Stability Analysis of Discrete-Time Takagi-Sugeno Fuzzy Systems

Renata Pytelková and Petr Hušek

Department of Control Engineering, Faculty of Electrical Engineering
Czech Technical University,
Technická 2, 166 27 Praha 6, Czech Republic
{pytelko,husek}@control.felk.cvut.cz

Abstract. The paper presents a method analyzing asymptotic stability of discrete-time Takagi-Sugeno fuzzy systems. It is supposed that the plant is described by the Takagi-Sugeno fuzzy system with linear state-space submodels in the consequents of rules and the controller by the Takagi-Sugeno fuzzy system with linear state feedback submodels. These fuzzy systems can be considered as linear time-varying systems, where coefficients are varying in given intervals. The method tests stability by checking only products of vertex (extreme) matrices.

1 Introduction

Stability is one of the most important issues in analysis of control systems. In the case of fuzzy systems no particular stability theory exists. In fact the fuzzy system is a kind of nonlinear system with the disadvantage of having a complex analytical description.

There exist several particular methods designed for some special case of fuzzy systems. These methods are based on methods used in nonlinear control theory, for example small gain theorem and its related techniques (the circle criterion, conicity methods etc.), qualitative methods (stability indices) and the Lyapunov stability theory, among others. These methods usually put some constraints on the controller or to the model of the controlled system.

Special case of fuzzy systems are so-called Takagi-Sugeno fuzzy systems ([4]). In fuzzy control theory Takagi-Sugeno fuzzy systems with linear state-space or input-output submodels in the consequents of rules are used. The main advantage is, that this approach (sometimes called model based fuzzy approach) provides to combine together the knowledge of linear systems and fuzzy logic and use it for stability analysis. There exist two main approaches of stability analysis of such systems.

The first one is based on finding a common Lyapunov function for all subsystems used in consequents of rules. Problem of finding of this function is solved by LMI. This method states sufficient condition of stability and more details concerning this method can be found for example in [5]. However this algorithm is unable to decide about stability in many practical situations due to its big conservatism.

Another approach can be found in [3]. In this paper it was shown, that problem of stability analysis of Takagi-Sugeno fuzzy systems with linear submodels in consequents of rules can be transformed to robust stability analysis of a polynomial with polynomial structure of its coefficients. Stability is tested by the Modified Jury or the Modified Routh criterion. It states sufficient and necessary stability conditions for slowly varying systems.

In this paper the new method for stability analysis of discrete-time Takagi-Sugeno fuzzy systems, where the plant is described by the Takagi-Sugeno fuzzy system with linear state-space submodels in the consequents of rules and the controller by the Takagi-Sugeno fuzzy system with linear state feedback submodels.

The paper is organized as follows. In section 2 the Takagi-Sugeno fuzzy model of a plant and the PDC controller is described. In section 3 the computation procedure of vertex extreme matrices of the closed-loop Takagi-Sugeno fuzzy system is introduced. Section 4 is devoted to the stability analysis method. Illustrative example is shown in the section 5 and some concluding remarks are mentioned in section 6.

2 Takagi-Sugeno Fuzzy Systems

In this section Takagi-Sugeno fuzzy systems with linear submodels in the consequents of rules will be introduced. Linear state-space submodels are supposed.

The Takagi-Sugeno fuzzy system with the state-space submodels is considered in the following form:

$$\begin{aligned} R^i : & \text{IF } z_1(t) \text{ is } M_1^i \text{ and } \dots \text{ and } z_g(t) \text{ is } M_g^i \\ & \text{THEN } \mathbf{x}(t+1) = \mathbf{A}^i \mathbf{x}(t) + \mathbf{B}^i \mathbf{u}(t) \end{aligned} \quad (1)$$

for $i = 1, 2, \dots, R$ where $\mathbf{z}(t) = [z_1(t), \dots, z_g(t)]$ is a vector of variables of the premise (some measurable system variables), M_j^i are fuzzy sets, $\mathbf{x}(t) \in \mathbb{R}^n$ is the state vector, $\mathbf{u}(t) \in \mathbb{R}^m$ is the input vector, $\mathbf{A}^i \in \mathbb{R}^{n \times n}$ are linearized system matrices and $\mathbf{B}^i \in \mathbb{R}^{n \times m}$ are linearized input matrices.

