A Job-Shop Scheduling Problem with Fuzzy Processing Times

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Abstract. Job-shop scheduling is a difficult problem, both theoretically and practically. This problem is a combinatorial optimization of considerable industrial importance. Although the job-shop scheduling problem has often been investigated, very little of this research is concerned with the uncertainty characterized by the imprecision in problem variables. In this paper, we investigate the job-shop scheduling problem with imprecise processing times. We first use triangular fuzzy numbers to represent imprecise processing times, and then construct a fuzzy job-shop scheduling model. Our work intends to extend the deterministic job-shop scheduling problem into a more generalized problem that would be useful in practical situations.

1 Introduction

The job-shop scheduling problem is concerned with allocating limited resources to operations over time [3]. Between the operations precedence constraints for a job can be defined. Although job-shop scheduling has always had an important role in the field of production and operations management, it is a difficult problem in combinatorial optimization [12]. The difficulty is due to the high number of constraints, unfortunately unavoidable in the real-world applications [9]. The job-shop scheduling problem can be described as follows. We are given n jobs and m machines. Each job consists of a sequence of operations that must be processed on m machines in a given order. Each operation must be executed uninterrupted on a given machine for a given period of time and each machine can only handle at most one operation at a time. The problem is to find a schedule, an allocation of the operations of n jobs to certain time intervals on m machines, with a minimum overall time.

Solving the job-shop scheduling problem requires a high computational effort and considerable sophistication [2, 4]. Instead of investigating using optimal algorithms, it is often preferred to use approximation algorithms such as heuristics and meta-heuristics, e.g. simulated annealing, genetic algorithms, and tabu search [1, 5, 6-9, 15, 17, 19]. However, most of the methods proposed in the literature required the assumption that all time parameters are known exactly. This is a strong assumption, which may cause severe difficulties in practice. An example is the difficulty in estimating the exact processing times for all jobs on the machines. In fact, there are many vaguely formulated relations and imprecisely quantified physical data

values in real world descriptions since precise details are simply not known in advance. Stochastic methods exist, but not many address imprecise uncertainty. Although the job-shop scheduling problem has often been investigated, very few of these studies take uncertainty, typified by the imprecision or vagueness in time estimates, into account [10, 14, 15, 16, 18].

In this study, we investigate a job-shop scheduling problem with imprecise processing time and use fuzzy numbers to represent imprecise processing times in this problem. The main interest of our approach is that the fuzzy schedules obtained from Property 1 are the same type as those in the crisp job-shop scheduling problem. The fuzzy job-shop scheduling model, in the case of imprecise processing times, is then an extension of the crisp problem.

2 Job-Shop Scheduling Problem

The deterministic job-shop scheduling problem is stated as follows. There are n jobs to be scheduled on m machines. Each job consists of a sequence of operations that must be processed on m machines in a given order [13]. Each operation is characterized by specifying both the required machine and the fixed processing time. Several constraints on jobs and machines, which are listed as follows [11]:

- (1) Each job must pass through each machine once and only once.
- (2) Each job should be processed through the machine in a particular order.
- (3) Each operation must be executed uninterrupted on a given machine.
- (4) Each machine can only handle at most one operation at a time.

The problem is to find a schedule to determine the operation sequences on the machines in order to minimize the total completion time. Let c_{ik} denote the completion time of job i on machine k , and t_{ik} denote the processing time of job i on machine k . For a job i , if the processing on machine h precedes that on machine k , we need the following constraint:

$$c_{ik} - t_{ik} \geq c_{ih}$$

On the other hand, if the processing on machine k comes first, the constraint becomes

$$c_{ih} - t_{ih} \geq c_{ik}$$

Thus, we need to define an indicator variable x_{ihk} as follows:

$$x_{ihk} = \begin{cases} 1, & \text{processing on machine } h \text{ precedes that on machine } k \text{ for job } i \\ 0, & \text{otherwise} \end{cases}$$

We can then rewrite the above constraints as follows:

$$c_{ik} - t_{ik} + L(1 - x_{ihk}) \geq c_{ih}, \quad i = 1, 2, \dots, n, \quad h, k = 1, 2, \dots, m$$

where L is a large positive number. Consider two jobs, i and j , that are to be processed on machine k . If job i comes before job j , we need the following constraint:

$$c_{jk} - c_{ik} \geq t_{jk}$$

Otherwise, if job j comes first, the constraint becomes

$$c_{ik} - c_{jk} \geq t_{ik}$$

Therefore, we also need to define another indicator variable y_{ijk} as follows.

