Identification of MIMO Systems by Input-Output Takagi-Sugeno Fuzzy Models

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Abstract: The extraction of fuzzy information out of raw data is very important and contains saving potential for science and engineering applications. The fuzzy models from this information can be used for different purposes in various applications because of their transparency. A number of techniques have been introduced to construct fuzzy models from measured data. Relatively little attention has been devoted to the identification of Multi Input Multi Output (MIMO) fuzzy models from input-output data. This paper concentrates on the fuzzy modeling and identification of MIMO systems by TS (Takagi-Sugeno) fuzzy models. In this paper product-space fuzzy clustering with adaptive distance measure named as Gustafson-Kessel (GK) algorithm is used. With this approach, the knowledge of the physical structure can be easily incorporated in the structure of the model. Implementation in the MATALAB is briefly described with simulation examples.

1. Introduction

At the computational level, fuzzy models can be regarded as flexible mathematical structures that can approximate a large class of complex non-linear systems to a desired degree of accuracy. Significant non-linearity and time delays exhibited by the automotive systems are common problems. Recently, a great deal of research activity has focused on the development of methods to build or update fuzzy models. Conventional system theory relies on crisp mathematical models of systems, such as algebraic and differential or difference equations. For some simple systems, mathematical models can be obtained. This is because the physical laws governing the systems are well understood. For a complex, dynamic and non-linear system, however, the gathering of an acceptable degree of knowledge needed for physical modeling is a difficult, time-consuming and expensive or even impossible task. [1].

Model based predictive control from input output data using TS fuzzy model is discussed in [3]. The modeling of a gas turbine aero engine based on multiobjective genetic algorithm is described in [3]. A progressive fuzzy modeling for control of an automotive engine air inlet is proposed in [4]. Feed forward scheme of a dynamic model with look up table of air flow rate mapped against engine crankshaft speed and throttle plate angle is used to predict the air flow into the engine with fuzzy c-mean clustering [5]. The c-mean clustering for fuzzy identification based engine fault

detection and isolation is also discussed in [9]. So far most attention has been devoted to Single Input Single Output (SISO) or Multi Input Single Output (MISO) systems.

Relatively little attention has been devoted to the identification of MIMO fuzzy models from input-output data. The problem of developing a general methodology for intelligent system design has always been demanding. Fuzzy modeling is a framework in which different modeling and identification methods are combined, providing, on the one hand, a transparent interface with the designer or the operator and, on the other hand, a flexible tool for non-linear system modeling and control, comparable with other non-linear techniques. The rule-based character of fuzzy models allows for a model interpretation in a way that is similar to the one humans use. Such a compound system requires the identification of the local models from input-output data.

This work addresses a method for the fuzzy modeling and identification, which identifies the fuzzy rules describing the system behaviour from input-output data using TS models. In this paper product-space fuzzy clustering with adaptive distance measure i.e., GK algorithm is used. It gives a classical fuzzy model with local linear models in the consequent of each rule. The contribution of this paper is three-fold: First, it is shown that the fuzzy modeling & identification technique based on product-space fuzzy clustering can be extended to uncertain non-linear dynamical systems in a straight forwarded way. Second, a MATLAB implementation is presented which implements the presented techniques. Third, illustrative simulation examples of non-linear dynamic systems are given to prove that the approach is very simple and intuitively appealing.

2. TS Fuzzy Model Structure

TS fuzzy models are suitable to model a large class of non-linear systems [1]. Consider a MIMO system with n_i inputs: $u \in U \subset R^{n_i}$, and n_o outputs: $y \in Y \subset R^{n_o}$. This system will be approximated by a collection of coupled MISO discrete time fuzzy models. Denote q^{-1} the backward shift

operator: $q^{-1}y(k) \stackrel{\text{def}}{=} y(k-1)$ where y is a signal sampled in discrete time instants k . Denote by the capital letters A and B polynomials in q , e.g., $A = a_0 + a_1q + a_2q^2 + \Lambda$.. Given two integers, $m \leq n$, define an ordered sequence of

delayed samples of the signal y as: $\{y(k)\}_m^n \stackrel{\text{def}}{=} [y(k-m), y(k-m-1), \Lambda, y(k-n)]$. The MISO models are of the input-output NARX (Non linear Auto Regressive with Exogenous input) type:

$$y_l(k+1) = F_l(x_l(k)), \quad l = 1, 2, \Lambda, n_o, \quad (1)$$

where the regression vector $x_l(k)$ is given by:

$$x_l(k) = [\{y_1(k)\}_0^{n_{yl1}}, \{y_2(k)\}_0^{n_{yl2}}, \Lambda, \{y_{n_o}(k)\}_0^{n_{yl n_o}}, \\ \{u_1(k)\}_{n_{dl1}}^{n_{ul1}}, \{u_2(k)\}_{n_{dl2}}^{n_{ul2}}, \Lambda, \{u_{n_i}(k)\}_{n_{din_i}}^{n_{uin_i}}].$$