The total output of the fuzzy system is inferred as follows:

$$\begin{aligned} \mathbf{x}(t+1) &= \frac{\sum_{i=1}^R w_i(\mathbf{z}(t)) \{ \mathbf{A}^i \mathbf{x}(t) + \mathbf{B}^i \mathbf{u}(t) \}}{\sum_{i=1}^R w_i(\mathbf{z}(t))} \\ &= \sum_{i=1}^R h_i(\mathbf{z}(t)) \{ \mathbf{A}^i \mathbf{x}(t) + \mathbf{B}^i \mathbf{u}(t) \} \end{aligned} \quad (2)$$

where

$$w_i(\mathbf{z}(t)) = \prod_{j=1}^g M_j^i(z_j(t)) \quad (3)$$

$M_j^i(z_j(t))$ is the grade of membership of $z_j(t)$ in M_j^i . It is assumed that $w_i(\mathbf{z}(t)) \geq 0$, for $i = 1, 2, \dots, R$ and $\sum_{i=1}^R w_i(\mathbf{z}(t)) > 0$ for all t . Therefore

$$\sum_{i=1}^R h_i(\mathbf{z}(t)) = 1 \quad (4)$$

The PDC controller of the following form is considered:

$$\begin{aligned} R^i : & \text{IF } z_1(t) \text{ is } M_1^i \text{ and } \dots \text{ and } z_g(t) \text{ is } M_g^i \\ & \text{THEN } \mathbf{u}(t) = -\mathbf{K}^i \mathbf{x}(t) \end{aligned} \quad (5)$$

The total output of this fuzzy controller is

$$\begin{aligned} \mathbf{u}(t+1) &= \frac{-\sum_{i=1}^R w_i(\mathbf{z}(t)) \mathbf{K}^i \mathbf{x}(t)}{\sum_{i=1}^R w_i(\mathbf{z}(t))} \\ &= -\sum_{i=1}^R h_i(\mathbf{z}(t)) \mathbf{K}^i \mathbf{x}(t) \end{aligned} \quad (6)$$

By substituting (6) into (2) the total state space model of the fuzzy closed-loop system is obtained:

$$\mathbf{x}(t+1) = \sum_{i=1}^R \sum_{j=1}^R h_i(\mathbf{z}(t)) h_j(\mathbf{z}(t)) \{\mathbf{A}^i - \mathbf{B}^i \mathbf{K}^j\} \mathbf{x}(t) \quad (7)$$

3 Computation of Vertex Extreme Matrices

The presented stability analysis method is based on stability analysis of linear time-varying systems, where coefficients are varying in given intervals. The method tests stability by checking only products of vertex extreme matrices.

Let's define the system matrix of the closed-loop system obtained from (7) as follows:

$$\mathbf{G}(\mathbf{h}(\mathbf{z}(t))) = \sum_{i=1}^R \sum_{j=1}^R h_i(\mathbf{z}(t)) h_j(\mathbf{z}(t)) \{\mathbf{A}^i - \mathbf{B}^i \mathbf{K}^j\} \quad (8)$$

By substituting $h_R(\mathbf{z}(t)) = 1 - \sum_{i=1}^{R-1} h_i(\mathbf{z}(t))$ into (8), $\mathbf{G}(\mathbf{h}(\mathbf{z}(t)))$ can be expressed as:

$$\begin{aligned} \mathbf{G}(\mathbf{h}(\cdot)) &= \sum_{i=1}^{R-1} \sum_{j=1}^{R-1} h_i(\cdot) h_j(\cdot) (\mathbf{B}^j \mathbf{K}^R + \mathbf{B}^R \mathbf{K}^j - \mathbf{B}^i \mathbf{K}^j - \mathbf{B}^R \mathbf{K}^R) \\ &+ \sum_{j=1}^{R-1} h_j(\cdot) (\mathbf{A}^j - \mathbf{B}^j \mathbf{K}^R - \mathbf{A}^R - \mathbf{B}^R \mathbf{K}^j + 2\mathbf{B}^R \mathbf{K}^R) + \mathbf{A}^R - \mathbf{B}^R \mathbf{K}^R \end{aligned} \quad (9)$$