$$y_{ijk} = \begin{cases} 1, & \text{if job } i \text{ precedes job } j \text{ on machine } k \\ 0, & \text{otherwise} \end{cases}$$

We can then rewrite the above constraints as follows:

$$c_{jk} - c_{ik} + L(1 - y_{ijk}) \geq t_{jk}, \quad i, j = 1, 2, \dots, n, \quad k = 1, 2, \dots, m$$

The job-shop scheduling problem with a makespan objective can be formulated as follows:

$$\min \max_{1 \leq k \leq m} [\max_{1 \leq i \leq n} [c_{ik}]] \quad (1)$$

$$\text{s.t. } c_{ik} - t_{ik} + L(1 - x_{ihk}) \geq c_{ih}, \quad i = 1, 2, \dots, n, \quad h, k = 1, 2, \dots, m \quad (2)$$

$$c_{jk} - c_{ik} + L(1 - y_{ijk}) \geq t_{jk}, \quad i, j = 1, 2, \dots, n, \quad k = 1, 2, \dots, m \quad (3)$$

$$c_{ik} \geq 0, \quad i = 1, 2, \dots, n, \quad k = 1, 2, \dots, m \quad (4)$$

$$x_{ihk} = 0 \text{ or } 1, \quad i = 1, 2, \dots, n, \quad h, k = 1, 2, \dots, m \quad (5)$$

$$y_{ijk} = 0 \text{ or } 1, \quad i, j = 1, 2, \dots, n, \quad k = 1, 2, \dots, m$$

3 Constructing a Fuzzy Job-Shop Scheduling Model

3.1 Imprecise Processing Time

As in real life situations, some unexpected events may occur, resulting in small changes to the processing time of each job. Therefore, in many situations, the

processing time can only be estimated as being within a certain interval. Let us illustrate this point using one example. The example taken from [10] is an industrial case in a chemical environment. The chemical reaction depends not only on the pressure and the temperature but also on the quality of the components. It is very difficult to control this dependence. Therefore, the uncertainties of those reaction times must also be modeled. Because of this interval estimation feature, the representation of processing time for a job can be more realistically and naturally achieved through the use of a fuzzy number. As a result, decision-makers (DM) do not need to give a single precise number to represent the processing time of a job. We use t_{jk} , the processing time for job j on machine k , to denote the process time for each job in this paper. However, t_{jk} is just an estimate and its exact value is actually unknown.

3.2 A Fuzzy Job-Shop Sequencing Model

Consider the schedule for the job-shop problem is performed several times in practical situations. Obviously, the processing time for this schedule at different execution times is not necessarily the same. Therefore, an estimated processing time interval, i.e. $[t_{jk} - \Delta_{jk1}, t_{jk} + \Delta_{jk2}]$, should be given to represent the possible range of values for the processing time. Thus the use of interval $[t_{jk} - \Delta_{jk1}, t_{jk} + \Delta_{jk2}]$ is more appropriate than the use of a single estimate, t_{jk} , in practical situations. The DM should carefully determine the parameters Δ_{jk1} and Δ_{jk2} , which satisfy $0 < \Delta_{jk1} < t_{jk}$ and $0 < \Delta_{jk2}$, for defining an acceptable processing time range for any particular problem. After that, the DM can choose an appropriate value from the interval $[t_{jk} - \Delta_{jk1}, t_{jk} + \Delta_{jk2}]$ as an estimate for the processing time for job j on machine k . Obviously, when the estimate happens to be t_{jk} , which is the crisp processing time, the error rate is zero. When the estimate deviates from t_{jk} , the error rate will become larger. In fact, we can use the term “confidence level” instead of “error rate” while we consider processing time interval based on the fuzzy viewpoint. We can therefore say that the confidence level is one if the processing time estimate equals t_{jk} . Otherwise, when the processing time estimate deviates from t_{jk} , the confidence level will become smaller. Finally, when the estimate approaches one of the two ends of the interval, i.e. $t_{jk} - \Delta_{jk1}$ or $t_{jk} + \Delta_{jk2}$, the confidence level will be close to zero.

A level 1 triangular fuzzy number corresponding to the above interval $[t_{jk} - \Delta_{jk1}, t_{jk} + \Delta_{jk2}]$ is given as follows:

$$\begin{aligned} \tilde{t}_{jk} &= (t_{jk} - \Delta_{jk1}, t_{jk}, t_{jk} + \Delta_{jk2}; 1) \in F_N(1), \\ 0 &< \Delta_{jk1} < t_{jk}, \quad 0 < \Delta_{jk2}, \quad j = 1, 2, \dots, n, \quad k = 1, 2, \dots, m. \end{aligned} \quad (6)$$

Fig. 1. A triangular fuzzy number $\tilde{t}_{jk}$.

