \phi') = \text{next_state}(p, \nu, y \rightarrow x := \text{exp};)$ is identical to ν except that $\phi'(\phi(p)(y))(x) = \text{value}(p, \nu, \text{exp})$.
- $(\psi', \phi') = \text{next_state}(p, \nu, y := \text{ref};)$ is identical to ν except that $\phi'(p)(y) = \text{value}(p, \nu, \text{ref})$.
- $(\psi', \phi') = \text{next_state}(p, \nu, w := \text{ref};)$ is identical to ν except that $\psi'(w) = \text{value}(p, \nu, \text{ref})$.
- $(\psi', \phi') = \text{next_state}(p, \nu, w \rightarrow y := \text{ref};)$ is identical to ν except that $\phi'(\psi(w))(y) = \text{value}(p, \nu, \text{ref})$.
- $(\psi', \phi') = \text{next_state}(p, \nu, y_1 \rightarrow y_2 := \text{ref};)$ is identical to ν except that $\phi'(\phi(p)(y_1))(y_2) = \text{value}(p, \nu, \text{ref})$.

Given an action sequence $\text{act}_1 \dots \text{act}_n \in \text{Aseqset}_S$, we let $\text{next_state}(p, \nu, \text{act}_1 \text{act}_2 \dots \text{act}_n) = \text{next_state}(p, \text{next_state}(p, \nu, \text{act}_1), \text{act}_2 \dots \text{act}_n)$.

The *initial state* (ψ_0, ϕ_0) of an implementation SII must satisfies the following restrictions: (1) $\psi_0(z) = 0$ for all $z \in \text{Enu}_g$, (2) $\psi_0(w) = \text{null}$ for all $w \in \text{Ptr}_g$, (3) $\phi_0(p)(x) = 0$ for all $p \in II$ and $x \in \text{Enu}_l$, and (4) $\phi_0(p)(y) = \text{null}$ for all $p \in II$ and $y \in \text{Ptr}_l$. We assume that processes interact with *interleaving semantics* in the granularity of transitions, that is at any moment, at most one process can execute a transition. Interleaving semantics is well-accepted in verification theory for its simplicity. From the viewpoint of languages, such an atomicity may be difficult to implement. Since our focus is on verification and modelling, instead of implementation, we believe such an atomicity is important in modelling embedded systems in which natural world atomic transitions can only be emulated by several algorithm actions.

A *computation* of an implementation SII is a (finite or infinite) sequence $\rho = \nu_0 \nu_1 \dots \nu_k \dots$ of global states with $\nu_k = (\psi_k, \phi_k)$ for all $k \geq 0$ such that

- ν_0 is the initial state of SII ; and
- for each ν_k with $k > 0$, there is a $p \in II$ and transition from q to q' such that $p, \nu_{k-1} \models \tau(q, q')$ and $\text{next_state}(p, \nu_{k-1}, \pi(q, q')) = \nu_k$.

3.3 Safety Bound Problem and Its Undecidability

The computation definition of our algorithm implementations is independent of the real names used for each process in Π . Never the names of processes are used to affect the behaviors of our implementations. Instead, only the relation among processes in various process states is important. Thus it is better if we can present our safety analysis problem regardless of the actual names used for processes. Given a global state $\nu = (\psi, \phi)$ of an implementation $S\Pi$ and a process predicate $pred$, $\mathbf{count}_{pred}(\nu)$ is the number of processes satisfying $pred$ at ν , i.e. $|\{p \mid p \in \Pi; p, \nu \models pred\}|$. A computation $\rho = \nu_0 \nu_1 \dots \nu_k \dots$ of $S\Pi$ violates safety property $pred$ with bound $c \in \mathcal{N}$ iff there is a $k \geq 0$ such that $\mathbf{count}_{pred}(\nu_k) > c$.

