BROADCAST INTERACTIVE PROOFS

(Extended Abstract)

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Abstract

In this paper we extend the notion of (single-verifier) interactive zero-knowledge proofs to (multi-verifier) broadcast proofs. In our scheme the prover broadcasts messages to many verifiers simultaneously. We consider two cases: one for which the number of rounds of messages exchanged is unbounded (as a function of the length of the common input x), and one for which it is constant. Compared to repeated single-verifier proofs (one proof for each verifier), the saving in broadcast bits is of the order of the number of verifiers in the first case, provided there are enough verifiers. More precisely, if the number of verifiers exceeds $\log |x|$ then there is “practically” no extra cost in broadcast bits by further increasing the number of verifiers. In the second case the saving in the number of rounds is “practically” $|x|/\log |x|$. An added feature of broadcast proofs of the second type is that they are *sabotage-free*.

Our scheme makes use of a *network* which directs the messages of the verifiers to the prover. The universality of the scheme derives from the way in which the network handles collisions.

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1 Introduction

Suppose that an authority (say the president of the U.S.A.) wants to prove something to a large number of verifiers. The proof can be related, for example, to a command or some action. One obvious solution would be to prove it separately to each verifier, sequentially. This solution however could be very time consuming, certainly when the number of verifiers is large. Another solution is to perform all the proofs in parallel. However in both cases the number of bits communicated could be enormous. Our solution is completely different. The prover will use a broadcast station and the verifiers will call in their questions. We shall take into account aspects such as the time it will take to run the protocol and the *bandwidth*, and the total number of bits that must be broadcast, which must be small. For us broadcasting is expensive and phone calls are cheap. Furthermore we shall use a network which reduces substantially the number of operators required to answer incoming calls.

When the same message is sent to many receivers broadcasting is the logical solution. Recently broadcast has been receiving a lot of attention in the research on cryptography. Fiat discussed the protection of privacy and selective broadcast [7] and Desmedt-Yung discussed the issue of authenticating broadcasted messages [6]. In this paper we discuss how broadcasting can simplify the prover's effort when multi verifiers want to check interactively a zero-knowledge proof.

In our model the prover is P and the verifiers are $V_1, \dots, V_v$. P broadcasts to all the verifiers. Each V_i has a one-directional line to a (telephone) network W . P has n incoming lines originating from W . These lines are "read-only". The prover P communicates his messages to the V_i by broadcasting them, whereas the V_i will "phone-in" their "queries" through the network W . Each verifier selects a particular line of the prover. W directs incoming calls to P on a "first-come/first-serve" basis (and chooses randomly if there is a collision).

Our proof is a *multi-verifier* interactive zero-knowledge proof. Multi-prover interactive proofs were introduced in [3] in a theoretical context. We are mainly concerned here with practical aspects, and our approach is completely different. Multi-language zero-knowledge proofs were recently dis-

cussed in [12].

We remark that existing non-interactive zero-knowledge proofs, such as [4, 5], are not practical and many signature schemes are not zero-knowledge.

This note is organized as follows. In Section 2 we consider a formal setting for broadcast proofs. *We recommend that the reader who is satisfied with the informal model above should omit this section in a first reading.* In Section 3 we present a “sequential” broadcast zero-knowledge proof. In Section 4 we present a constant-round zero-knowledge broadcast proof. which is sabotage-free.

2 A formal setting: definitions and notations

For a formal setting we use the framework in [9, pp. 4–10].

Definition 1 A Turing machine which is equipped with a read-only input tape, a work tape, a random tape, a history tape, i read-only communication tapes and j write-only communication tapes, is called an (i, j) -*interactive Turing machine*.

Observe that the interactive Turing machines in [9] have *one* read-only communication tape and *one* write-only communication tape. Below we use the abbreviations, ITM for an “interactive probabilistic Turing machine”, and MITM for an (i, j) -interactive probabilistic Turing machine, when there is no ambiguity about i and j .

Definition 2 A *broadcast interactive protocol* $(P, W, V_1, \dots, V_v)$ consists of v ITM’s $V_1, \dots, V_v$, one $(n, 1)$ -ITM and one (v, n) -ITM, denoted by P and W respectively, which are such that:

- P and $V_1, \dots, V_v$ share a common communication tape which is write-only for P and read-only for all the V_i . When P writes a string on this tape, we say that P *broadcasts* that string.
- P and W share n communication tapes which are read-only for P and write-only for W . We call these tapes: *incoming phone lines*.

