Interactive Deformation of Irregular Surface Models

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Abstract. Interactive deformation of surface models, which consist of quadrilateral (regular) and non-quadrilateral (irregular) surface patches, arises in many applications of computer animation and computer aided product design. Usually a model is mostly covered by regular patches such as Bézier or B-spline patches and the remaining areas are blended by irregular patches. However, the presence of irregular surface patches has posed a difficulty in surface deformation. Although regular patches can be easily deformed, the deformation of an irregular patch, however, has proven much trickier. This is made worse by having to maintain the smoothness conditions between regular and irregular patches during the process of deformation. By inserting extra control points, we have proposed a technique for the deformation of irregular surface patches. By setting continuity conditions as constraints, we also allow a surface model of an arbitrary topology, consisting of both regular and irregular surface patches, to be deformed smoothly.

1 Introduction

Interactive deformation of surface models is an important research topic in surface modelling, with numerous applications in computer animation, virtual reality and computer aided product design. Traditionally geometric models are represented by parametric surfaces, such as Bézier and B-spline surfaces. NURBS is so popular that it has become a de-facto standard. However, these surface models suffer from one significant setback — an inability of coping with surfaces of irregular topology, such as holes and branches. To remedy this weakness, two main alternatives have been proposed and also have taken increasing popularity in geometric modelling: subdivision surfaces [2], [4] and surfaces with combined regular (quadrilateral) and irregular (non-quadrilateral) patches. The latter is to blend regular patches with irregular patches (known as the blending surface patches) to form an overall smooth surface model. In this paper, our interest is in the interactive deformation of an irregular surface model (possibly with holes and branches) represented using the second approach.

Blending surface has long been an important research subject in geometric modelling. It is one of the most often used surface types for the representation of aesthetic features of computer generated models. A large number of methods have been developed which include the rolling ball method [23], the cyclide based method [24] and the partial differential equation method [22]. Another large family of techniques which also attract enormous attention are the control point based irregular surfaces. With this approach, a model is mostly covered by regular patches such as Bézier or NURBS patches. The remaining areas are blended by irregular n-sided patches. One technique is to use several regular control point patches to generate an irregular blending patch [6], [11], [12], [15], [18]. Another scheme is to produce a complete n-sided patch for the blending task. Loop and DeRose [14] proposed a method using S-patches

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for an n-sided hole. Sabin [17] tackled the same problem with a B-spline like control point patch. Zheng and Ball [20] generalised Sabin's patches to an arbitrary m degree.

A necessary feature of a modern geometric modelling system is the facility for interactive deformation of both regular and irregular surface models. A high degree of user control and interactivity is a practical requirement on nowadays geometric modelling systems, especially in computer animation and product design where people are expecting more and more visual realism from the CG models and characters.

Given a surface model consisting of both regular and irregular surface patches with at least G^1 continuity, interactive deformation involves the shape change of both types of patches, with possibly the following user-controlled deformation operations:

- moving control points of a patch;
- specifying geometric constraints for a patch, such as positional interpolation, i.e. letting the surface interpolate a given point;
- deforming a patch by exerting virtual forces, which can act as sculpting tools.

All three operations have been studied by various researchers for a regular patch. Things will however get more complicated when those operations are applied to irregular patches. But by far the most difficult task is to allow all these operations for both types of surface patches without violating their connection smoothness, such as the continuity conditions between a regular patch and an irregular patch. In this paper we propose a technique which will enable all these operations.

2 Background

Surface deformation is a desirable facility in both computer animation and product design. To date the majority of research has focused on the deformation of regular surface patches.

Deforming regular surfaces

One useful sculpting operation is to deform a surface patch by specifying positional constraints, i.e. spatial points that the surface has to interpolate. By moving these interpolated points interactively, one can deform the surface with a greater degree of direct user control, compared with the ordinary control-point based deformation approach. With the original configuration of control points, however, one often finds that there are not enough degrees of freedom (DOF) to satisfy the sufficient number of constraints. This difficulty can be overcome by producing extra control points for a surface patch. Here let us briefly review two types of most often used regular surfaces in this context: Bézier and B-spline surfaces.