Here n_y and n_u are the number of delayed outputs and inputs, respectively, and n_d is the number of pure transport delays from the input to the output. n_y is a $n_o \times n_o$ matrix, and n_u, n_d are $n_o \times n_u$ matrices. F_l are rule based fuzzy models of the TS type:

$$R_{li} = \text{If } x_l(k) \text{ is } S_{li} \text{ then } y_{li}(k+1) = A_{li}y(k) + B_{li}u(k) + C_{li}, \quad i = 1, 2, \Lambda, K_l \quad (2)$$

Here S_{li} is the antecedent fuzzy set of the i^{th} rule, and $A_{li} = [A_{li1}, \Lambda, A_{lin_o}]$, $B_{li} = [B_{li1}, \Lambda, B_{lin_i}]$, are vectors of polynomials. K_l is the number of rules in the l^{th} model. The fuzzy sets S can be defined by multivariate membership functions $w(x(k)): R^{p_l} \rightarrow [0, 1]$, where $p_l = \sum_{j=1}^{n_o} n_{ylj} + \sum_{j=1}^{n_i} n_{ulj} + 1$ is the dimension of the antecedent space. Alternatively, the antecedent of (2) can be given in the conjunctive form:

$$R_{li} = \text{If } x_{l1}(k) \text{ is } S_{li1} \text{ and } \Lambda \text{ and } x_{lp}(k) \text{ is } S_{lip} \\ \text{then } y_{li}(k+1) = A_{li}y(k) + B_{li}u(k) + C_{li}, i = 1, 2, \Lambda, K_l. \quad (3)$$

Both these forms can be constructed from data, as shown in next section. The output of the TS model is computed by the weighted mean:

$$y_l(k+1) = \frac{\sum_{i=1}^{K_l} w_{li}(x_l(k)) y_{li}(k+1)}{\sum_{i=1}^{K_l} w_{li}(x_l(k))}. \quad (4)$$

3. Identification Method

The structure of the model, i.e., the matrices n_y , n_u and n_d are determined by the user on the basis of prior knowledge and/or by comparing several candidate structures in terms of the prediction error or other suitable criteria. Once the structure of the model is fixed, the parameters of the SISO, MISO and SIMO models (2) are estimated. These parameters are the antecedent membership functions and the consequent polynomials. An additional structural parameter is the number of rules, which can be specified or sought by the user. The identification GK algorithm proceeds in four steps as under, the theoretical details can be seen in [1-2]:

- Form the input-output sequences, $\{(u(k), y(k))\}_{k=1}^{N_d}$, form the non-linear regression problem of (1), using the user specified structural parameters n_y , n_u and n_d .
- Partition the data into the set of local linear sub models by using fuzzy clustering in the Cartesian product space $X \times Y$.
- Compute the antecedent membership function from the cluster parameters.
- Given the antecedent membership functions, estimate the consequent parameters by the least squares method.

4. MATLAB Implementation

The identification algorithm presented in the previous section was implemented in MATLAB. The core of the identification routine is the GK clustering algorithm. The function in the algorithm to construct a SISO, MISO, SIMO and MIMO fuzzy model from data is named *fmclust* and it has the following synopsis:

$$FM = fmclust(U, Y, c, FMtype, m, tol, seed, ny, nu, nd)$$

The data sequences are given in columns of the matrices U and Y , respectively. The number of required clusters, c is a scalar for MISO systems and a vector for MIMO systems (each MISO model may have a different number of clusters. The remaining parameters are optional. *FMtype* specifies whether the antecedent membership functions of the resulting model are computed analytically in the antecedent product space or are derived by projection. Product-space membership functions give faster but often less accurate models. In the algorithm, the parameter m is the fuzziness exponent. When $m \neq 1$, with the default value $m = 2$. Larger values imply fuzzier (more overlapping) clusters. Currently, there is no theoretical basis for an optimal choice for the value of m . However, it is established that the algorithm converges for any $m \in (1, \infty)$. The termination tolerance for the clustering algorithm can be given in *tol* (default *tol* = 0.01). The fuzzy partition matrix is initialised at random. In order to obtain reproducible results, the random generators may be seeded by supplying the seed parameter. Default value is `sum(100 * clock)`. The *ny*, *nu* and *nd* parameters are the delay matrices defined in section (2).