Each element $g_{ij}(\mathbf{h}(\mathbf{z}(t)))$ of the matrix $\mathbf{G}(\mathbf{h}(\mathbf{z}(t)))$ is a polynomial function of the second order of the time-varying parameter $\mathbf{h}(\mathbf{z}(t))$ and can be rewritten to the following form:

$$g_{ij}(\mathbf{h}) = \mathbf{h}^T \mathbf{C}^{(ij)} \mathbf{h} + (\mathbf{d}^{(ij)})^T \mathbf{h} + v^{(ij)} \quad (10)$$

$$\mathbf{C}^{(ij)} = [\mathbf{c}^{(ij)}] \in \mathbb{R}^{R-1, R-1}, \mathbf{d}^{(ij)} \in \mathbb{R}^{R-1}, v^{(ij)} \in \mathbb{R}$$

and

$$\mathbf{h} \in H \subset \mathbb{R}^{R-1} \quad (11)$$

$$H = \{\mathbf{h} \in H \subset \mathbb{R}^{R-1} : h_k \in [0, 1], \sum_{k=1}^{R-1} h_k \leq 1, k = 1, \dots, R-1\}$$

where $\mathbf{h}$ denotes $(\mathbf{h}(\mathbf{z}(t)))$.

The system matrix $\mathbf{G}(\mathbf{h})$ can be overbounded by the interval matrix $\mathbf{G}^I$ in the following form

$$\mathbf{G}^I = \begin{bmatrix} [g_{11}^{\min}, g_{11}^{\max}] & \cdots & [g_{1n}^{\min}, g_{1n}^{\max}] \\ \vdots & & \vdots \\ [g_{n1}^{\min}, g_{n1}^{\max}] & \cdots & [g_{nn}^{\min}, g_{nn}^{\max}] \end{bmatrix} \quad (12)$$

The problem of finding extreme values of $g_{ij}(\mathbf{h})$ on a box H is a task of mathematical programming ([1]). General formulation of a task of mathematical programming is as follows.

Let us consider the problem of minimization of a function $f_0(\mathbf{x})$, where the constraints are given in the form of inequalities

$$\min \{f_0(\mathbf{x}) \mid f_j(\mathbf{x}) \leq b_j, j = 1, \dots, m\} \quad (13)$$

Necessary conditions of extreme values can be determined by the following theorem.

Definition 1. Let a point ${}^0\mathbf{x}$ satisfy all constraints of (13). Let $J({}^0\mathbf{x})$ be the set of indices, for which the corresponding constraints are active (e.g. inequality changes to equality):

$$J({}^0\mathbf{x}) = \{j \mid f_j({}^0\mathbf{x}) = b_j\} \quad (14)$$

The point ${}^0\mathbf{x}$ is said to be a regular point of the set X given by constraints in (13), if the gradients $\nabla f_j({}^0\mathbf{x})$ are linearly independent $\forall j \in J({}^0\mathbf{x})$.

Theorem 1 (Kuhn-Tucker). Let ${}^*\mathbf{x}$ be a regular point of a set X and a function $f_0(\mathbf{x})$ has in some neighbourhood of ${}^*\mathbf{x}$ continuous first partial derivatives. If the function $f_0(\mathbf{x})$ has in the point ${}^*\mathbf{x}$ the local minimum on X , then there exists a (Lagrange) vector ${}^*\lambda \in \mathbb{R}^m$ that

$$\begin{aligned} \nabla f_0({}^*\mathbf{x}) + \sum_{r=1}^m {}^*\lambda_r \nabla f_r({}^*\mathbf{x}) &= 0 \\ {}^*\lambda_j (f_j({}^*\mathbf{x}) - b_j) &= 0 \\ {}^*\lambda_j &\geq 0 \end{aligned} \quad (15)$$

hold $\forall j = 1, \dots, m$.

Remark 1. For maximization of a function $f_0(\mathbf{x})$ the last inequality of (15) is replaced by $^*\lambda_j \leq 0$.