A well-known property of Bézier surfaces is that the same surface can be described with a higher order representation. The process of obtaining a higher order representation is called degree elevation [25]. For instance, a cubic Bézier patch can be equivalently represented by a quartic patch without changing the geometry of the original surface. As a result of degree elevation, more control points are produced, which in effect provide more DOF for the surface. These extra degrees of freedom can then satisfy extra constraints.

B-spline surfaces, especially NURBS, again can be treated to satisfy extra constraints in a similar fashion. A NURBS surface patch can be refined by inserting node points in its node vector. The inserted node points are used to calculate the corresponding control points. The number of node points to be inserted depends on the number of constraints to be satisfied.

At a slightly higher level, a designer might find the ability to perform virtual sculpting operations attractive, whereby the designer deforms a surface by applying virtual forces [3], [21], [26].

Deforming irregular surfaces

To our knowledge, the deformation of irregular surfaces is a neglected research topic. One possible reason is that irregular surfaces are more difficult to deal with. It is also true that irregular surfaces, such as blending surfaces, are not as often encountered as regular surfaces, and in the past people were easily satisfied by a crude look of a CG model. With the rapid improvement of hardware performance, development of computer graphics and perfection of rendering techniques, however, the pursuit of realism of CG models has become a common requirement. Irregular patches whose roles are often to smoothly connect regular patches are no longer inferior to the mainstream regular surfaces.

In our previous attempt [21], we have tried to deform an irregular patch by applying virtual forces on it. Although the result is promising, we only considered a simple scenario that there is only one irregular patch and it is assumed there are enough DOF to start with. To make this technique practically useful, this model is clearly too simple and restrictive.

Outline of the proposed research

In this paper, we propose a technique for the deformation of a surface model that consists of both regular and irregular surface patches. This technique will have the following special contributions:

- no assumption is made for the degrees of freedom of an irregular patch. If extra DOF is needed, they will be produced without altering the original geometry;
- both regular and irregular surface patches can be deformed in the unified form by both geometric constraints and virtual forces;
- during deformation process, the smoothness conditions between patches (regular and irregular) will be maintained.

Since the deformation of a regular patch is already in the public knowledge, in this paper we will concentrate on the issues of irregular surface patches and the connection between different patches. For the modelling of irregular patches, we employ the model proposed by Zheng and Ball [20], which represents a generic n-sided surface patch. This surface model is control-point based and to a large extent similar to Bézier surfaces. To ensure enough degrees of freedom are present, we will formulate an explicit formula to elevate the degree of the surface and to insert a necessary number of extra control points. To guarantee that the smooth conditions between surface patches are not violated, they will be incorporated into our deformation formula as constraints which have to be satisfied.

3. Introduction to Zheng-Ball Patches

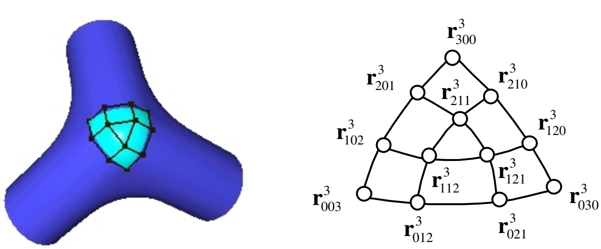

Fig. 1. 3-sided cubic Zheng-Ball patch with its control points.

As it is the basic model we employ for the proposed technique of surface deformation, in this section the basics of the n-sided Zheng-Ball patches are presented. For details, the reader is referred to [20]. An n-sided patch of degree m is defined by the following equation:

$$\mathbf{r}^m(\boldsymbol{u}) = \sum_{j=0}^{[m/2]} \sum_{\min\lambda=j} B_\lambda^m(\boldsymbol{u})\mathbf{r}_\lambda^m \tag{1}$$

where $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_n)$ represents the n-ple subscripts, $\lambda_i \le n$. $\boldsymbol{u} = (u_1, u_2, \cdots, u_n)$ are the n parameters. $\mathbf{r}_\lambda^m$ denotes the control points in 3D space $\mathbf{R}^3$, as shown in Figure 1, and $B_\lambda^m(\boldsymbol{u})$ are the associated basis functions. The parametrisation of $\boldsymbol{u} = (u_1, u_2, \cdots, u_n)$ can be in [20].