The output of *fmclust* is the *FM* model matrix, which contains the parameters of the obtained fuzzy model (under MATLAB, *FM* is defined as a structure). The obtained fuzzy model can be validated by using function *fmsim* with the following synopsis:

$$[Y_m, VAF] = fmsim(U, Y, FM)$$

This function simulates the fuzzy model *FM* from the input data U and plots the simulated output Y_m along with the true output Y . The first u_y values of Y are used

to initialize Y_m . The output argument percentile Variance Accounted For (VAF) between two signals is the performance index of the model is given by:

$$VAF = 100\% \left[1 - \frac{\text{var}(Y - Y_m)}{\text{var}(Y)} \right] \quad (5)$$

The VAF of two equal signals is 100%. If the signals differ, VAF is lower. The VAF index is used to assess the quality of a model, by comparing the true output with the output of the fuzzy model. As additional output arguments, the degrees of fulfilment and the local outputs of the individual rules can be obtained.

The output obtained on the screen for approximation of $y = \sin(x)$ is shown in Figures 1–2 with VAF and three rules are as:

If x is S_1 then $y = 4.3137x + 0.1329$,

If x is S_2 then $y = -5.0666x + 2.2756$

If x is S_3 then $y = 5.4982x - 4.9211$

5 Simulation Examples

5.1 Identification without data clustering

In the this example, fuzzy modelling & identification of a simple non-linear dynamic system without data clustering is described by a first-order difference equation:

$$y(k+1) = y(k) + u(k)e^{-3[y(k)]} \quad (6)$$

We use a stepwise input signal to generate with this equation a set of 300 input–output data pairs. The complexity of the system behaviour is typically not uniform. Some operating regions can be well approximated by a model, while other regions require fine partitioning. Comparatively fuzzy data clustering can capture the non-uniform behaviour of the system to obtain an efficient representation of identification, which follows in the next section. The identified model is shown in Fig 3.

5.2 Identification of internal combustion engine models

In the this example, fuzzy modelling & identification of the simple SISO, MISO and SIMO models are implemented with the above said algorithm by the TS fuzzy model with three rules. This example presents a model of a four-cylinder spark ignition automotive IC engine air inlet system, which demonstrates identification capabilities to model an IC engine. In the throttle body, the control input is the angle of the throttle plate [11]. The rate at which the model introduces air into the intake manifold can be expressed as the product of two functions: one, an empirical function of the throttle plate angle only; and the other, a function of the atmospheric and manifold

pressures. In cases of lower manifold pressure (greater vacuum), the flow rate through the throttle body is sonic and is only a function of the throttle angle.

$$\dot{m}_{at} = f(\alpha)g(p_m) \quad (7)$$

$$f(\alpha) = 0.5642 - 0.05231\alpha + 0.10299\alpha^2 - 0.00063\alpha^3$$

where $\dot{m}_{at}$ = Mass flow rate into manifold (g/s), α = Throttle angle (deg),
 p_m = Manifold pressure (bar)

$$g(p_m) = \begin{cases} 1, & p_m \leq \frac{p_{amb}}{2} \\ \frac{2}{p_{amb}} \sqrt{p_m p_{amb} - p_m^2}, & \frac{p_{amb}}{2} \leq p_m \leq p_{amb} \\ -\frac{2}{p_m} \sqrt{p_m p_{amb} - p_{amb}^2}, & p_{amb} \leq p_m \leq 2p_{amb} \\ -1, & p_m \geq 2p_{amb} \end{cases}$$

p_{amb} = Ambient atmospheric pressure (bar). The difference in the incoming and outgoing mass flow rates represents the net rate of change of air mass with respect to time. This quantity, according to the ideal gas law, is proportional to the time derivative of the manifold pressure.

