Orthogonal and Straight-Line Drawings of Graphs with Succinct Representations^{*}

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The concept of a *hierarchical design* has played an important role in various areas of computer science and engineering, including software engineering, CAD, among others. Graphs with hierarchical structures, which are capable of describing large-scale regular structures in a succinct manner, have naturally become an interesting and important modeling tool for facilitating such a hierarchical design. In the literature, a number of succinct models for representing graphs with hierarchical structures have been proposed. See, e.g., [4,5]. In each of such hierarchical graph models, a succinct description is capable of representing a graph (which can be thought of as the *expansion* of the description) whose size is exponential in the length of the description.

A *hierarchical graph* [4,5] $\Gamma = (G_1, \dots, G_k)$ of *depth* k contains k *cells* $G_1, \dots, G_k$, each of which is a graph consisting of two types of vertices, namely, *terminals* and *nonterminals*. Intuitively speaking, nonterminals are those that are going to be replaced by cells of smaller indices during the expansion process. Suppose $G_i = (V_i, E_i)$, $1 \leq i < k$, where V_i and E_i are the sets of vertices and edges of G_i , respectively. For each G_i , there is a *pin assignment function* $\rho_i : \{1, \dots, d_i\} \rightarrow V_i$ (where d_i is a positive integer), which specifies the way vertices are connected to the upper layer. The d_i is called the *degree* of G_i , and each of the vertices in $\rho_i(\{1, \dots, d_i\})$ is called a *frontier vertex* (*f-vertex*, for short). Each nonterminal inside G_i is specified as (m, G_j) where m (an integer) represents the unique *name* of the nonterminal, and G_j , $1 \leq j < i$, denotes the *type* of the nonterminal. A nonterminal of type G_j has degree d_j , and each incident edge is labeled by a unique integer in $\{1, \dots, d_j\}$. The *expansion* of G_i , denoted by $E(G_i)$, is obtained by expanding all of its subcells $G_1, \dots, G_{i-1}$ recursively, and then replacing each nonterminal of type G_j ($1 \leq j < i$) inside G_i by a copy of $E(G_j)$ in such a way that each incident edge labeled l ($1 \leq l \leq d_j$) is connected to the f-vertex $\rho_j(l)$. The *expansion* of a hierarchical graph $\Gamma = (G_1, \dots, G_k)$, denoted by $E(\Gamma)$, is defined to be $E(G_k)$.

In this research, our goal is to ‘draw’ hierarchical planar graphs on *grids* in the styles of *straight-line drawing* (i.e., each edge is a straight-line segment) and *orthogonal drawing* (i.e., each edge is a chain of horizontal and vertical segments) [2]. Since the size of the expansion of a hierarchical graph can be exponential in the length of its succinct representation, care must be taken when defining

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the output of a graph drawing algorithm. In our setting, the outputs of our drawing algorithms are succinct representations of drawings and not of drawings themselves. (That is, we require that the drawing of a hierarchical graph be expressible succinctly as well.) To this end, the drawings of two copies of the same cell must be ‘identical’ in the sense that one drawing can be obtained by the operations of *flipping* with respect to the y-axis and/or *rotations* of 90° , 180° or 270° from the other. A hierarchical graph is said to have a *planar* straight-line (resp., orthogonal) grid drawing if its expansion can be drawn on grids in a straight-line (resp., orthogonal) fashion without edge crossings subject to the above constraints regarding copies of each cell. It should be noted that our *hierarchical graph model* differs from that of ‘hierarchical graphs’ used in, e.g., [3]. The latter refers to graphs whose vertices are assigned to layers, and the so-called ‘hierarchical drawing’ is to place all the vertices of a hierarchical graph on a set of equally-spaced horizontal lines.

The main contributions of this paper include the design and analysis of orthogonal and straight-line drawing algorithms which operate on the succinct descriptions of hierarchical graphs directly, and output succinct representations of drawings. (For related results, the reader is referred to [1].) We do not require that the hierarchical graphs be expanded in order for our algorithms to work. To the best of our knowledge, conventional graph drawing algorithms operate only on completely specified graphs. For hierarchical graphs $(G_1, \dots, G_k)$ with the number of outgoing connections in each G_i bounded by 2, we derive a planar straight-line grid drawing algorithm which runs in time $\sum_{i=1}^k |G_i|$, where $|G_i|$ denotes the number of vertices in G_i . (Notice that $\sum_{i=1}^k |G_i|$ is linear in the size of the succinct representation.) The drawing area of the expanded graph is bounded by $O(2^{3k} \prod_{i=1}^k (|G_i| - 2)^6)$. For orthogonal grid drawings of hierarchical planar graphs, we present an algorithm which runs in $O(\sum_{i=1}^k |G_i|^2)$ time, provided that the input graph is 2-connected (i.e., a graph which remains connected even if a single vertex is removed). The drawing area and the total number of bends of the expanded graph are bounded by $O(n^2)$ and $(\max_{i=1, \dots, k} \{|G_i|\})^k$, respectively, where n is the number of vertices in the expanded graph. Our algorithm can also be used to report whether the input graph exhibits a planar straight-line (or orthogonal) grid drawing.

References

1. F. Brandenburg, Designing Graph Drawings by Layout Graph Grammars, *Graph Drawing'94*, LNCS 894, pp. 416-427, 1994.
2. G. Di Battista, P. Eades, R. Tamassia, and I. Tollis, *Graph Drawing: Algorithms for the Visualization of Graphs*, Prentice-Hall, 1999.
3. P. Eades, X. Lin, and R. Tamassia, An Algorithm for Drawing a Hierarchical Graph, *Internat. J. Comput. Geom. Appl.*, Vol. 6, pp. 145-156, 1996.
4. T. Lengauer, Efficient Algorithms for Finding Minimum Spanning Forests of Hierarchically Defined Graphs, *J. Algorithms*, Vol. 8, pp. 260-284, 1987.
5. T. Lengauer, Hierarchical Planarity Testing Algorithms, *J. ACM*, Vol. 36, No. 3, pp. 474-509, 1989.