For example, for a cubic triangular patch, i.e. n=3, m=3, we have

$$B_\lambda^3(\boldsymbol{u}) = F_\lambda^3(\boldsymbol{u}) + \binom{3}{\lambda_{i-1}}\binom{3}{\lambda_i}\prod_{j=1}^{3} u_j^2 P_\lambda^3(\boldsymbol{u}) \tag{2}$$

in which

$$F_{3\delta_i}^3(\boldsymbol{u}) = u_i^3(1 - u_{i+1}u_{i+2}), \quad F_{\delta_i+2\delta_{i+2}}^3(\boldsymbol{u}) = 3u_{i+2}^3 u_i\left(1 - (1+u_i)u_{i+1}\right), \tag{3a}$$

$$F_{\delta_i+\delta_i+2\delta_{i+2}}^3(\boldsymbol{u}) = 9u_i u_{i+1} u_{i+2}^2, \quad F_{\delta_i+\delta_i+2\delta_{i+2}}^3(\boldsymbol{u}) = 9u_i u_{i+1} u_{i+2}^2 \tag{3b}$$

$$P_{3\delta_i}^3(\boldsymbol{u}) = 2(u_i - 1), \quad P_{\delta_i+2\delta_{i+2}}^3(\boldsymbol{u}) = \tfrac{2}{3}, \quad F_{\delta_i+\delta_i+2\delta_{i+2}}^3(\boldsymbol{u}) = -\tfrac{2}{3} \quad i = 1,2,3. \tag{4}$$

Here the 3-ple indices $\delta_i = (\delta_{i1}, \delta_{i2}, \delta_{i3})$, $\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \ne j \end{cases}$ and the subscript i is circularly taken modulo 3.

This patch model can have any number of sides and is able to smoothly blend the surrounding regular patches. In Figure 1, the 3-sided cubic patch, with 12 control points, is connected with the surrounding patches. However, once such an n-sided patch is constructed, there is no room for further deformation, i.e. there are no spare DOF are present. This fact will pose an even greater difficulty if one wants to maintain the smoothness at the patch boundaries during deformation.

4. Explicit Formula of Degree Elevation

A useful feature of this n-sided patch is its similarity to a Bézier patch. Control points can be inserted by elevating the degree of the surface. Therefore it allows more control points to be generated without changing the geometry of the surface. But before this feature is made of use, we have to solve one problem. That is to obtain a degree elevation formula such that the inserted control points are explicitly generated, as [2] only provides an implicit, recursive formula. In the interests of space, in the following we will derive the formulas of degree elevation for a 3-sided patch. Those of other patches can be similarly obtained.

Elevating the degree of a 3-sided cubic patch once gives the expression of the resulting quartic patch,

$$\mathbf{r}^4(\boldsymbol{u}) = \sum_{j=0}^{2} \sum_{\min\lambda=j} B_\lambda^4(\boldsymbol{u})\mathbf{r}_\lambda^4 \tag{5}$$

where the basis functions are defined by

$$B_\lambda^4(\boldsymbol{u}) = F_\lambda^4(\boldsymbol{u}) + \binom{4}{\lambda_{i-1}}\binom{4}{\lambda_i}\prod_{j=1}^{3} u_j^2 P_\lambda^4(\boldsymbol{u}) \tag{6}$$

and the functions $F_\lambda^4(\boldsymbol{u})$ are defined by

$$F_{4\delta_i}^4(\boldsymbol{u}) = u_i^4(1 - 4u_{i+1}u_{i+2}), \quad F_{3\delta_i+\delta_{i+2}}^4(\boldsymbol{u}) = 4u_i^3 u_{i+2}\left(1 - (1+2u_{i+2})u_{i+1}\right), \tag{7a}$$