$$\dot{p}_m = \frac{RT}{V_m} (\dot{m}_{at} - \dot{m}_{ao}) \quad (8)$$

where R = Specific gas constant, T = Temperature, V_m = Manifold volume m^3

$\dot{m}_{ao}$ = Mass flow rate of air out of the manifold (g/s), $\dot{p}_m$ = Rate of change of manifold pressure (bar/s). The mass flow rate of air that the model pumps into the cylinders from the manifold is described in (9) by an empirically derived equation. This mass rate is a function of the manifold pressure and the engine speed [12].

$$\dot{m}_{ao} = -0.366 + 0.08979np_m - 0.0337np_m^2 + 0.0001n^2p_m \quad (9)$$

where n = engine speed (rad/s). In this example, we develop simple dynamic models for the relationship between the throttle angle, speed of an engine and manifold pressure for automotive IC engine air inlet. We will obtain a fuzzy model of the following structure: $y(k+1) = f(y(k), u(k))$, where $y(k)$ and $u(k)$ are the state and the input at time k , respectively, and f is a static function, called the state-transition function.

For SISO model the input is the throttle plate angle $u = [\alpha]^T$ and the output is the engine crankshaft speed $y = [n]^T$. For MISO model the inputs are $u = [\alpha, n]^T$ and

the output is manifold pressure $y = [p_m]^T$. For SIMO model input is $u = [\alpha]^T$ and outputs are $y = [n, p_m]^T$. Now we load the data set, split it in halves and use the first half for identification and the second half for validation. The objective is to control the air inlet subsystem of an engine by implementing the above exposed algorithm. In this automotive application, fuzzy modelling & identification of the throttle angle and the speed of an engine functions is implemented with the above said algorithm by the TS fuzzy model with three rules. The output obtained on the screen is shown in Figs 4 –5. The output obtained for MISO and SIMO model are shown in Figs 6 –7.

5.3 Identification of Water Tank Application

A tank with a pipe flowing in and a pipe flowing out is shown in Fig 8. The water that flows in, the outflow rate depends on the diameter of the outflow pipe (which is constant) and the pressure in the tank (which varies with the water level). The system has some very non-linear characteristics. A controller for the water level in the tank needs to know the current water level and it needs to be able to set the valve. The input is the water level and its output will be the rate at which the valve is opening or closing. The system is represented by the equation:

$$\dot{h} = 1/S (F_{in} - \beta \sqrt{h}) \quad (10)$$

where S is the horizontal cross sectional area, h is the level of the liquid in the tank, β is a flow coefficient, and F_{in} is the inlet liquid flow rate. The identification is shown in Fig.9-10 and the rules of identified fuzzy model are as:

R_1 : If F_{in} is S_{11} and h is S_{12} then $\dot{h} = 0.8912F_{in} + 0.0949h - 0.0186$

R_2 : If F_{in} is S_{21} and h is S_{22} then $\dot{h} = 0.9285F_{in} + 0.0958h - 0.0310$

R_3 : If F_{in} is S_{31} and h is S_{32} then $\dot{h} = 0.9451F_{in} + 0.0945h - 0.0403$

Fig.1. Static function and its approx. by a TS fuzzy model (VAF=99.67%)

Fig. 2. Obtained local models (top), membership functions (bottom).

Fig 3. System / model (solid/dashed line)

Fig.4. SISO model and its approx.
by fuzzy model (VAF=99.67%)

Fig. 5. Individual local models (top) and membership functions (bottom)

Fig.6. MISO model and its approx. by a fuzzy model (VAF=98.51%)

Fig.7. SIMO model and its approx. by fuzzy model (VAF=99.67% and 57%)

Fig 8. Water tank

Fig. 9. Model and approx fuzzy model (VAF=99.58%)

Fig 10. Obtained individual local model (top) and membership functions (bottom).

6 Conclusion

In this paper, fuzzy modelling & identification is presented for approximation of non-linear SISO, MISO, SIMO & MIMO dynamic system by means of product space fuzzy clustering with adaptive distance measure i.e., GK algorithm. The input-output TS fuzzy models are used, which are suitable to model a large class of non-linear systems. The approach is applied to simple applications like, an automotive IC engine air inlet based on throttle angle, engine speed and manifold pressure and single and cascaded water tank systems. The Identified example illustrates the definition of the various parameters with implementation in MATLAB. Identification with data clustering shows superior performance as compared with identification without data clustering. Because of the significant non-linearity and time delays exhibited non-linear dynamic systems, the control might be improved in the future by employing identified fuzzy models. Due to lack of space, the all output of example could not be presented here, however the simulation examples demonstrate the superior performance.

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