- W and $V_1, \dots, V_v$ share v communication tapes which are read-only for W and write-only for $V_1, \dots, V_v$. We call these: *outgoing phone lines*.

W is the network. Its purpose is to direct incoming calls from the V_i to P and to prevent “collisions”: if more than one call is addressed to any particular line, then W selects randomly which verifier receives the line.

Let $(P, W, V_1, \dots, V_v)$ be a broadcast interactive protocol, x be the input, and σ be the concatenation of the strings written on the communication tapes by $P, W, V_1, \dots, V_v$ and the random coin tosses of $V_1, \dots, V_v$. We say that $(P, W, V_1, \dots, V_v)$ outputs σ on input x . $(P, W, V_1, \dots, V_v)(x)$ is the probability space which assigns to σ the probability that $(P, W, V_1, \dots, V_v)$ outputs σ on input x . $\text{Joint-View}_{(P, W, V_1, \dots, V_v)}(x)$, is the joint probability space generated by $(P, W, V_1, \dots, V_v)(x)$. Below we will use the notation $\tilde{P}$ or $\tilde{V}_i$ to indicate that P or V_i do not necessarily adhere to the program of $(P, W, V_1, \dots, V_v)$. We use the notation P, V_i only for participants who adhere to the program. We call such participants *honest*. We shall assume that the network W is *not corrupted*. In other words, the *anonymity* of the callers is guaranteed. So a dishonest prover cannot single out a particular verifier.

Definition 3 Let $L \subset \{0, 1\}^*$ be a language, $(P, W, V_1, \dots, V_v)$ a broadcast interactive protocol, x the common input. We say that $(P, W, V_1, \dots, V_v)$ is a *broadcast proof* for L if,

- *Completeness* (If $x \in L$ and if all the verifiers are honest then every verifier will accept the proof.) For any c , for all sufficiently large x , if $x \in L$ then $\forall i : \text{Prob}(V_i \text{ accepts } x) \geq 1 - |x|^{-c}$ (overwhelming).
- *Soundness* (If $x \notin L$ then an honest verifier will not accept.) For any c , for all sufficiently large x , for any dishonest proof system $(\tilde{P}, W, \tilde{V}_1, \dots, \tilde{V}_v)$: if $x \notin L$ then for each *honest* $\tilde{V}_j : \text{Prob}(\tilde{V}_j \text{ accepts } x) \leq |x|^{-c}$ (negligible).

Definition 4 A broadcast proof $(P, W, V_1, \dots, V_v)$ for L is *zero-knowledge* if it is possible to simulate the joint-view of the verifiers when $x \in L$. Formally, for any $x \in L$, for any ITM's $\tilde{V}_1, \dots, \tilde{V}_v$ (possibly cheating), for

any “histories” $\widetilde{H}_1, \dots, \widetilde{H}_v$, there exist a probabilistic Turing machine M running in expected polynomial time, such that the family of probability spaces $\{M(x; \widetilde{H}_1, \dots, \widetilde{H}_v)\}$ is indistinguishable from the family of probability spaces $\{\text{Joint-View}_{(P, W, \widetilde{V}_1, \dots, \widetilde{V}_v)}(x; \widetilde{H}_1, \dots, \widetilde{H}_v)\}$. As in the case of single verifier proofs we have *perfect, statistical, computational* zero-knowledge proofs corresponding to the level of distinguishability [9].

Definition 5 Let $(P, W, V_1, \dots, V_v)$ be broadcast proof for L . It is *sabotage-free* if

- For any c , for all sufficiently large x , for any dishonest proof system $(\tilde{P}, W, \tilde{V}_1, \dots, \tilde{V}_v)$, with $\tilde{P} = P$: if $x \in L$ then for every *honest* $\tilde{V}_j$: $\text{Prob}(\tilde{V}_j \text{ accepts } x) \geq 1 - |x|^{-c}$.

Remarks

- (i) For us, v and n are functions of the input length $|x|$, and v is polynomially bounded. Our formal model does not allow for this. In the final paper we will present a model which solves this problem.
- (ii) One may wonder why we have introduced the sabotage-free aspect. Completeness implies that if $x \in L$, V_j accepts x *only* if all the other verifiers are honest. Because of the way our network is set up, it is not excluded that an honest prover will fail. When $x \in L$, dishonest verifiers could conspire against a honest verifier such that the last one will never accept.
- (iii) Our model implicitly assumes that a dishonest verifier cannot eavesdrop on the questions asked by honest verifiers. To enforce this, privacy protection could be used when necessary.
- (iv) We have avoided to use a model which is too restrictive. On the n phone lines at the prover’s premises many phone calls could be received before the prover answers, or the prover could just answer the first one, or he could wait a certain time and then answer when no further calls are received. Our model allows for this.