$$F_{2\delta_i+2\delta_{i+2}}^4(\boldsymbol{u}) = 6u_i^2 u_{i+2}^2(1 - 2u_{i+1}), \quad F_{3\delta_i+\delta_{i+1}+\delta_{i+2}}^4(\boldsymbol{u}) = 16u_i^3 u_{i+1} u_{i+2} \tag{7b}$$

$$F^4_{2\delta_i+\delta_{i+1}+2\delta_{i+2}}(\boldsymbol{u}) = 24u_i^2 u_{i+1} u_{i+2}^2\,, \quad i = 1,2,3\,, \quad F^4_{222}(\boldsymbol{u}) = 36u_1^2u_2^2u_3^2\,, \tag{7c}$$

The remainder functions $P^4_\lambda(\boldsymbol{u})$ are unknown. To derive them, we first calculate the following auxiliary functions $R^3_\lambda(\boldsymbol{u})$:

$$R^3_{121}(\boldsymbol{u}) = \frac{1}{9\prod_{t=1}^{3} u_t^2}\left(F^3_{121}(\boldsymbol{u}) - \tfrac{1}{16}\left(9F^4_{131}(\boldsymbol{u}) + 6F^4_{122}(\boldsymbol{u}) + 6F^4_{221}(\boldsymbol{u}) + \tfrac{4}{3}F^4_{222}(\boldsymbol{u})\right)\right)$$

$$= \frac{1}{9\prod_{t=1}^{3} u_t^2}\left(9u_1u_2^2u_3 - \tfrac{1}{16}\left(9\cdot 16u_1u_2^3u_3 + 6\cdot 24u_1u_2^2u_3^2 + 6\cdot 24u_1^2u_2^2u_3 + \tfrac{4}{3}36u_1^2u_2^2u_3^2\right)\right) = -\frac{1}{3}(1+6u_2) \tag{8a}$$

$$R^3_{021}(\boldsymbol{u}) = \frac{1}{3\prod_{t=1}^{3} u_t^2}\left(F^3_{021}(\boldsymbol{u}) - \tfrac{1}{16}\left(12F^4_{031}(\boldsymbol{u}) + 3F^4_{131}(\boldsymbol{u}) + 8F^4_{022}(\boldsymbol{u}) + 2F^4_{122}(\boldsymbol{u})\right)\right)$$

$$= \frac{1}{3\prod_{t=1}^{3} u_t^2}\left(3u_2^2u_3(1-(1+u_3)u_1) - \tfrac{1}{16}\left(12\cdot 4u_2^3u_3(1-(1+2u_3)u_1) + 3\cdot 16u_1u_2^3u_3\right.\right.$$

$$\left.\left. + 8\cdot 6u_2^2u_3^2(1-2u_1) + 2\cdot 24u_1u_2^2u_3^2\right)\right) = 0 \tag{8b}$$

$$R^3_{030}(\boldsymbol{u}) = \frac{1}{\prod_{t=1}^{3} u_t^2}\left(F^3_{030}(\boldsymbol{u}) - \tfrac{1}{16}\left(16F^4_{040}(\boldsymbol{u}) + 4F^4_{130}(\boldsymbol{u}) + 4F^4_{031}(\boldsymbol{u}) + F^4_{131}(\boldsymbol{u})\right)\right)$$

$$= \frac{1}{\prod_{t=1}^{3} u_t^2}\left(u_2^3(1-3u_1u_3) - \tfrac{1}{16}\left(16u_2^4(1-4u_1u_3) + 4\cdot 4u_1u_2^3(1-u_3(1+2u_1))\right.\right.$$

$$\left.\left. + 4\cdot 4u_2^3u_3(1-u_1(1+2u_3)) + 16u_1u_2^3u_3\right)\right) = 4u_2^2 \tag{8c}$$

We are now in a position to formulate functions $P^4_\lambda(\boldsymbol{u})$.