Fig. 1 shows a level 1 triangular fuzzy number $\tilde{t}_{jk}$. From fig. 1 we can see that the membership grade at t_{jk} in $\tilde{t}_{jk}$ is 1. However, the more the fuzzy number deviates from the t_{jk} position, the lesser the fuzzy numbers membership grade in $\tilde{t}_{jk}$. The membership grade at $t_{jk} - \Delta_{jk1}$ or $t_{jk} + \Delta_{jk2}$ is zero. Of course, we can see that the confidence level for an estimate in the interval corresponds to the membership grade of a fuzzy number in the fuzzy sets. This concept naturally leads to the use of fuzzy numbers for the processing time in the job-shop scheduling problem.

Let $\tilde{c}_{ik} = (c_{ik}, c_{ik}, c_{ik}; 1) = (\tilde{c}_{ik})_1 \in F_N(1)$. (7)

The signed distance of level λ fuzzy number is defined as follows. For each $\lambda \in [0, 1]$ and $\tilde{A} = (a, b, c; \lambda) \in F_N(\lambda)$. The signed distance of $\tilde{A}$ measured from $\tilde{0}_1$ is defined by $d(\tilde{A}, \tilde{0}_1) = \frac{1}{4}(2b + a + c)$. Then $d(\tilde{c}_{ik}, \tilde{0}_1) = c_{ik}$ is the signed distance from $\tilde{0}_1$ to $\tilde{c}_{ik}$. Since $d(\tilde{c}_{ik}, \tilde{0}_1) = c_{ik} > 0$, it is a positive distance from $\tilde{0}_1$ to $\tilde{c}_{ik}$. Therefore $c_{ik} = d(\tilde{c}_{ik}, \tilde{0}_1)$ is the completion time measured from the origin 0.

The relations $\succ, \approx$ are the ranking as defined on $F_N(1)$. We then fuzzify (1) - (4) using (6) and (7) to obtain the following fuzzy job-shop scheduling problem:

$$\min \max_{1 \leq k \leq m} [\max_{1 \leq i \leq n} [d(\tilde{c}_{ik}, \tilde{0}_1)]] \quad (8)$$

$$\text{s.t. } \tilde{c}_{ik} \ominus \tilde{t}_{ik} \oplus (L(\tilde{1} - x_{ihk}))_1 \leq \tilde{c}_{ih}, \quad i = 1, 2, \dots, n, \quad h, k = 1, 2, \dots, m \quad (9)$$

$$\tilde{c}_{jk} \ominus \tilde{c}_{ik} \oplus (L(\tilde{1} - y_{ijk}))_1 \leq \tilde{t}_{jk}, \quad i, j = 1, 2, \dots, n, \quad k = 1, 2, \dots, m \quad (10)$$

$$\tilde{c}_{ik} \leq \tilde{0}_1, \quad i = 1, 2, \dots, n, \quad k = 1, 2, \dots, m \quad (11)$$

$$x_{ihk} = 0 \text{ or } 1, \quad i = 1, 2, \dots, n, \quad k = 1, 2, \dots, m \quad (12)$$

$$y_{ijk} = 0 \text{ or } 1, \quad i, j = 1, 2, \dots, n, \quad k = 1, 2, \dots, m$$

Note that $(L(\tilde{1} - x_{ihk}))_1$ is a fuzzy point at $L(1 - x_{ihk})$.

$$\text{Since } d(\tilde{c}_{ik}, \tilde{0}_1) = c_{ik}, \quad (13)$$

$$\text{and } d(\tilde{t}_{jk}, \tilde{0}_1) = t_{jk} + \frac{1}{4}(\Delta_{jk2} - \Delta_{jk1}), \text{ let } t_{jk}^* = d(\tilde{t}_{jk}, \tilde{0}_1). \quad (14)$$

We summarize (9)-(14) to obtain the following Property 1.

Property 1. The job-shop scheduling problem with imprecise processing time modeled by fuzzy number is as follows:

$$\min \max_{1 \leq k \leq m} [\max_{1 \leq i \leq n} [c_{ik}]] \quad (15)$$

$$\text{s.t. } c_{ik} - t_{ik}^* + L(1 - x_{ihk}) \geq c_{ih}, \quad i = 1, 2, \dots, n, \quad h, k = 1, 2, \dots, m \quad (16)$$

$$c_{jk} - c_{ik} + L(1 - y_{ijk}) \geq t_{jk}^*, \quad i, j = 1, 2, \dots, n, \quad k = 1, 2, \dots, m \quad (17)$$

$$c_{ik} \geq 0, \quad i = 1, 2, \dots, n, \quad k = 1, 2, \dots, m \quad (18)$$

$$x_{ihk} = 0 \text{ or } 1, \quad i = 1, 2, \dots, n, \quad h, k = 1, 2, \dots, m \quad (19)$$

$$y_{ijk} = 0 \text{ or } 1, \quad i, j = 1, 2, \dots, n, \quad k = 1, 2, \dots, m$$