The *safety bound problem* instance $\mathbf{SBP}(S, pred, c)$ is to determine if for all finite sets Π of processes and all computation ρ of $S\Pi$, ρ does not violate safety property $pred$ with bound c . Such a problem framework can be used to verify process state reachability problem[14] which is a special case of $\mathbf{SBP}(S, pred, c)$ with $c = 1$. Also mutual exclusion problem can be formulated with $c = 1$. Reader-Writer problem can be formulated with c set to the number of readers.

Example 2. : Consider the modified MCS algorithm in example 1. The critical section is in mode 4. A process can enter the critical section only if its local variable **locked** is true. Thus the safety bound problem for the mutual exclusion to critical section can be formulated as $\mathbf{SBP}(S, \mathbf{locked} = true, 1)$ which answers “true” iff along any computation ρ of all implementations, no more than one process can have **locked** = true simultaneously. $\parallel$

However, such a problem is extremely difficult to answer. In fact, we can show $\mathbf{SBP}(S, pred, 1)$ for a given S and $pred$ is undecidable, i.e. there is no computer with finite amount of memories capable of answering $\mathbf{SBP}(S, pred, 1)$. Lemma 1 proves this by reducing two-counter machine halting problem[15] to $\mathbf{SBP}(S, pred, 1)$. A two-counter machine M has a finite-state control and two counters which can hold any natural numbers. The finite-state control can increment a counter, decrement a counter, or transit between finitely many operation modes by testing whether a particular counter contains zero. It is known that two-counter machine can emulate Turing machine whose halting problem cannot be answered by any computers with finite amount of memories.

Lemma 1. : *Two-counter machine halting problem is reducible to $\mathbf{SBP}(S, pred, 1)$.*

Proof : Due to page-limit, we shall only give a sketch of the proof. Suppose we are given a two-counter machine M . We want to show that we can construct a concurrent algorithm S such that M reaches its final state iff there is an implementation $S\Pi$ such that there is a process in Π which also reaches its corresponding local final state. We shall implement two stacks to emulate the two counters respectively with pointers linking together adjacent elements in the stacks. The first (second) stack’s height of $S\Pi$ emulates the content of the first (second) counter of M . Suppose in the computation of M to reach its

final state, counter 1 and 2 have maximum contents v_1 and v_2 respectively. Then there exists a Π , of $3 + v_1 + v_2$ processes, which makes the construction works.

The halting state of M is encoded in $pred$. The first transiting process in the computation will be used to emulate the finite-state control. The second and third transiting processes in the computation will respectively be used to emulate the stack bottoms for the two counters. Then each increment operation of a counter will need one process to be pushed onto the corresponding stacks. If there is not enough number of processes for the increment operations in the implementation, then the computation simply halts in a state without satisfying $pred$. Each decrement operation of a counter will need the top process in the corresponding stack to be popped. Testing for zero value of a counter can be implemented by asking if the stack top process is equal to the stack bottom process for the corresponding counter. In this way, we can construct S and $pred$ such that $SBP(S, pred, 1)$ answers true iff M reaches its halting state. $\parallel$

4 Collective Image Set

With lemma 1 and many similar complexity results[1, 2, 14, 22, 23], it is clear that classic verification technology is not able to handle the complexity incurred by verification problems for concurrent algorithms with sophisticate data-structures. However, we have observed that classic verification theory does not distinguish "well-behaved" systems from "bad" systems. In many algorithms for concurrent systems, the number of processes is usually not a crucial factor in the correctness of systems.

Our CIS is a set whose elements are global state images which are finite recordings of multisets of process information patterns. A multiset is conceptually a set which allows an element to repeat many times. Mathematically, it is a mapping from a domain to $\mathcal{N}$. We say that process p *points to* process p' (and p' is called a *reference* of p) in state (ψ, ϕ) if there is either a $y \in \text{Ptr}_l$ such that $\phi(p)(y) = p'$ or a $w \in \text{Ptr}_g$ such that $\psi(w) = p'$. State of each process p is collapsed down to a *PDSI* (*process data-structure image*) which only records information that process p can read from the local variables of itself and its global or local references.