3 Broadcast proofs: A first approach

3.1 An introduction

We first present a new “single-verifier” zero-knowledge proof on which we will build our scheme. This proof is based on concepts proposed by Fiat-Shamir [8] and Guillou-Quisquater [10].

Let a, b , be constants, $0 < a < b$. The common input is $x = (I, u, N)$, where N be the product of large prime numbers, $I \in Z_N^*$, and u is a prime with $|N|^a \leq u \leq |N|^b$, where $|N|$ is the binary length of N . Observe that $|N| = \Theta(|x|)$. The prover proves that there exists an s such that: $Is^u \equiv 1 \pmod{N}$. We call s the “secret”. Let $n = \lfloor \log_2 u \rfloor$ and $s_1 = s, s_2 = s^2, \dots, s_n = s^{2^{n-1}}, I_1 = I, I_2 = I^2, \dots, I_n = I^{2^{n-1}}$. We assume that P and V have calculated all the I_j from I and N before the protocol starts.

Observe that one generally assumes that it is hard to decide if a number I has a u -th root modulo N [1]. No correlation between the complexity of this problem and the choice for u have been found so far. Therefore it is reasonable to restrict u to be a prime with $|N|^a \leq u \leq |N|^b$. In the final paper we shall exhibit a protocol for a more general setting.

The protocol¹ has as follows.

First V verifies if u is a prime and if $|N|^a \leq u \leq |N|^b$. Then, repeat $t = |x|$ times:

Step 1 P sends V the number $z \equiv r^u \pmod{N}$ where $r \in_R Z_N^*$.

Step 2 V sends P the bit string $(b_1, \dots, b_n) \in_R \{0, 1\}^n$ as a query.

Step 3 P halts the protocol if the query is not of the correct form; otherwise P sends V the number $y \equiv r \cdot \prod_{b_j=1} s_j \pmod{N}$.

Step 4 The verifier V checks that $y \in Z_N^*$ and that $z \equiv y^u \cdot \prod_{b_j=1} I_j \pmod{N}$.

If this is not so the protocol is halted.

If V has not halted the protocol then V accepts x .

¹In the protocol, “ $\in_R$ ” indicates a random selection with uniform distribution, independently of earlier choices.

Lemma 1 *The protocol above defines a zero-knowledge proof of membership for the language $L = \{(I, u, N) \mid u \text{ is a prime, } |N|^a \leq u \leq |N|^b, I \in (Z_N^*)^u\}$.*

Proof. The proof is straightforward and will be given in the final paper. $\square$

3.2 A broadcast proof

We will use the protocol above to describe a broadcast proof. The main difference is that we assign to each s_j one incoming phone line. We assume that P and all the V_i have calculated the I_j from I and N before the protocol starts. The network in this case operates, essentially, on a “first-come/first-serve” basis. However the caller does not know if his call has been accepted. The prover allows some time between broadcasts for the verifiers to make their calls. The broadcast protocol has as follows.

Repeat T times:

Step 1 P sets $f_i^J = 0$ for $i = 0, 1$ and $J = 1, \dots, n$. P broadcasts the number $z \equiv r^u \pmod{N}$ where $r \in_R Z_N^*$.

Step 2 Each V_i , $i = 1, \dots, v$, sends: W a line number $J_i \in_R \{1, \dots, n\}$ and P (via W) the query $b_{J_i} \in_R \{0, 1\}$. P reads (simultaneously during this step) the incoming queries (scanning line after line). If on some line J , the incoming query $q \notin \{0, 1\}$ then P ignores it, else $f_q^J = f_q^J + 1$.

Step 3 P broadcasts the number $y \equiv r \cdot \prod_{\bar{b}_j=1} s_j \pmod{N}$ where $\bar{b}_j$ have been randomly selected by P such that $\text{Prob}(\bar{b}_j \text{ is selected}) = f_{\bar{b}_j}^j / (f_0^j + f_1^j)$ and in case that both f_0^j and f_1^j are zero $\bar{b}_j$ is chosen randomly with uniform probability distribution.