Considering the symmetry of the functions, we can choose

$$P^4_{222}(\boldsymbol{u}) = 0\,, \qquad P^4_{2j2}(\boldsymbol{u}) = 0 \qquad j = 0,1 \tag{9}$$

Following the recursive degree elevation algorithm [2], we have

$$P^4_{131}(\boldsymbol{u}) = P^3_{121}(\boldsymbol{u}) + R^3_{121}(\boldsymbol{u}) - P^4_{221}(\boldsymbol{u}) - P^4_{122}(\boldsymbol{u}) - \tfrac{4}{3}P^4_{222}(\boldsymbol{u}) \tag{10a}$$

$$P^4_{031}(\boldsymbol{u}) = P^3_{021}(\boldsymbol{u}) + R^3_{021}(\boldsymbol{u}) - P^4_{131}(\boldsymbol{u}) - P^4_{122}(\boldsymbol{u}) - P^4_{022}(\boldsymbol{u}) \tag{10b}$$

$$P^4_{040}(\boldsymbol{u}) = P^3_{030}(\boldsymbol{u}) + R^3_{030}(\boldsymbol{u}) - P^4_{031}(\boldsymbol{u}) - P^4_{130}(\boldsymbol{u}) - P^4_{131}(\boldsymbol{u}) \tag{10c}$$

Inserting (4c), (8a) and (9) into (10a), we get

$$P^4_{131}(\boldsymbol{u}) = -(1+2u_2) \tag{11a}$$

Inserting (4b), (8b), (9) and (11a) into (10b), we obtain the following

$$P^4_{031}(\boldsymbol{u}) = \tfrac{5}{3} + 2u_2 \tag{11b}$$

By symmetry, we have

$$P^4_{130}(\boldsymbol{u}) = \tfrac{5}{3} + 2u_2 \tag{11c}$$

Substituting (4a), (8c), (11b), (11c) and (10a) into (10c), we get

$$P^4_{040}(\boldsymbol{u}) = 4u_2^2 - \tfrac{13}{3} \tag{11d}$$

By symmetry, we have the following complete expression for the functions $P^4_\lambda(\boldsymbol{u})$:

$$P^4_{4\delta_i}(\boldsymbol{u}) = 4u_i^3 - \frac{13}{3}, \quad P^4_{3\delta_i+\delta_{i+1}}(\boldsymbol{u}) = \frac{5}{3} + 2u_i, \quad P^4_{3\delta_i+\delta_{i+1}+\delta_{i+2}}(\boldsymbol{u}) = -(1+2u_i),$$

$$P^4_\lambda(\boldsymbol{u}) = 0, \text{ for other } \lambda\text{'s} \tag{12}$$

So far, we have obtained the explicit expression of the remainder functions $P^4_\lambda(\boldsymbol{u})$. Substituting formulae (7) and (12) into (6) results in the complete expression of the basis functions $B^4_\lambda(\boldsymbol{u})$ of degree 4.

The control points $\mathbf{r}^4_\lambda$ of the quartic surface (5) are shown in Figure 2 and are expressed in terms of the control points $\mathbf{r}^3_\lambda$ for cubic surface (1), as following:

$$\mathbf{r}^4_{\lambda_i^{k,j}} = \frac{1}{16}\left(kj\mathbf{r}^3_{\lambda_i^{k-1,j-1}} + k(4-j)\mathbf{r}^3_{\lambda_i^{k-1,j}} + (4-k)j\mathbf{r}^3_{\lambda_i^{k,j-1}} + (4-k)(4-j)\mathbf{r}^3_{\lambda_i^{k,j}}\right) \tag{13a}$$

$$k \le 2, j < 2, \qquad i = 1, \cdots n,$$

$$\mathbf{r}^4_{\lambda_i^{2,2}} = \frac{1}{n}\sum_{i=1}^{n}\mathbf{r}^3_{\lambda_i^{1,1}} \tag{13b}$$

where $\lambda_i^{k,j} = k\delta_i + j\delta_{i+1} + \lambda_{i+2}\delta_{i+2}$.