A global state image, called *GDSI* (*global data-structure image*), is treated as a finite recording of a multiset of PDSI's of the participating processes. It is finite because when more than B , a constant chosen by users, processes have the same PDSI in a global state, they are only recorded by a flag (∞ here) which denotes that the number of processes in that PDSI exceeds B . By viewing the GDSI's as a multiset, we treat all processes in a symmetric way without the price for the management of process identifier permutation [13]. With the constant B , we are able to map states of infinitely many implementations down to finitely many global state images. To choose a value for B , users should have the intuition that when more than B processes are of a PDSI, the actual number of such processes is not important. For example, for many mutual exclusion algorithms, if there are either 3 or 4 processes in the waiting queue, the processes in each PDSI's

exhibit the same behavior pattern. For example, to pass over the right to the critical section, the process in the critical section (of one PDSI) passes the right to the head process in the waiting queue (of another PDSI). The behavior pattern only depends on whether there is a process in the waiting queue. According to our observation, many algorithms work with such reasoning. For a lot of mutual exclusion protocols, small value of B like 1 will work and the CIS's exhibit simple regularity.

In the following subsections, we shall define rigorously the image mapping of global states of implementations. Then we shall define the *transitions*, among GDSI's, which corresponds to transition rules described in $A(P)$.

We need the following conventions regarding number systems respecting a bound B . Let $\mathcal{N}^\infty = \mathcal{N} \cup \{\infty\}$ where ∞ means any number greater than B . For any $c \in \mathcal{N}$, $c < \infty$. For any $c, d \in \mathcal{N}^\infty$, $c \leq \infty$ and $c + \infty = \infty + d = \infty$.

Given any two numbers c and B in $\mathcal{N}^\infty$, we let $c^{(B)} = c$ if $c \leq B$; $c^{(B)} = \infty$ if $c > B$. Finally, $[0, B]^{(\infty)} = \{0, 1, \dots, B\} \cup \{\infty\}$.

4.1 Pointer Data-Structure Images

The global-state images in our method is characterized by finite sets of propositional atoms. We shall first define *PSI* (*process state images*) as building blocks to construct PDSI. PSI represents the observation a process can make without going through pointers. The PSI of process p at state $\nu = (\psi, \phi)$, in symbols $\text{PSI}(p, \nu)$, is a finite set of atoms constructed in the following way.

$$\begin{aligned} & \{x = c \mid x \in \text{Enu}_l; c \in D_x; c = \phi(p)(x)\} \\ & \cup \{z = c \mid z \in \text{Enu}_g; c \in D_x; c = \psi(z)\} \\ & \cup \{P = y \mid y \in \text{Ptr}_l; p = \phi(p)(y)\} \\ & \cup \{P = w \mid w \in \text{Ptr}_g; p = \psi(w)\} \\ \text{PSI}(p, \nu) = & \cup \{y = \text{null} \mid y \in \text{Ptr}_l; \phi(p)(y) = \text{null}\} \\ & \cup \{w = \text{null} \mid w \in \text{Ptr}_g; \psi(w) = \text{null}\} \\ & \cup \{y = w \mid y \in \text{Ptr}_l; w \in \text{Ptr}_g; \phi(p)(y) = \psi(w)\} \\ & \cup \{y_1 = y_2 \mid y_1, y_2 \in \text{Ptr}_l; \phi(p)(y_1) = \phi(p)(y_2)\} \\ & \cup \{w_1 = w_2 \mid w_1, w_2 \in \text{Ptr}_g; \psi(w_1) = \psi(w_2)\} \end{aligned}$$

Here we use P to symbolically represent the address p of the corresponding process. Note that we conveniently define PSI's to also record information on global variables. Thus in our GDSI defined later, PSI's of all processes must all agree on the informations of those global variables. Conveniently, we shall let PSIset be the set of all PSI's.

