FUZZY OBJECT BLENDING IN 2-D

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Abstract: In this paper, a new curve modeling method using fuzzy logic is presented for blending in 2-D. For this goal, approximation curves are produced by means of fuzzy logic techniques. The base of the study is to obtain least three internal points of the curve by fuzzy logic, in case of knowing the start and end control points. After internal points are obtained, all points including start and end points of the approximation curve are created. As application, the method is applied to blending process and some sample blended objects are given in 2-D.

Key words: Fuzzy logic technique, object blending and curve modeling

1 Introduction

The curve modeling is the one of the basic subject in computer graphics. There are generally two curves in computer graphics: approximation and interpolation curves. While a curve is produced, different methods are implemented. These are; producing the curve via control points, creating the curve to be satisfied C^1 continuous at starting and ending points and generating curve by knowing curvature radius [1]. In this study, the curve producing technique by control points is improved. It is easier than other methods, because first and second order derivation computations don't need.

It is well - known that it is easy to construct the approximation or interpolation curves where start and end points are determined. In this study, the approximation curves are preferred for blending. The basis of work is that: in case of knowing the start and end points, least three points, that are computed by the means of original points and their auxiliary points, are produced by the of fuzzy logic technique and approximation curves are generated by using all of points. And then, blend surfaces were produced by means of curves and some blended surfaces are given.

The one of the most popular problem in the solid modeling is to achieve objects not having certain mathematical form. To overcome this problem, one of the applied methods is to model the new objects by joining smooth surfaces or curves based on given primary objects. This process is called as blending on computer graphics [1,2]. Blending can be defined as joining of objects without sharp edges or corners in the intersection regions of the objects [3,4]. Another definition can be given as follows; intersecting or non-intersecting objects can be joined with curves and surfaces having smooth curves and surfaces [8]. The general blend surfaces has been worked by Rockwood [9], constant-radius blending techniques were studied by Rossignac and Requicha [6], Choi and Ju[3], cyclide blends were studied by Schene [10], Dutta [7].

The first literature on fuzzy logic can be found in Zadeh [12]. Fuzzy logic was used in different areas such as sound detection, pattern recognition, control applications, image processing,..., etc. The works about computer graphics by using fuzzy logic can be found in [5,11].

The structure of the paper is as follows; Section two is concerned about a method, which are developed to create curve in 2-D and fuzzy logic technique. In section three, how to make blending process is mentioned and some sample blended objects are given. Lastly, conclusion are offered.

2 Fuzzy Curve Modelling Method

The control points have to be known, while the approximation and interpolation curves are created by means of control points. In this study, when start and end points are known, the curve is produced by means of other points, which are calculated by these points. The basis of study is to compute the interval points by fuzzy logic technique, where start and end points are known. While that computation was done, at least three points except start and end points are used to create curve. The reason is that, local changes on the created curve must be able to satisfy at least three points. Figure 1 depicts this effect. Let $A(u,v)$ and $B(u,v)$ be start and end points given in 2-D. Infinite curves can be created

by means of these two points. If a controlled curve is wanted to be created, then the location of auxiliary points (second points) is important.

Figure 1. $A(u, v), B(u, v)$ are two points and infinite curves can be produced by means of these points.

So, some parameters will be needed to create a controlled curve as well. Therefore, the shape of curve can be satisfied. It is required to use different control points to create curve 1 on Figure 1. Similarly, curve 2 and 3 require different control points. To compute location of these points by fuzzy logic, we use points called as auxiliary points. The reason of using auxiliary points is that, the control mechanism, that will satisfy local changing on the curve, is to obtain. These points will determine shape of curve and set the basis of blending.

2.1 Developed Method

Let $A(u, v)$ and $B(u, v)$ be two points in 2-D. Infinite curves can pass from these two points. Let us define two other points to create curve that will satisfy property to be requested and passing from these points. Let $A'(u, v)$ and $B'(u, v)$ be two auxiliary points. Auxiliary points are points at distance δ of original $A(u, v)$ and $B(u, v)$ points. Let δ be very small such as $\delta \ll 1$. Let α and β be two angles. (Figure 2.)