Step 4 Each V_i checks that $y \in Z_N^*$ and $z \equiv y^u \cdot I_1^{b_{J_i}} \dots I_{J_i}^{b_{J_i}} \dots I_n^{b_n} \pmod{N}$, by using his own b_{J_i} and by allowing all possible values for the other queries $b_j \in \{0, 1\}$ (this requires only $2^{n-1} < u \leq |N|^b = \Theta(|x|^b)$ operations). If one of these verifications is successful, then the verifier V_i performs: **success:=success+1**.

Regardless of the outcome, the protocol is repeated for a maximum of T times. At the end of the protocol a verifier will accept if and only if $\text{success} \geq t$, where $t = |x|$ is the number of iterations of the single verifier protocol.

3.3 Theorem and proof

Theorem 1 *Suppose that:*

- (i) $n = O(\log |x|)$,
- (ii) $d \geq n/v \geq b > 0$, where $d = O(\log |N|)$ and $b = 8 \cdot |x|^{-\frac{1}{4}}$.
- (iii) $T = t/(p(1 - c))$ is an integer, where $p = 1/2 + b(1 - e^{-1})/2$ and c is such that $c > t^{-\frac{1}{4}}$ and

$$2p(1 - 2c) > 1. \quad (1)$$

If the number of iterations of the broadcast protocol is (precisely) T , then $(P, W, V_1, \dots, V_v)$ is a zero-knowledge broadcast proof for the language $L = \{(I, u, N) \mid u \text{ is a prime, } |N|^a \leq u \leq |N|^b, I \in (Z_N^)^u\}$.*

Remarks

- (i) A crude approximation for T is $2t$. So the honest prover can prove that $x \in L$ by “running twice” the single-verifier protocol, “irrespectively” of the number of verifiers.
- (ii) Observe that for the broadcast proof of Section 3.2 *each* verifier asks only a *one* bit question, while in the single-verifier proof of Section 3.1 *the* verifier asks an n -bit question.
- (iii) In the final paper we will generalize Theorem 1. The main difference is that the queries b_i of the verifiers V_i are selected from a set $\{1, \dots, q\}$, where $q \geq 2$.

Proof of Theorem 1. (It is suggested to skip this in a first reading.)

We shall require the following lemmas:

Lemma 2 *If the prover P is honest and if all the verifiers are honest, then the probability that the query of a verifier will be answered correctly is*

$$\bar{p} = \frac{1}{2} + \frac{n}{2v} (1 - (1 - n^{-1})^v). \quad (2)$$

Proof. Will be given in the full paper. □

Lemma 3 $\bar{p} \geq p$.

Proof. Will be given in the full paper. □

Lemma 4 *Suppose that $x \notin L$. If the honest verifier V_i asks the query b_i , then the probability $\tilde{p}$ that a (cheating) prover $\tilde{P}$ (any $\tilde{P}$) can answer this query correctly is no better than $1/2$.*

Proof. Will be given in the full paper. □

Lemma 5 *We can choose the c in (1) and in such a way that $c > t^{-\frac{1}{t}}$ and $T = t/(p(1 - c))$ is an integer.*

Proof. Will be given in the full paper. □

We can now prove the main theorem.

Completeness (*When all the verifiers are honest, the honest prover will answer correctly every verifier, for at least t iterations out of T , with overwhelming probability.*) We shall use Okamoto's bound [11, p. 69] for the tail probabilities of the binomial distribution. Let $\bar{z}$ be the number of times in T iterations that the honest prover P will answer correctly the query of an honest verifier V . Then $\bar{p}$ is the expected value of $\bar{z}/T$ and,

$$\text{Prob}\{|\bar{z}/T - \bar{p}| \geq \varepsilon\} < \exp[-2T\varepsilon^2], \text{ for any } \varepsilon > 0. \quad (3)$$

We take $\varepsilon = cp$. By Lemma 5, $T\varepsilon^2 = tc^2p/(1 - c) \geq wt^{\frac{1}{t}}$, w a constant, since $p > 1/2$. We are assuming that $t = |x|$, so e^{-t} is negligible. Thus $\exp[-2T\varepsilon^2]$ is negligible, and from (3), $\text{Prob}\{\bar{z} \leq \bar{p}T - \varepsilon T\}$ is negligible. By Lemma 5, $t = pT(1 - c) = pT - cpT = pT - \varepsilon T \leq \bar{p}T - \varepsilon T$, since $\bar{p} \geq p$ by Lemma 3. So $\text{Prob}\{\bar{z} \leq t\}$ is negligible, and therefore $\text{Prob}\{\bar{z} \geq t\}$ is overwhelming.