Our process image, the PDSI (*process data-structure image* (*PDSI*)) of a process p in a state $\nu = (\psi, \phi)$ with bound B , in symbols $\mathbf{PDSI}^{(B)}(p, \nu)$, is graphically shown in figure 2 and only records

- the PSI's of p and p 's global and local references;
- the equality among p and p 's references' references (i.e. if the references point back); and
- the multiset of incoming local pointers from peer processes to p with bound B ($\text{ILM}^{(B)}$ for *incoming link multiset* with bound B , will be defined later).

Fig. 2. Information of PDSI

Intuitively, PDSI of a process p records the information that p can infer by at most one dereferencing through its local and global pointers. In the following, we shall make the definition of PDSI more precise.

The *incoming link multiset* (ILM) of process p at state $\nu = (\psi, \phi)$, in symbols $\text{ILM}(p, \nu)$, records those peer processes which have p as their global or local references. When a transiting process p changes the PSI of itself and its references, the PDSI's of processes with p as their references will also be changed. To propagate such waves of PDSI changes, we need information in $\text{ILM}(p, \nu)$ to enumerate the possibilities. Formally speaking, for all $\lambda \in \text{PSIset}$ and $y \in \text{Ptr}_l$ in an algorithm implementation SII ,

$$\text{ILM}(p, \nu)(\lambda, y) = |\{(p', y) \mid p' \in \Pi; \text{PSI}(p', \nu) = \lambda; \phi(p')(y) = p\}|$$

To respect bound B , we let $(\text{ILM}(p, \nu))^{(B)}$ be a mapping from the domain to $[0, B]^{(\infty)}$ such that for all $\lambda \in \text{PSIset}$ and $y \in \text{Ptr}_l$, $(\text{ILM}(p, \nu))^{(B)}(\lambda, y) = (\text{ILM}(p, \nu)(\lambda, y))^{(B)}$.

Given an algorithm, it is clear that the number of different possible PDSI's at states of all implementations is finite. We let $\text{PDSIset}^{(B)}$ be the set of all possible PDSI's with bound B .

The *global data-structure image* (GDSI) χ of a state $\nu = (\psi, \phi)$ with bound B , in symbols $\mathbf{GDSI}^{(B)}(\nu)$, is a mapping from $\text{PDSIset}^{(B)}$ to $[0, B]^{(\infty)}$ such that for all $\mu \in \text{PDSIset}^{(B)}$, if $\mathbf{GDSI}^{(B)}(\nu)(\mu) \neq \infty$, it means that there are exactly $\mathbf{GDSI}^{(B)}(\nu)(\mu)$ processes in ν whose PDSI's are μ ; otherwise, it means that there are more than B processes in ν whose PDSI's are μ . Since GDSI's are constructed with finite set of atomic propositions and constant B , it is clear that the number of GDSI's is finite.

Notationally, we let $\text{GDSIset}^{(B)}$ be the set of all distinct GDSI's with B .

4.2 Transitions among GDSI's

We let $\mathbf{x}tion_e(\chi, \chi')$ be the relation, between χ and χ' , which is true iff from GDSI χ , we can transit to GDSI χ' through transition rule e . $\mathbf{x}tion_e(\chi, \chi')$ can be computed by the following nondeterministic procedure.

-
- A. Nondeterministically choose PDSI's in χ corresponding to the transiting process and its references.
 - B. If they are inconsistent, then answer *false* and stop.
 - C. Change the PSI's and PDSI's of those chosen PDSI's according to the meaning of transition rule e .
 - D. Nondeterministically propagate the changes to other unchosen PDSI's according to the ILM recordings of those chosen PDSI's.
 - E. After all propagations are done, if χ is changed to χ' , answer *true*; else answer *false*.
-

The steps are written in nondeterministic style for simplicity. If and only if one of the executions end up with return value *true*, then χ can go to χ' with transition rule e . In practice, they have to be implemented with recursive procedure-calls. In each level of the recursion, a choice is made and recorded in the recursion stack.