Figure 2. Case of $A(u, v), B(u, v), A'(u, v)$, and $B'(u, v)$ points

There may be different cases on location of points. Because angles can change interval $[0^0, 360^0]$. Figure 3 depicts created curves for different α and β values.

Figure 3. Creating curves with respect to different α and β values.

Having α and β determined, it is not enough to take only these angles to be able to use fuzzy logic technique. The reason is that, if the distance among points is not determined and only angle is evaluated then, undesired cases may occur. So, the curves can not be produced. This case was graphically depicted in figure 3.

Figure 4. The angles α and β are same, but the produced curves are different.

If noticed, although angles are the same, the curves may have different shape. Because curves require different control points. For that reason, when fuzzy logic technique is applied to, angles α , β and distance among points are used as input parameters. These distances are d_1 and d_2 . Figure 5 depicts d_1 and d_2 . It is easy to compute distance d_1 and d_2 in analytical.

Figure 5. Distance d_1 and d_2 .

Figure 6. Cases of m_1, m_2, m_3, t_1, t_2 and t_3 .

If it is noticed to figure 6, the points $M_1(u, v)$, $M_2(u, v)$ and $M_3(u, v)$ are points to be computed. While locations of these points are computed, lines n_1, n_2 and n_3 are used. These lines are located as perpendicular to the line connecting the points $A(u, v)$ and $B(u, v)$. The distances m_1, m_2, m_3 and n_1, n_2 and t_1, t_2 and t_3 are important. Simply, distances m_1, m_2, m_3 and n_1, n_2, n_3 are computed by means of fuzzy logic technique. If these distances and points C_1, C_2, C_3 are determined then, to compute exact points $M_1(u, v)$, $M_2(u, v)$ and $M_3(u, v)$ can be computed in fuzzy inference machine. Center of sums is used as inference machine. (Equation 1).

$\mu(u_i)$: membership functions.

u_i : The value of membership functions.

$$u^* = \frac{\sum_{i=1}^l u_i \cdot \sum_{k=1}^n \mu(u_i)}{\sum_{i=1}^l \sum_{k=1}^n \mu(u_i)} \quad (1)$$

The fourth degree approximation curve can be created by equation (2). The $M_1(u, v)$, $M_2(u, v)$ and $M_3(u, v)$ points are intermediate points computing by the means of developed method.

$$P(u, v) = (1-u)^4 A(u, v) + 4(1-u)^3 M_1(u, v) + 6(1-u)^2 u^2 M_2(u, v) + 4u^3 (1-u) M_3(u, v) + u^4 B(u, v) \quad (2)$$

Figure 7 shows curves produced by equation (2).

Figure 7. Curves produced by equation (2)

2.2 Rules for Fuzzy Logic and Construction of Membership Functions.

In this section, membership functions selected by the means of parameters determined in section 2.1 and produced rule sets by these functions are presented. It is known that, the parameters that will be evaluated for fuzzy logic technique are angle α , β and distances d_1, d_2 . However, there is a

problem to select d_1 and d_2 . Because distances d_1 and d_2 can change in a very large interval. Instead of d_1 and d_2 , we use division of d_1 into d_2 . Let p be division of d_1 into d_2 .

$$\begin{aligned} p &= \frac{d_1}{d_2} & d_1 > d_2 \\ p &= \frac{d_2}{d_1} & d_1 < d_2 \end{aligned} \tag{3}$$

So, both mathematical operations and the one of input parameters of fuzzy logic are decreased. Noticed that, there is no case that value p is very big. Because distance among $A(u,v)$ and $A'(u,v)$ is very small and distance among $B(u,v)$ and $B'(u,v)$ is very small, too. For that reason, distance d_1 is almost equal to distance d_2 , and so, it will be selected in interval $[0-1.25]$ of p . Figure 7 shows membership functions used as input parameters and figure 8 shows membership functions selected as output parameters.

Figure 8. Membership functions used as input parameters.