Soundness (*The probability that a dishonest prover will answer correctly for t or more iterations out of T is negligible.*) In this case $x \notin L$, but the prover $\tilde{P}$ wants to make an honest verifier V accept the proof. From Lemma 4 the probability $\tilde{p}$ that $\tilde{P}$ will succeed in having success=1 after the first round is at most $1/2$. If $\tilde{z}$ is the number of times out of T that $\tilde{P}$ will increase success then, as in the previous case, $\text{Prob}\{\tilde{z}/T - \tilde{p} \geq \epsilon\} = \text{Prob}\{\tilde{z} \geq \tilde{p}T + \epsilon T\}$ is negligible for $\epsilon = cp$. Indeed, $\tilde{p}T + \epsilon T = \tilde{p}T + cpT \leq (\frac{1}{2} + cp)T < p(1-c)T = t$, by (1). So $\text{Prob}\{\tilde{z} \geq t\}$ is negligible.

Zero-Knowledge (*If the prover is honest then it is possible to simulate the joint-view of all the verifiers.*) This is straightforward, since the number of queries is polynomially bounded. Indeed, $2^n < u \leq |N|^b = \Theta(|x|^b)$. $\square$

Remark

The number of users v of the broadcast proof can be as much as $nb^{-1} = \frac{1}{8}|x|^{\frac{1}{4}} \log |x|$. We can get v to be linear in $|x|$ if we take the number of rounds to be $t = c(|x|/\log |x|)^4$, c a constant.

4 A broadcast proof based on the constant round proof of Bellare-Micali-Ostrovsky

4.1 Introduction

Recently Bellare, Micali and Ostrovsky proposed a 5 step perfect zero-knowledge proof for graph isomorphism [2]. Their protocol can be used to design a sabotage-free broadcast proof. Below we present a variation of this protocol. This is a 5 step zero-knowledge proof for quadratic residuosity². The common input is $x = (I; N)$, where $Is^2 \equiv 1 \pmod{N}$ and N is the product of two large primes. We define $I_i \equiv I \pmod{N}$ and $s_i \equiv s \pmod{N}$, $i = 1, \dots, k$. Here $k = |x|$.

²Bellare-Micali-Ostrowski [2] mention that they have a 5 step perfect zero knowledge interactive proof for quadratic residuosity, but do not describe their protocol. We presume that it is similar to the one given here.

4.2 A 5 step zero-knowledge interactive proof for quadratic residuosity

Step 1 P sends V the number $\chi \equiv \rho^2(\text{mod } N)$ where $\rho \in_R Z_N^*$.

Step 2 V sends P the numbers $\beta_i \equiv \mu_i^2 \chi^{q_i}(\text{mod } N)$ where $\mu_i \in_R Z_N^*$ and $q_i \in_R \{0, 1\}$, $i = 1, \dots, k$.

Step 3 P sends V the numbers $z_i \equiv r_i^2(\text{mod } N)$ where $r_i \in_R Z_N^*$, $i = 1, \dots, k$.

Step 4 V sends P the bits q_i and the numbers μ_i for $i = 1, \dots, k$.

Step 5 P checks that the q_i are bits, that $\mu_i \in Z_N^*$, and that $\beta_i \equiv \mu_i^2 \chi^{q_i}(\text{mod } N)$ for $i = 1, \dots, k$. The protocol is halted if this is not so. Otherwise P sends V the numbers ρ and $y_i \equiv r_i s_i^{q_i}(\text{mod } N)$, for $i = 1, \dots, k$.

Step 5' V checks that the ρ and y_i belong to Z_N^* , that $\chi \equiv \rho^2(\text{mod } N)$, and that $z_i \equiv y_i^2 I_i^{q_i}(\text{mod } N)$ for all $i = 1, \dots, k$. If this is the case he accepts, otherwise he rejects. (V did not send).

4.3 A 5 step zero-knowledge sabotage-free broadcast proof for quadratic residuosity

The broadcast proof is essentially a parallelized version of the Fiat-Shamir scheme, but its novelty is that it is zero-knowledge in 5 rounds. (In the final paper we will present an adjusted version for which there is a substantial saving in broadcast bits.) Below we use the notation $g(|x|) \prec h(|x|)$ to indicate that the functions g, h of $|x|$ are bounded by $\lim_{|x| \rightarrow \infty} g(|x|) / h(|x|) = 0$.