To apply Theorem 1 for solving our problem it is necessary to check whether the preconditions of this theorem are satisfied. As $g_{ij}(\mathbf{h})$ is a polynomial function of the second order, its first partial derivatives are continuous $\forall \mathbf{h} \in H$ and the second assumption is satisfied. In our case

$$\begin{aligned} f_0(\mathbf{h}) &= g_{ij}(\mathbf{h}) \\ f_j(\mathbf{h}) &= (-1)^{j+1} h_i \\ b_j &= 0 \text{ for } j \text{ even} \\ b_j &= 1 \text{ for } j \text{ odd} \\ f_{2R-1}(\mathbf{h}) &= \sum_{k=1}^{R-1} h_k \\ b_{2R-1} &= 1 \end{aligned} \quad (16)$$

where $i = 1, \dots, R-1$, $j = 1, \dots, 2(R-1)$, $j = 2i-1, 2i$.

Then

$$\begin{aligned} \nabla f_j(\mathbf{h}) &= (-1)^{j+1} \mathbf{e}^{(i)}, \quad \forall \mathbf{h} \in H, \quad j = 1, \dots, 2(R-1) \\ i &= \frac{j+1}{2} \text{ for } j \text{ odd } i = \frac{j}{2} \text{ for } j \text{ even} \\ \nabla f_{2R-1}(\mathbf{h}) &= [1, \dots, 1]^T \end{aligned} \quad (17)$$

where $\mathbf{e}^{(i)} = [0, \dots, 0, 1, 0, \dots, 0]^T$ with 1 on the i -th position. Because for some $\mathbf{h} \in H$ only even or only odd constraints (or none of them) can be active $\forall i = 1, \dots, R-1$, $\nabla f_j(\mathbf{h})$, are linearly independent $\forall \mathbf{h} \in H$, $j \in J(\mathbf{h})$. It means that all points $\mathbf{h} \in H$ are regular ones.

According to Theorem 1 it is necessary to determine the gradient $\nabla g_{ij}(\mathbf{h})$. The components are

$$\nabla g_{ij}(\mathbf{h}) = \left[\frac{\partial g_{ij}(\mathbf{h})}{\partial h_1} \quad \dots \quad \frac{\partial g_{ij}(\mathbf{h})}{\partial h_{R-1}} \right]^T \quad (18)$$

follows from (10)

$$\frac{\partial g_{ij}(\mathbf{h})}{\partial h_l} = 2c_{il}^{(ij)} \cdot h_l + \sum_{\substack{r=1 \\ r \neq l}}^{R-1} (c_{lr}^{(ij)} + c_{rl}^{(ij)}) \cdot h_r + d_l^{(ij)} \quad (19)$$

$$i, j, l = 1, \dots, R-1$$

After substituting (16), (17), (18) and (19) to (15) the following system of equations and inequalities is obtained:

$$\begin{bmatrix} 2c_{11}^{(ij)} & c_{12}^{(ij)} + c_{21}^{(ij)} & \cdots & c_{1,R-1}^{(ij)} + c_{R-1,1}^{(ij)} \\ c_{21}^{(ij)} + c_{12}^{(ij)} & 2c_{22}^{(ij)} & \cdots & c_{2,R-1}^{(ij)} + c_{R-1,2}^{(ij)} \\ \vdots & \vdots & \ddots & \vdots \\ c_{R-1,1}^{(ij)} + c_{1,R-1}^{(ij)} & c_{R-1,2}^{(ij)} + c_{2,R-1}^{(ij)} & \cdots & 2c_{R-1,R-1}^{(ij)} \end{bmatrix} \cdot \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_{R-1} \end{bmatrix} +$$

$$+ \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & \cdots & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 & 0 & \cdots & 0 & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & 1 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_{2R-1} \end{bmatrix} = \begin{bmatrix} d_1^{(ij)} \\ d_2^{(ij)} \\ \vdots \\ d_{R-1}^{(ij)} \end{bmatrix} \quad (20)$$

$$\begin{aligned} \lambda_1(h_1 - 1) &= 0 \\ \lambda_2 h_1 &= 0 \\ \lambda_3(h_2 - 1) &= 0 \\ \lambda_4 h_2 &= 0 \\ &\vdots \\ \lambda_{2 \cdot R-3}(h_{R-1} - 1) &= 0 \\ \lambda_{2 \cdot (R-1)} h_{R-1} &= 0 \\ \lambda_{2 \cdot R-1} \left(\sum_{k=1}^{R-1} h_k - 1 \right) &= 0 \end{aligned} \quad (21)$$