We shall demonstrate the working of the above-mentioned procedure for the transition rule from mode 0 in the modified MCS algorithm (figure 1). The rule in guarded command language is

$$\mathbf{mode} = 0 \rightarrow (\mathbf{mode} := 1; \mathbf{prev} := L; L := P;)$$

In step A, in case we choose $\mu \in \text{PDSIset}^{(B)}$, with $\chi(\mu) \neq 0$, for either the transiting process or one of its references, what we actually do is to create a new PDSI μ' in χ with $\chi(\mu') = 1$ to represent the chosen process. Then we have to decrement the value of $\chi(\mu)$ respecting bound B . That is, if $\chi(\mu) \neq \infty$, then the decremented value can either be ∞ or B nondeterministically; otherwise, the decremented value is $\chi(\mu) - 1$.

Suppose that we pick μ_P as the transiting process with $\chi(\mu_P) = \infty$. According to our modification to MCS algorithm, the two local pointers: **prev** and **next** of μ_P both point to **null**. For convenience, we shall adopt notations like $\mu_P \rightarrow \mathbf{prev}$ for the PDSI which represents the reference **prev** of PDSI μ_P in a GDSI. $\mu_P \rightarrow \mathbf{mode}$ also has the same intuition. The only other reference which μ_P can make is through global pointer L . Suppose that we pick μ_L with $\mu_L \rightarrow \mathbf{mode} = 2$ as the global reference L . It can be shown from our CIS construction that $\chi(\mu_L) = 1$. This is natural because a global pointer can only point to one process. Thus after step A, we have the following pictorial description of those chosen PDSI's.

Note since our modification to MCS algorithm eliminates strayed pointers (see appendix), $\mu_L \rightarrow \mathbf{prev}$, if not null, will be unique in χ , i.e., $\chi(\mu_L \rightarrow \mathbf{prev}) = 1$. Suppose after a nondeterministic choice of μ_P and μ_L , χ is changed to χ_A .

In step B, consistency among the chosen PDSI's has to be checked. For example, if μ_L requires that the local pointer **next** of PDSI $\mu_L \rightarrow \text{prev}$ does not point back to PDSI μ_L , then we have to check whether this requirement is compatible with the recordings in $\mu_L \rightarrow \text{prev}$.

After step C, the transition is executed, and the configuration among the chosen PDSI's changes to the following for the above-mentioned Modified MCS algorithm scenario.

Till this step, there is no need to update χ_A .

In step D, the propagation may go in three directions. First, we have to non-deterministically modify the reference information of those unchosen processes with references as one of the chosen processes. In this scenario, since global reference L is reassigned to μ_P , the global side-effect of this step is to update the global references of all PDSI's in χ_A so that every PDSI knows that μ_P has become the new L . Especially, we have to check if there is any PDSI $\bar{\mu}$ with $\bar{\mu} \rightarrow \text{next}$ points to the unique previously globally referenced μ_L and changes the recording of $\bar{\mu} \rightarrow \text{next}$.

Second, we have to modify the ILM recordings of the references' of those chosen PDSI's if those chosen PDSI's are PSI-modified. In our scenario, we have to nondeterministically pick a PDSI corresponding to $\mu_L \rightarrow \text{prev}$ and changes its ILM recordings. With the side-effect nondeterministically propagated, we assume that χ_A is changed to χ_D to reflect the propagation.

Third, the new PDSI's of the chosen have to be collapsed with the PDSI's in χ_D respecting bound B . In this scenario, there will be two such PDSI's, one modified from μ_P (say μ'_P) and the other from μ_L (say μ'_L), to be added into χ_D again. In case that $\chi_D(\mu'_L) = B$ or ∞ , then $\chi_D(\mu'_L)$ should be changed to ∞ ; otherwise, $\chi_D(\mu'_L)$ is incremented by one.

In Step E, if and only if we found that χ_D is changed to be equal to χ' , we shall answer *true*. The performance of the procedure can be enhanced with symbolic manipulation techniques[6, 8]. We are still investigating the details.

5 Safety Bound Verification with CIS

The following lemma shows that CIS is indeed a conservative approximation of all reachable states from the initial states of all implementation.