Let h be such that $\log |x| \prec h \prec |x| / \log |x|$ and let $n = k/h$. (The case when h is *not* a factor of $|x|$ can be easily accommodated by taking $k = \Theta(|x|)$; we shall do this in the final paper.) We partition the secrets into subsets

$$(s_{11}, \dots, s_{1h}), \dots, (s_{i1}, \dots, s_{ih}), \dots, (s_{n1}, \dots, s_{nh}).$$

Each subset is assigned to an incoming phone line. So the number of queries per line, $q = 2^h$, is superpolynomial. Then the probability with

which a dishonest prover can “guess” the correct answer, $1/q$, is negligible. The public numbers are

$$(I_{11}, \dots, I_{1h}), \dots, (I_{i1}, \dots, I_{ih}), \dots, (I_{n1}, \dots, I_{nh}).$$

For the broadcast protocol we use a “first-come, first-serve” network. In this case the *lines are exclusive and the verifiers know when their call is selected*. In what follows we only consider verifiers which know that they have been selected. So our network model is different from the one we used earlier. We assume that if a verifier’s call is selected in Step 2, then the line is reserved for the remaining 3 steps. The protocol has as follows:

- Step 1** P sends all the V_i , $i = 1, \dots, v$, the number $\chi \equiv \rho^2 \pmod{N}$ where $\rho \in_R Z_N^*$.
- Step 2** Each V_i sends: W the line-number $J_i \in_R \{1, \dots, n\}$, and P (via W) $\beta_{J_i1}, \dots, \beta_{J_in} \pmod{N}$, where $\beta_{J_im} \equiv \mu_{J_im}^2 \chi^{q_{J_im}} \pmod{N}$ with $\mu_{J_im} \in_R Z_N^*$ and $q_{J_im} \in_R \{0, 1\}$, $m = 1, \dots, h$.
- Step 3** For each line $J \in \{1, \dots, n\}$, P selects a verifier $V_l \in_R \{V_j \mid V_j \text{ has selected line } J\}$ and broadcasts all these l . Then P broadcasts the numbers $z_i \equiv r_i^2 \pmod{N}$ where $r_i \in_R Z_N^*$, $i = 11, \dots, nh$.
- Step 4** Each acknowledged V_l sends P the bits $q_{J_l1}, \dots, q_{J_lh}$ and the numbers $\mu_{J_l1}, \dots, \mu_{J_lh}$.
- Step 5** P checks that the q_j are bits, that $\mu_j \in Z_N^*$ and that $\beta_j \equiv \mu_j^2 \chi^{q_j} \pmod{N}$, $j = 11, \dots, nh$. P broadcasts ρ and Y_j where: $Y_{J_i1} = 0, \dots, Y_{J_ih} = 0$ if any one of these checks has failed for $j = J_im$; otherwise $Y_j \equiv r_j s_j^{q_j} \pmod{N}$.
- Step 5'** Each acknowledged V_i checks that the ρ and Y_{J_im} belong to Z_N^* , that $\chi \equiv \rho^2 \pmod{N}$, and that $z_{J_im} \equiv Y_{J_im}^2 \cdot I_{J_im}^{q_{J_im}} \pmod{N}$, $m = 1, \dots, h$. If this is the case he accepts, otherwise he rejects.

Theorem 2 *The protocol described above defines a 1-call sabotage-free broadcast zero-knowledge proof $(P, W, V_1, \dots, V_v)$ for quadratic residuosity.*

Proof. **Completeness** (The honest prover will answer correctly, with overwhelming probability, the first call of every verifier, even when all the

other verifiers cheat.) This follows directly from the completeness of the single verifier protocol.

Soundness (*The probability that a dishonest prover will answer correctly a single call of any honest verifier is negligible.*) This follows from the fact that the number of queries per line is superpolynomial. Consequently a dishonest prover can only guess the answer with a negligible probability of success.

Zero-Knowledge (*If the prover is honest then it is possible to simulate the joint-view of all the verifiers.*) This follows immediately from the fact that the single-verifier proof is zero-knowledge. $\square$

5 Conclusion

Broadcast protocols are more efficient and have lower bandwidth requirements than repeated single-verifier proofs. (In the final paper we will discuss the bandwidth needed for these proofs.) We have assumed throughout that the network is honest (the telephone companies should be pleased!).

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