4 Computational Results

Consider an example from [11]. We compare the results obtained from Property 1 with that of the crisp job-shop scheduling problem using the Johnson's constructive algorithm. Johnson's algorithm for, $n/2/G/F_{\max}$ (i.e. n jobs, two machines, general job-shop problem, to minimize makespan) problem is stated briefly as follows:

Suppose that the set of n jobs $\{J_1, J_2, \dots, J_n\}$ may be partitioned into four types of jobs.

Type A: Those to be processed on machine M_1 .

Type B: Those to be processed on machine M_2 .

Type C: Those to be processed on both machines in the order M_1 then M_2 .

Type D: Those to be processed on the machine in the order M_2 then M_1 .

Then the construction of an optimal schedule is straightforward, as follows:

- (1) Schedule the jobs of type A in any order to give the sequence S_A .
- (2) Schedule the jobs of type B in any order to give the sequence S_B .
- (3) Schedule the jobs of type C according to Johnson's algorithm for $n/2/F/F_{\max}$ (n jobs, 2 machines, flow-shop problem, to minimize makespan) problems to give the sequence S_C .
- (4) Schedule the jobs of type D according to Johnson's algorithm for $n/2/F/F_{\max}$ problems to give the sequence S_D .

The example is the $9/2/G/F_{\max}$, nine jobs and two machines, with times and processing order as given in Table 1.

Table 1. Processing order and times.

Job	First Machine	Second Machine
1	M_1 8 ($= t_{11}$)	M_2 2 ($= t_{12}$)
2	M_1 7 ($= t_{21}$)	M_2 5 ($= t_{22}$)
3	M_1 9 ($= t_{31}$)	M_2 8 ($= t_{32}$)
4	M_1 4 ($= t_{41}$)	M_2 7 ($= t_{42}$)
5	M_2 6 ($= t_{52}$)	M_1 4 ($= t_{51}$)
6	M_2 5 ($= t_{62}$)	M_1 3 ($= t_{61}$)
7	M_1 9 ($= t_{71}$)	—
8	M_2 1 ($= t_{82}$)	—
9	M_2 5 ($= t_{92}$)	—

An optimal crisp sequence for Table 1, obtained by Johnson's algorithm is

Processing Sequence of jobs	
Machine M_1	(4, 3, 2, 1, 7, 5, 6)
Machine M_2	(5, 6, 8, 9, 4, 3, 2, 1)

We can see that the total time $F_{\max} = 44$ for the optimal sequence. From Property 1, we consider the following fuzzification for processing time t_{jk} .

$$\tilde{t}_{11} = (8 - 2, 8, 8 + 4), \tilde{t}_{21} = (7 - 1, 7, 7 + 4), \tilde{t}_{31} = (9 - 3, 9, 9 + 1), \tilde{t}_{41} = (4 - 1, 4, 4 + 2),$$

$$\tilde{t}_{51} = (4 - 0.5, 4, 4 + 1), \tilde{t}_{61} = (3 - 0.6, 3, 3 + 1.5), \tilde{t}_{71} = (9 - 3, 9, 9 + 2),$$

$$\tilde{t}_{12} = (2 - 0.8, 2, 2 + 1), \tilde{t}_{22} = (5 - 1, 5, 5 + 2), \tilde{t}_{32} = (8 - 1, 8, 8 + 4), \tilde{t}_{42} = (7 - 2, 7, 7 + 3),$$

$$\tilde{t}_{52} = (6 - 2, 6, 6 + 1), \tilde{t}_{62} = (5 - 0.9, 5, 5 + 2), \tilde{t}_{82} = (1 - 0.5, 1, 1 + 0.6), \tilde{t}_{92} = (5 - 2, 5, 5 + 1)$$

By (14) we obtain the following fuzzy processing times.

$$t_{11}^* = 8.5, t_{21}^* = 7.75, t_{31}^* = 8.5, t_{41}^* = 4.25, t_{51}^* = 4.125, t_{61}^* = 3.225, t_{71}^* = 8.75, t_{12}^* = 2.05,$$

$$t_{22}^* = 5.25, t_{32}^* = 8.75, t_{42}^* = 7.25, t_{52}^* = 5.75, t_{62}^* = 5.275, t_{82}^* = 1.025, t_{92}^* = 4.75.$$

Then we find an optimal schedule as follows:

Type A jobs: job 7 on M_1 .