Lemma 2. *Given a concurrent algorithm S with X as the set of GDSI's of initial states of all implementations, the minimal set *Reachable* satisfying the following conditions:*

- $X \subseteq \text{Reachable}$; and
- for all $\chi \in \text{Reachable}$ and $\chi' \in \text{GDSIset}$ such that $\mathbf{xtion}_e(\chi, \chi') = \text{true}$ for some transition rule e , $\chi' \in \text{Reachable}$.

is a super set of GDSI's of all reachable states of all implementations of S .

Proof : Our GDSI overapproximates a global state by filtering out information over bound B . While nondeterministically computing $\mathbf{xtion}_e(\chi, \chi')$ for some GDSI's χ, χ' and transition rule e , overapproximation is also done by considering all possibilities with regard to the precision bound setup by B . Thus in the construction of *Reachable*, only overapproximation has been done and underapproximation is never done. Considering all these, we infer that *Reachable* is indeed a superset of GDSI's of reachable states of all implementations of S . $\parallel$

By just naive enumerating all the GDSI's reachable from the initial state in an algorithm implementation, we will easily bump into combinatorial explosion of complexity because each PDSI can be mapped to any number in $[0, B]^{(\infty)}$. However, we can take advantage of our interleaving semantics to eliminate much of such complexity. The idea is based on lemma 3. A GDSI χ' contains another GDSI χ , in symbols $\chi \subseteq \chi'$, if for every PDSI $\mu \in \text{PDSIset}^{(B)}$, $\chi(\mu) \leq \chi'(\mu)$.

Lemma 3. *Suppose we have two GDSI's $\chi \subseteq \chi'$. Then for every GDSI sequence $\chi_0\chi_1\dots\dots$ with $\chi_0 = \chi$, we can construct another GDSI sequence $\chi'_0\chi'_1\dots\dots$ with $\chi'_0 = \chi'$ such that for all $k \geq 0$, $\chi_k \subseteq \chi'_k$ and χ_k and χ'_k may go to χ_{k+1} and χ'_{k+1} respectively with the same transition rule.*

Proof : A pictorial explanation of this fact is in figure 3. The relation can happen because in a concurrent system without invariance conditions specified for the operation modes, we can withhold those PDSI's in χ' but not in χ from firing transitions. $\parallel$

Now we shall present our approximation algorithm for safety bound problem with finite GDSI set as our CIS. We shall take advantage of lemma 3 such that two GDSI's χ, χ' will not be in the CIS simultaneously if $\chi \subseteq \chi'$.

Fig. 3. Containing relation between GDSI sequences

Given a $\chi \in \text{GDSIset}^{(B)}$, $\text{count}_{pred}(\chi)$ is also the number, respecting bound B , of PDSI's satisfying $pred$ in χ . Formally speaking,

$$\text{count}_{pred}(\chi) = \left(\sum_{(\theta, \beta) \in U_A^{(B)}; (\bigwedge_{a \in \theta} a) \Rightarrow pred \chi((\theta, \beta))} \right)^{(B)}$$

Now we have the procedure **Safety_Bound()** in table 1 to embody our safety bound verification method in details. Note in statements (2.2.2) and (2.2.3), we delete those GDSI's contained by other GDSI's in V according to lemma 3.