The generated extra control points will be used to modify the shape of the 3-sided patch as shown in Figure 2. With G^1 continuity being satisfied by the control points near the boundaries, the extra central control point can be moved freely to modify the shape of the blending surface. And the process of degree elevation can continue to generate further extra control points until enough degrees of freedom are created.

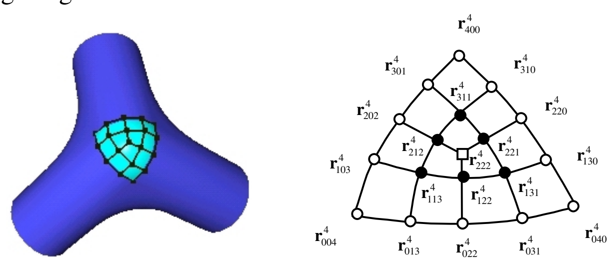

Fig. 2. Quartic patches with control points after degree elevation. The circles represent the control points contributing to the C^0 condition, the black dots represent the control points contributing to the G^1 condition, and the square in the middle represents the free central control point.

Comparing Figure 1 with Figure 2 it is clear that, after degree elevation, one central control point is obtained, which has provided an extra degree of freedom. Since this control point does not contribute to the current continuity conditions, moving this control point will deform the shape of the blending patch intuitively without violating the continuity conditions with the surrounding patches.

5. Surface Deformation

To deform an irregular patch, [21] proposed a technique based on minimising the following energy functional:

$$E = \mathbf{V}^T\mathbf{K}\mathbf{V} - 2\mathbf{V}^T\mathbf{F}, \tag{14}$$

where $\mathbf{V}$ is the control vector whose entries are 3D control point vectors for an n-sided patch. $\mathbf{K}$ is the stiffness matrix whose entries are functions of Zheng-Ball base functions. $\mathbf{F}$ is the force vector whose entries are functions of both Zheng-Ball base functions and the physical forces applied.

To deform all regular and irregular surface patches, the same idea is applied, with the boundary conditions being coded as constraints.

For an arbitrary patch Π_i — regular or irregular — we may define a similar energy functional to (14):

$$E_i = \mathbf{V}_i^T \mathbf{K}_i \mathbf{V}_i - 2\mathbf{V}_i^T \mathbf{F}_i \tag{15}$$

where $\mathbf{V}_i$, $\mathbf{K}_i$ and $\mathbf{F}_i$ are the control point vector, stiffness matrix and force vector, respectively, with respect to each patch Π_i. For the whole surface model, we have

$$\mathbf{V} = (\mathbf{V}_1, \mathbf{V}_2, \cdots)^T, \quad \mathbf{K} = \begin{pmatrix} \mathbf{K}_1 & \mathbf{0} & \\ \mathbf{0} & \mathbf{K}_2 & \\ & & \ddots \end{pmatrix}, \quad \mathbf{F} = (\mathbf{F}_1, \mathbf{F}_2, \cdots)^T \tag{16}$$

Thus the new global energy functional is given by

$$E = \mathbf{V}^T \mathbf{K} \mathbf{V} - 2\mathbf{V}^T \mathbf{F} \tag{17}$$

The continuity constraints are defined by the following linear matrix equation

$$A\mathbf{V} = \mathbf{b} \tag{18}$$

Minimising the quadratic form (17) subject to constraint (18) leads to the production of a deformed model consisting of both regular and irregular patches.

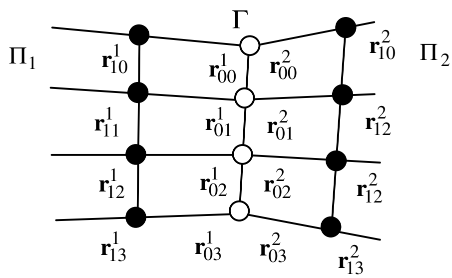

Fig. 3. Two cubic patches Π_1 and Π_2 share a common boundary Γ.