Type B jobs: jobs 8 and 9 on M_2 in an arbitrary order (8,9).

Type C jobs: jobs 1, 2, 3, and 4 require M_1 first and then M_2 . Using Johnson's algorithm we obtain the relations $t_{31}^* < t_{32}^*$, $t_{41}^* < t_{42}^*$, and $t_{41}^* < t_{31}^*$; $t_{11}^* > t_{12}^*$, $t_{21}^* > t_{22}^*$, and $t_{12}^* < t_{22}^*$. Thus the sequence obviously is (4, 3, 2, 1).

Type D jobs: jobs 5 and 6 require M_2 first and then M_1 . Again, using Johnson's algorithm we obtain the relations $t_{51}^* < t_{52}^*$, $t_{61}^* < t_{62}^*$, and $t_{61}^* < t_{51}^*$. Note that M_1 now becomes the second machine. The sequence is therefore (5, 6). Finally, an optimal sequence for Property 1 is

	Processing Sequence of Jobs
Machine M_1	(4, 3, 2, 1, 7, 5, 6)
Machine M_2	(5, 6, 8, 9, 4, 3, 2, 1)

The processing sequence for jobs obtained from the fuzzy case is the same as that for the crisp case. The total time is $F_{\max}^* = 45.1$ for the optimal sequence of Property 1. We compare the result of Property 1 with that of crisp case as follows:

$$\frac{F_{\max}^* - F_{\max}}{F_{\max}} \times 100 = 2.5\%.$$

5 Conclusion

In this paper, we have investigated the fuzzy job-shop scheduling problem with imprecise processing times. In conclusion, we point out that our work has produced the following main results for the job-shop scheduling problem with fuzzy processing times modeled as fuzzy numbers. In Section 3, we presented a fuzzy job-shop scheduling problem based on triangular fuzzy numbers. In (14), if $\Delta_{jk2} = \Delta_{jk1}, \forall j, k$, obviously, Fig. 1 is an isosceles triangle and we obtain $t_{jk}^* = t_{jk}, \forall j, k$. Thus, the equations (15)-(19) will become the same as (1)-(5). If $\Delta_{jk2} = \Delta_{jk1} = 0, \forall j, k$, the job-shop scheduling in the fuzzy sense of Property 1 will become the crisp job-shop scheduling of equations (1)-(5). Therefore, the job-shop scheduling in the fuzzy sense of Property 1 is an extension of the crisp scheduling of (1)-(5).

The interpretation of Fig. 1 is as follows: When $\Delta_{jk2} > \Delta_{jk1}, \forall j, k$, the triangle is skewed to the right-hand side, thus obtaining $t_{jk}^* > t_{jk}, \forall j, k$. This means that the completion time in the fuzzy sense is longer than in the crisp case. Conversely, when $\Delta_{jk2} < \Delta_{jk1}, \forall j, k$, the triangle is skewed to the left-hand side, thus obtaining $t_{jk}^* < t_{jk}, \forall j, k$, indicating that the completion time in the fuzzy sense is shorter than the crisp case. From (14), we can see that $t_{jk}^* = t_{jk} + \frac{1}{4}(\Delta_{jk2} - \Delta_{jk1})$ is the processing time in the fuzzy sense, where t_{jk}^* equals the crisp processing time, t_{jk} , plus some variants due to the inclusion of fuzzy values.

The comparison of [10] with our work is as follows. Fortemps used six-point fuzzy numbers to represent fuzzy durations and fuzzy makespan. In his approach, the resulting fuzzy framework was not an extension of the original crisp problem. He used a simulated annealing technique to solve the fuzzy model obtaining the optimization sequence of the problem. In our approach, however, the crisp duration x become an interval $[x - \Delta_1, x + \Delta_2]$, $0 < \Delta_1 < x$, $0 < \Delta_2$, represents an allowable range of duration. We let the fuzzy number $\tilde{x} = (x - \Delta_1, x, x + \Delta_2; 1)$ correspond to the interval $[x - \Delta_1, x + \Delta_2]$. Then we used a signed distance method for ranking fuzzy numbers to obtain a fuzzy job-shop scheduling problem. After defuzzifying the fuzzy problem using the signed distance ranking method, we obtained a job-shop scheduling problem in the fuzzy sense. The resulting fuzzy framework in our approach is an extension of the original crisp problem. Therefore, the algorithms that were used for solving the crisp job-shop sequencing problem can also be used for solving the fuzzy problem.

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