The complexity of the method is polynomial to the number of GDSI's of states which then depends on $A(P)$ and B . A rough complexity analysis follows. The equivalence relation among pointers in a PDSI basically partitions global pointers, local pointers of P , the local pointers of references of P , the local pointers of references of those process pointed to by global pointers. The total number of pointers involved is $H = 1 + (|\text{Ptr}_g| + |\text{Ptr}_l|)(1 + |\text{Ptr}_l|)$ which is square to the size of S . The number of different partitions on these many pointers is roughly in the complexity of factorial to H . This will be the dominating factor in the complexity. Considering the values of $\text{ILM}^{(B)}$, we can deduce that the number of different PDSI's is roughly $O((B+1)^{|\text{PSIset}|2^H}) = O(2^{|S|^2 \log B})$. Since GDSI

```

/*  $S$  is an algorithm with transition rule set  $E$ .
/*  $pred$  is a process predicate describing the dangerous property.
/*  $C$  is the number of processes allowed in critical section.
/*  $B$  is the bound used in GDSI's. It is assumed  $C \leq B$ . */
Safety_Bound( $S, pred, C, B$ ) {
(1) Generate the initial GDSI  $\chi_0$ ; let  $V := \{\chi_0\}; W := V$ ;
(2) Repeat until  $W = \emptyset$ . {
  (1) If there is a  $\chi \in V$  such that  $\mathbf{count}_{pred}(\chi) > C$ , report “don’t know” and
  stop.
  (2) Let  $W := \{\chi_2 \mid \chi_1 \in W; \chi_2 \in \text{GDSIset}^{(B)}; \exists e \in E(\mathbf{x\!tion}_e(\chi_1, \chi_2))\} - V$ ;
  (3) Let  $V := V \cup W$ ;
}
(3) Report “SAFE!”
}

```

Table 1. Safety analysis with CIS

are mappings from PDSI's to $[0, B]^{(\infty)}$, the total number of different GDSI's is then $(B+2)^{O(2^{|S|^2 \log B})} = 2^{2^{O(|S|^2 \log B + \log B \log \log B)}} = 2^{2^{O(|S|^2 \log B)}}$. Thus our approach in each iteration of B value is of complexity doubly exponential to $|S|^2 \log B$.

For a lot of mutual exclusion protocols, small value of B like 1 will work and the CIS's exhibit simple regularity. The complexity analyses for MCS mutual-exclusion algorithm in section 6 shall justify our claim.

6 On Mellor-Crummy & Scott's Algorithm

We shall prove that our method indeed can verify Mellor-Crummy & Scott's (MCS) locking algorithm[18] for mutual exclusion in concurrent systems. MCS locking algorithm is a provenly correct algorithm requiring little shared memory. We believe that our method can verify many such algorithms with small B values regardless of the number of processes. The following lemma shows that our method can verify the modified version. A similar one can be used to prove for the original version.

Lemma 4. : *In the CIS constructed for MCS locking algorithm as shown in figure 1 with $B = 1$ and lemma 3, there is no image χ with $\mathbf{count}_{\text{mode}=4}(\chi) > 1$.*

Proof : A transition rule e is executed by a process in exactly the same way as step C of $\mathbf{x\!tion}_e$ is operated on the corresponding PDSI's. Steps A, B, D, E are nondeterministically performed to properly maintain the counts in the multisets and in the ILM's to consider all possibilities respecting the bound B . Only the transiting PDSI with the local `locked = true` can make its-successor-in-the-queue's `locked` true. With $B = 1$, we know in a GDSI whether there is only one PDSI with the local `locked = true`.

Initially, all PDSI's are in **mode** 0 with their local **locked** = *false*. From that point on, we want to make sure that given two GDSI's χ, χ' and a transition rule e with $\mathbf{x}tion_e(\chi, \chi') = true$, if $\mathbf{count}_{\mathbf{locked}=true}(\chi) \leq 1$, then $\mathbf{count}_{\mathbf{locked}=true}(\chi') \leq 1$. The first PDSI detecting **prev** = **null** while leaving **mode** 1, will enter **mode** 4 with **locked** set to **true**. This is the only situation when we can go from $\mathbf{count}_{\mathbf{locked}=true}(\chi) = 0$ to $\mathbf{count}_{\mathbf{locked}=true}(\chi') = 1$.