Remarks. Suppose two cubic patches Π_1 and Π_2 (either regular or irregular) share a common boundary Γ as show in Fig. 3. Typical G^1 continuity constraints for the two patches Π_1 and Π_2 can be expressed by the following linear equation which can be embedded into (18):

$$A_1 \tilde{\mathbf{V}} = \mathbf{b}_1 \tag{19}$$

where $\tilde{\mathbf{V}} = (\mathbf{r}^1_{00}, \mathbf{r}^1_{01}, \mathbf{r}^1_{02}, \mathbf{r}^1_{03}, \mathbf{r}^1_{10}, \mathbf{r}^1_{11}, \mathbf{r}^1_{12}, \mathbf{r}^1_{13}, \mathbf{r}^2_{00}, \mathbf{r}^2_{01}, \mathbf{r}^2_{02}, \mathbf{r}^2_{03}, \mathbf{r}^2_{10}, \mathbf{r}^2_{11}, \mathbf{r}^2_{12}, \mathbf{r}^2_{13})^T$,

$$A_1 = \begin{pmatrix} 1 & & & & & & & & -1 & & & & & & & \\ & 1 & & & & & & & & -1 & & & & & & \\ & & 1 & & & & & & & & -1 & & & & & \\ & & & 1 & & & & & & & & -1 & & & & \\ 1 & & & & -1 & & & & -1 & & & & 1 & & & \\ & 1 & & & & -1 & & & & -1 & & & & 1 & & \\ & & 1 & & & & -1 & & & & -1 & & & & 1 & \\ & & & 1 & & & & -1 & & & & -1 & & & & 1 \end{pmatrix}, \qquad \mathbf{b}_1 = \mathbf{0} \tag{20}$$

A surface model with irregular patches is given in Figure 4. There are eight triangular patches on the outer corners of the model, and eight pentagonal patches on the inner corners of the model. All the remaining parts are covered by regular bi-cubic Bézier patches (the blue ones). The original model is on the left of Figure 4. The model is deformed by the algorithm presented above. The resulting models are shown on the middle and right of Figure 4. We can see that both regular patches and irregular blending patches are deformed.

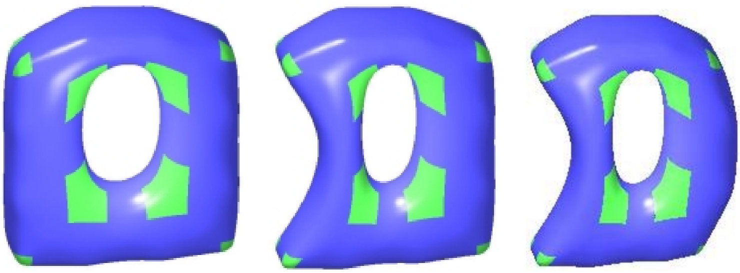

Fig. 4. Model with 3- and 5-sided patches (green patches). (Middle and Right) Deformed models.

6. Algorithm for Interactive Deforming

In surface deformation, linear constraints are also useful deformation tools, such as to let the surface interpolate specified points, curves and norms [3], [21]. Linear constraints can be expressed as:

$$A\mathbf{V} = \mathbf{b} \tag{21}$$

where $\mathbf{V}$ is the column vector whose entries are the 3D co-ordinates of l control points. A is a $k \times l$ matrix of coefficients. k is the number of constraints. Each row of the matrix A represents a linear constraint on the surface. Assuming that redundant constraints in (21) are eliminated.

If physical forces are applied to the surface, the following linear system is generated by minimising the quadratic form (18):

$$\mathbf{KV} = \mathbf{F}_0 + \iint \mathbf{Z} \cdot \Delta \boldsymbol{f}(\boldsymbol{u})\, du dv \tag{22}$$

where $\mathbf{K}$ is an $l \times l$ positive semi-definite stiffness matrix. $\mathbf{Z}$ is the vector whose entries are the base functions. $\mathbf{F}_0$ is generated by the initial forces. $\Delta f(x)$ is the density of the distributed forces used to deform an initial n-sided surface to a new shape with new vector $\mathbf{V}$. As mentioned in [21], if the matrix $\mathbf{K}$ is singular, then least square solution will be used.