Figure 9. Membership functions used as output parameters

Table 1. Fuzzy control rules

Inputs			Outputs					
α	β	p	m_1	m_2	m_3	t_1	t_2	t_3
1	1	1	1	2	1	1	2	1
1	1	2	1	1	2	1	2	2
1	2	1	2	3	2	1	2	2
1	2	2	3	1	2	2	1	1
1	3	1	1	2	2	1	3	2
1	3	2	2	3	3	2	3	1
2	1	1	3	2	2	2	1	2
2	1	2	1	1	2	2	1	2
2	2	1	2	3	1	1	1	3
2	2	2	1	2	2	2	2	2
2	3	1	2	2	1	1	2	3
2	3	2	2	1	2	3	1	2
3	1	1	3	2	3	3	1	2
3	1	2	2	1	3	2	3	1
3	2	1	1	2	2	1	2	2
3	2	2	1	2	2	1	3	2
3	3	1	3	1	1	2	1	1
3	3	2	1	3	1	1	2	1

Note 1. For α and β ; 1, 2 and 3 values show little, medium and large, respectively.

2. For p ; the 1, 2 values show little and large, respectively.

3. For m_1, m_2, m_3 and t_1, t_2, t_3 ; the 1, 2, 3 values show little, medium and large, respectively.

3 Object Blending in 2-D

Applying the method presented in section 2 to blending process require to know exactly some parameters and points $A(u, v)$, $A'(u, v)$ and $B(u, v)$, $B'(u, v)$. The method can apply to blending process, after these points and parameters are known.

Let $f(u, v)$ and $g(u, v)$ be two surfaces in 2-D and let $f'(u, v)$ and $g'(u, v)$ be their offset surfaces. It is known that an offset surface is a translated surface at distance d to the surface normal direction (Figure 10).

Figure 10. Original surfaces; $f(u, v)$ and $g(u, v)$, their offset surfaces; $f'(u, v)$ and $g'(u, v)$ and surface normals $\vec{n}_1$ and $\vec{n}_2$.

According to developed method, it is not important whether surfaces intersect or not.

$$f(u, v) - g'(u, v) = 0 \quad (4)$$

$$f'(u, v) - g(u, v) = 0 \quad (5)$$

If equation (4) and (5) is solved by means of Newton-Raphson method, then intersection curves, namely points $A(u, v)$ and $B(u, v)$, can be found. By the help of these points, $A'(u, v)$ and $B'(u, v)$ can be computed. For this purpose, offset surfaces are translated to distance δ . Hence, the points $A'(u, v)$ and $B'(u, v)$ are obtained. However, δ must be very little. Because δ gives only the information on shape of original surface $f(u, v)$ and $g(u, v)$.

δ should be very little value. Because δ must be able to give an information about shape of original surfaces $f(u, v)$ and $g(u, v)$. If it is selected big value, then it won't give an information shape of original surfaces. Therefore, while blending process is done, blending process having smooth curves and surfaces can not realized. For that reason, δ must be little enough.

In this study, δ is selected in range $0.0001 < \delta < 0.1$, especially for $\delta = 0.02$. Figure 11 shows the computation steps.

Figure 11. Computing of points $A(u, v)$, $B(u, v)$ and $A'(u, v)$, $B'(u, v)$.

After the points $A(u, v)$, $B(u, v)$ and $A'(u, v)$, $B'(u, v)$ are computed, three points should be computed. At the beginning, $f(u, v)$ and $g(u, v)$ are translated toward to each for other by their surface normal. This process is maintained by arising intersection. Intersection curves namely $A'(u, v)$ and $B'(u, v)$ are computed by means of equations 4 and 5. It can be solved by the help of Newton—Raphson method.

Figure 12 depicts some blended objects and their blending points. The case before blending process is on the left side, and the case after blending process is on the right side.

Figure 12. Some blended objects produced by means of developed method and their blending points.

4 Conclusion

In this paper, a new technique to create an approximation curve has been presented. The basic of the developed method is to create an approximation curve by the help of fuzzy logic technique. Produced curves have been used for blending in 2-D. As a future work, the method will be tried to be applied on blending in 3-D.

5 References

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