According to the algorithm, only the PDSI with **locked** = *true* can set the **locked** of other PDSI's to *true*. Immediately before setting the other PDSI's to *true* with transition rule

$$\mathbf{mode} = 7 \rightarrow \left(\begin{array}{l} \mathbf{mode} := 0; \\ \mathbf{locked} := false; \\ \mathbf{next} \rightarrow \mathbf{locked} := true; \\ \mathbf{next} := null; \end{array} \right)$$

the PDSI must first reset its own **locked**. With $B = 1$, we know that $\mathbf{count}_{\mathbf{locked}=true}(\chi)$ and $\mathbf{count}_{\mathbf{locked}=true}(\chi')$ will both be one. Considering both the two cases when a new PDSI has its local **locked** set to *true*, we find that $\mathbf{count}_{\mathbf{locked}=true}(\chi') = 1$. We thus concluded that our CIS does not have a GDSI in which more than one PDSI's have their **locked** = *true*. $\parallel$

We want to point out that our proof for lemma 4 is very much like a human proof for MCS algorithm. This shows that our method indeed reasons at an abstractness level similar to that of humans. Now we proceed to analyze the size of the CIS with $B = 1$ for the modified MCS locking algorithm, in figure 1, which has the good property that once a process is in **mode**=0, its local pointers will all be set to null and no other processes will have local pointers pointing to it again. This make the data-structure pretty much “clean” without “stray” pointers. Thus the number of different GDSI's solely depends on the different possibilities of PDSI's near the queue head, noted by H , and tail as shown in figure 4. We thus have the following case analysis.

- *In case there is only one PDSI in the queue.* Then $H \rightarrow \mathbf{mode} \in \{1, 4, 5\}$ and there are 3 possibilities.
- *In case there are two PDSI's in the queue.* The values of $H \rightarrow \mathbf{next}$ and $L \rightarrow \mathbf{prev}$ depend on $H \rightarrow \mathbf{mode}$ and $L \rightarrow \mathbf{mode}$. When $H \rightarrow \mathbf{mode} \in [1, 6]$, $L \rightarrow \mathbf{mode} \in [1, 3]$. When $H \rightarrow \mathbf{mode} = 7$, $L \rightarrow \mathbf{mode} = 3$. This accounts for $6 \times 3 + 1 = 19$ possibilities.
- *In case there are three PDSI's in the queue.* Similar to the reasoning in last item, we have $(6 \times 3 + 1) \times 3 = 57$ possibilities.
- *In case there are more than three PDSI's in the queue.* Assume the second PDSI in the queue is H' while the last second is L' . The values of $H' \rightarrow \mathbf{next}$ and $L' \rightarrow \mathbf{prev}$ depends on the modes of the third and the last third PDSI's in the queue. Also the ILM's of H' and L' also depend on the third and the last third PDSI's in the queue. Moreover, all other PDSI's will be mapped to ∞ according to lemma 3. Again, we have $(6 \times 3 + 1) \times 3 \times 3 \times 3 \times 3 = 1539$ possibilities.

Summing up all the possibilities in addition to the initial GDSI, we have $3 + 19 + 57 + 1539 + 1 = 1619$ different GDSI's in our final CIS where the “1” represents the initial GDSI which maps the initial PDSI to ∞ and everything else to zero.

Fig. 4. PDSI pattern for modified MCS algorithm

7 Conclusion

With the known worst-case complexities of most verification problems in theory, it is apparent that the current technology of model-checking is incapable of verifying nontrivial software systems. We believe such a dilemma results from the fact that current verification theory does not distinguish “good” design from “bad” design. We argue our CIS technology is a successful example to verify well-designed concurrent systems in which relations among different PDSI groups are more important than both the actual numbers of processes in each PDSI group and the actual values of all pointers. We feel hopeful our technology can be extended to verify well-designed concurrent systems with other types of infinite behaviors.

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A Original MCS Locking Algorithm

The modification follows good programming practice and can significantly reduce verification complexity. Two modifications in the modified MCS algorithm are

Fig. 5. original MCS locking algorithm

made. First, *true* and *false* for variable **locked** is interchanged in the modified version to be consistent with our initial state restrictions.

Second, local pointers are set to null as soon as the contents of the local pointers will not be used again. This is consistent with good programming practice. For example, in our modified algorithm, when a process releases the lock, it then also set its **next** to null because it is not meant in the queue already. However, in the original algorithm, this local **next** can still outdate point to some random process. Without cautious management, such “stray” local pointers can be mistakenly used.