$\lambda_1, \dots, \lambda_{2 \cdot R-1} \geq 0$ for minimization

$\lambda_1, \dots, \lambda_{2 \cdot R-1} \leq 0$ for maximization

The important fact is that the equation (20) is linear. The computational procedure of solving (20), (21) runs as follows. At first all solutions of (21) (nonlinear) are determined. This corresponds to determining of all the parts of the box H - the interior and all the parts of the boundary of H (all manifolds with the dimension i , $i = 0, \dots, R-2$ containing only points on the boundary of H). Each solution of (21) corresponds to $2R-1$ linear equations (from (21) it follows that at least one of $\lambda_{2i-1}, \lambda_{2i} \forall i = 1, \dots, R-1$ has to equal zero; if $\lambda_{2i-1} = 0$ then either $\lambda_{2i} = 0$ or $h_i = 0$, if $\lambda_{2i} = 0$ then either $\lambda_{2i-1} = 0$ or $h_i = 1$, $\forall i = 1, \dots, R-1$; λ_{2R-1} can be zero anytime). These $2R-1$ equations together with $R-1$ equations of (20) form $3R-2$ linearly independent linear equations for $3R-2$ unknown variables. It means that there exists a unique solution $(*\lambda, *\mathbf{h})$ (for each solution of (21)) of (20) and (21). As the number of manifolds with the dimension i , $i = 0, \dots, R-1$ containing only points on the boundary of H is $\binom{R}{i}$, the total number n_s of all solutions of (20) and (21) is given by

$$n_s = \sum_{i=0}^{R-1} \binom{R}{i} = 2^R - 1 \quad (22)$$

In the next step is checked whether $*\lambda_j^{(l)} \geq 0$ ($*\lambda_j^{(l)} \leq 0$), $\forall j = 1, \dots, 2(R-1)$ and $*\mathbf{h}^{(l)} \in H$ for each $l = 1, \dots, n_s$. Denote by $L_{\min}(L_{\max})$ the set of l for which these conditions are satisfied.

$$\begin{aligned} L_{\min} &= \{l : *\mathbf{h}^{(l)} \in H, *\lambda_j^{(l)} \geq 0, \forall j = 1, \dots, 2(R-1)\} \\ L_{\max} &= \{l : *\mathbf{h}^{(l)} \in H, *\lambda_j^{(l)} \leq 0, \forall j = 1, \dots, 2(R-1)\} \end{aligned} \quad (23)$$

Then

$$\begin{aligned} g_{ij}^{\min}(\mathbf{h}) &= \min_{l \in L_{\min}} (g_{ij}(*\mathbf{h}^{(l)})) \\ g_{ij}^{\max}(\mathbf{h}) &= \max_{l \in L_{\max}} (g_{ij}(*\mathbf{h}^{(l)})) \end{aligned} \quad (24)$$

Now let's define the set of extreme matrices $\mathbf{G}^E \subset \mathbf{G}^I$ such that

$$\mathbf{G}^E = \{[g_{ij}^E] \mid g_{ij}^E = g_{ij}^{\min} \text{ or } g_{ij}^{\max}; i, j = 1, \dots, R\} \quad (25)$$

Obviously number of elements of $\mathbf{G}^E$ is 2^p , where $p = n^2$ element of $\mathbf{G}^E$ will be referred as $\mathbf{G}_j^E$ with $j \in \{1, \dots, 2^p\}$.

Let denote the set of all products of extreme matrices of length k by

$$\overline{\mathbf{P}}_k^E = \{\mathbf{P}_k^E(i_1, \dots, i_k) = \mathbf{G}_{i_1}^E \cdot \dots \cdot \mathbf{G}_{i_k}^E \mid (i_1, \dots, i_k) \in \{1, \dots, 2^p\}^k\} \quad (26)$$

where $\{1, \dots, 2^p\}^k$ is the k -th cross product of $\{1, \dots, 2^p\}$. Therefore the number of elements of $\overline{\mathbf{P}}_k^E$ is 2^{pk} .