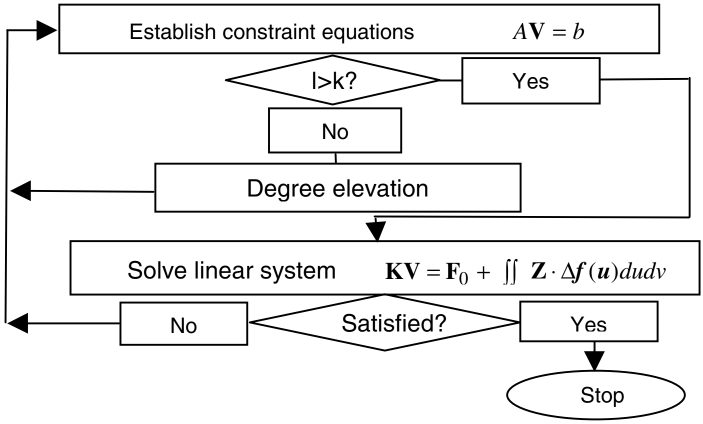

Fig. 5. Algorithm of interactive deformation.

The process of deforming the surface is equivalent to solving linear system (22) with respect to vector $\mathbf{V}$ subject to constraint (21).

In the literature, it is always assumed that there are enough degrees of freedom with respect to **V** in constraint (21). In fact it is not always so. There are two different cases:

- $l>k$. There are free variables left in (21). So linear system (22) can be solved.
- $l \leq k$. There is no free variable left in (21). So linear system (22) is not solvable.

In the latter case, extra degrees of freedom are needed to solve linear system (22). This is achieved by degree elevation. Since it is not known beforehand how many extra degrees of freedom are needed, the process will be iterative, as illustrated by the above diagram in Figure 5:

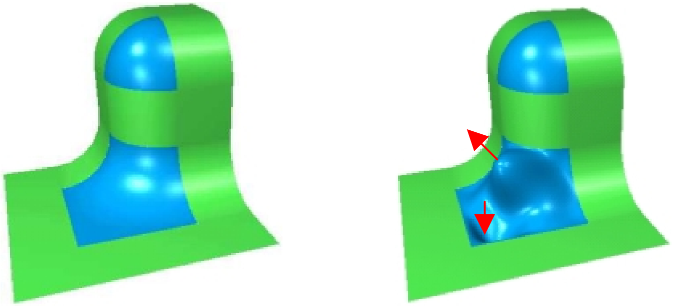

Fig. 6. A smooth model with 3- and 5-sided cubic surface patches (left). Deformed model after twice degree elevation (right). Arrows indicate the forces applied on the surface points.

Figure 6 shows a smooth model with 3- and 5-sided cubic surface patches. Without degree elevation, the 5-sided patch cannot be deformed (left). With degree elevation, the model is deformed. The boundary conditions are maintained during deformation.

7. Conclusions

The deformation of a quadrilateral surface model is a routine method for surface modelling. But in the literature, no reported methods have been able to deform a surface model interactively if *n*-sided surface patches are involved without compromising the smoothness conditions with the surrounding surfaces.

There are two main contributions in this paper. Firstly, we have proposed a surface deformation technique, which is able to deform a surface model consisting of connected regular and irregular surface patches without violating the smoothness conditions at the patch boundaries. Secondly, we have derived an explicit formula for degree elevation of irregular patches, and this derivation is applied to the deformation of irregular patches. As most CAD systems produce quadrilateral cubic patches, an irregular patch can be initially set as an *n*-sided cubic patch, which satisfies G^1 continuity condition when connecting to other regular patches. This irregular patch is then degree elevated to produce a required number of extra control points for it to be deformed together with the others while still maintaining the smoothness constraints between the patches. The algorithm of degree elevation has been given as an iterative procedure to satisfy the user requirements in the applications of interactive model deformation.

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