4 Stability Analysis

Let us define ([2]) the following theorem and lemma:

Theorem 2. $\|\prod_{n=1}^k \mathbf{G}(n)\| < 1$ for all choices of $\mathbf{G}(n) \in \mathbf{G}^I$, $n = 1, \dots, k$, iff $\|\mathbf{P}_k^E(i_1, \dots, i_k)\| < 1$ for all $\mathbf{P}_k^E(i_1, \dots, i_k) \in \overline{\mathbf{P}}_k^E$.

The symbol $\|\cdot\|$ denotes the induced matrix 1-norm or the ∞ -norm.

Lemma 1. Let $\mathbf{G}(n) \in \mathbf{G}^I$. Then the time-variant Takagi-Sugeno fuzzy system (7) is asymptotically stable if there exists a finite k such that $\|\prod_{n=1}^k \mathbf{G}(n)\| < 1, \forall \mathbf{G}(n) \in \mathbf{G}^I, n = 1, \dots, k$.

The theorem together with the lemma can be used to formulate the following corollary:

Corollary 1. The time-variant Takagi-Sugeno fuzzy system (7) is asymptotically stable if there exists a finite k such that $\|\mathbf{P}_k^E(i_1, \dots, i_k)\| < 1, \forall (i_1, \dots, i_k) \in \{1, \dots, 2^p\}^k$

Proofs of the theorem, lemma and corollary can be found in [2].

Stability test algorithm

1. Compute the interval matrix $\mathbf{G}^I$
2. Set $k = 1$, compute the set of extreme matrices $\mathbf{G}^E$
3. Generate all 2^{pk} products $\mathbf{P}_k^E(i_1, \dots, i_k)$ and compute the norm of all products.
4. If all norms are smaller than 1, the system is stable. In other case set $k = k+1$ and repeat step 3.

5 Example

Define the Takagi-Sugeno fuzzy system described by two following rules:

$$R^1 : \text{IF } x_1(t) \text{ is } M_1^1 \text{ THEN } \mathbf{x}(t+1) = \mathbf{A}^1 \mathbf{x}(t) + \mathbf{B}^1 \mathbf{u}(t)$$

$$R^2 : \text{IF } x_1(t) \text{ is } M_1^2 \text{ THEN } \mathbf{x}(t+1) = \mathbf{A}^2 \mathbf{x}(t) + \mathbf{B}^2 \mathbf{u}(t)$$

and define controller described by the following rules:

$$R^1 : \text{IF } x_1(t) \text{ is } M_1^1 \text{ THEN } \mathbf{u}(t) = \mathbf{K}^1 \mathbf{x}(t)$$

$$R^2 : \text{IF } x_1(t) \text{ is } M_1^2 \text{ THEN } \mathbf{u}(t) = \mathbf{K}^2 \mathbf{x}(t)$$

where

$$\mathbf{A}^1 = \begin{bmatrix} -0.2 & 0.4 \\ 0.5 & 0.5 \end{bmatrix} \mathbf{B}^1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \mathbf{K}^1 = \begin{bmatrix} 0.4 & 0.3 \end{bmatrix}$$

$$\mathbf{A}^2 = \begin{bmatrix} -0.4 & 0.2 \\ 0 & 0.5 \end{bmatrix} \mathbf{B}^2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \mathbf{K}^2 = \begin{bmatrix} -0.2 & 0.1 \end{bmatrix}$$

The interval matrix $\mathbf{G}^I$ is

$$\mathbf{G}^I = \begin{bmatrix} [-0.6, -0.2] & [0.1, 0.1] \\ [0.1, 0.2] & [0.2, 0.4] \end{bmatrix}$$

In the following table the extreme product norms are shown:

Table 1. Extreme product norms

k	$\max \ \cdot\ _1$	$\max \ \cdot\ _\infty$
1	0.47	0.42

Table 1 shows that the Takagi-Sugeno system is stable.

6 Conclusions

In the paper a new method for stability analysis of discrete-time Takagi-Sugeno fuzzy systems is presented. Linear state space submodels are supposed in rule consequents of the model of the plant and the controller is described by the linear state feedback submodels (so called PDC controller). The method states a sufficient condition of stability and it is based on the stability analysis of time-varying systems, where the system matrix is varying in given interval. This condition allows to test stability by checking only products of extreme matrices. This condition is simple to implement, but the computational complexity increases exponentially with the the dimension n .

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